

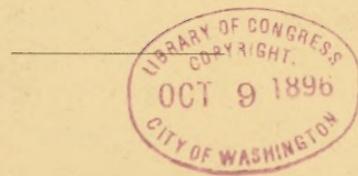
INFALLIBLE LOGIC

A VISIBLE AND AUTOMATIC
SYSTEM OF REASONING

BY

THOMAS D. HAWLEY

OF THE CHICAGO BAR

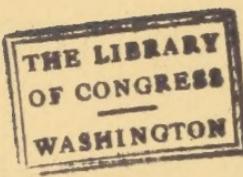


52597-B-1

ROBERT SMITH PRINTING COMPANY
LANSING, MICHIGAN

1896

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P R E F A C E.

This book describes a new system of logic by which reasoning can be carried on by an infallible process. All the implied meanings of sentences and of collections of facts, can be as infallibly and easily determined, by this new method, as the interest on a promissory note can be ascertained by mathematical rules.

The new system is based on axiomatic principles and governed by infallible laws. Its method consists in the repeated use of a few processes which are performed in a mechanical manner, and the results appear automatically.

It is easy to learn, and probably the time is not far distant when it will take the place of the syllogistic and algebraic systems of logic now in current use.

As a means of discovering the truth in regard to any disputed question, whether of words or facts, it acknowledges no equal.

Its tools are a few simple signs, namely, the capital letters of the alphabet to represent positive terms, the small letters to represent negative terms; the mathematical sign of equality, $=$, for "is"; a short perpendicular mark, |, for "or" and a square for the "universe of discourse."

When a square is subdivided into the proper number of sections it is called a Reasoning Frame. By the use of the Reasoning Frame every proposition which can possibly be made with the letters used, is set before us. We then eliminate every proposition which is inconsistent with the given proposition or state of facts.

The uneliminated propositions which automatically remain in the Reasoning Frame will then give us every iota of truth which our data will yield.

It is beyond dispute, that if we make every proposition which it is possible to make with the given terms and then eliminate the inconsistent propositions, the consistent ones must remain.

THOMAS D. HAWLEY.

6107 Madison Ave., Chicago, Ill.

September 1, 1896.

INFALLIBLE LOGIC.

INTRODUCTION.

1. This work is for the use of lawyers, ministers, teachers, students and for everyone who is interested in the art of reasoning, whatever may be the general or special object he has in view.

2. One of my chief objects has been to economize the time of the reader and therefore I have contented myself with the briefest and most diagrammatic account of logical facts and theories.

3. So far as the work contains a description of the Reasoning Frame, its methods, results and the discoveries which it has led me to make, it is original, but with regard to the old logic, I have borrowed liberally from the works of the best writers on that subject.

In such a subject as logic, it is hardly possible to have any ideas, the germs of which are not to be found in the works of preceding writers.

My acknowledgments are due in the first place to Keynes' "Formal Logic." I also owe much to the following works: Venn's "Symbolic Logic;" Jevon's "Lessons in Logic;" Miss Jones' "Elements of Logic;" "Studies in Logic," by members of Johns Hopkins' University; Bain's "Logic" and "Whately's "Elements of Logic."

4. While I have the greatest respect for the authors of these works, still I have thought there was room for another work describing fully a new system of reasoning.

5. In order to avoid elaborate descriptions, I have introduced a large number of diagrams to explain the working of the new system. It is a merit of this system that the eye can see that its results are correct. Thinking is made visible. I have aimed to be plain, concise and intelligible.

6. I have written for the first beginner in logic and for the most advanced logician. The first may think that some parts of the work are too obtruse; the other, that some parts are

too puerile, but I hope that each will find something interesting and profitable.

7. It differs from all other works on logic in this; it explains a graphic and diagrammatic system of logic, which, so far as I know, has never before been exhaustively described.

8. The object of the system is to teach a person how to reason correctly, how to detect fallacies, how to deduce the latent meanings of propositions, how to draw at one operation all the conclusions which necessarily follow from a large number of premises containing a large number of terms, no matter whether the propositions are categorical, disjunctive, hypothetical, affirmative or negative, simple or complex.

9. When propositions which are contradictory to each other are expressed in the Reasoning Frame, it immediately reveals their contradictoriness. It also enables us to perceive intuitively what propositions in the Reasoning Frame are inconsistent with the premises.

10. It makes inductive reasoning as easy and simple as deductive and thus enables us to tell exactly what the facts in any given case establish. It also enables us, given any proposition, to find all the propositions which are equivalent or contradictory to the given proposition.

11. If the reader will turn to the chapters on Examples and on Fallacies, he will get a good idea of the difficult problems which this system of logic is capable of solving.

12. I advise the reader to work the examples given. You may understand the rules laid down but practice is necessary in order to remember them. You should draw every figure and strike out the combinations which are to be eliminated until you can repeat the process without looking at the book. Begin slowly and carefully and in a short time with practice you can eliminate inconsistent combinations at sight and then you will find that your reputation for ability to reason correctly will enable you to rank with the best thinkers of the day.

I shall be happy at any time to answer any questions on the new Logic that may occur to my readers. I do not profess, however, to know much about the old logic.

CHAPTER I.

LOGIC.

13. Logic is the science of Interpretation. It is the art of interpreting, explaining, and expounding that which is not obvious. Its function is to make plain, clear and intelligible the implied meanings of propositions. It unfolds that which is hidden and latent. Logic has been defined as the science of reasoning, but this definition is too broad, because it covers numerical reasoning, and I regard numerical reasoning as a different kind of reasoning. In mathematics, the axioms, the laws, the rules, and the problems are of a different nature from those of logic. Logic is not capable of solving numerical problems. Again the definition is too narrow, because logic is an art as well as a science. Miss Jones in her book, "Elements of Logie," defines Logic as the "Science of Propositions," but I think this definition is indefinite. Keynes, in his "Formal Logic," says that "Formal logic may be defined as the science which investigates those regulative principles of thought that have universal validity whatever may be the particular objects about which we are thinking. It is a science which is concerned with the form as distinguished from the matter of thought," but I think that the matter of thought is an important item in logic. The definition which I like is: Logic is the Science and Art of Interpretation.

14. In reasoning, from the given proposition or propositions, we infer all the other propositions which must be true if the given propositions, which are called premises, are true. In other words we bring to light all the latent meanings of the premises. Sir W. Hamilton says, "Reasoning is the showing explicitly that a proposition not granted or supposed, is implicitly contained in something different, which is granted or supposed." In reasoning logically we show in the conclusion how much has been admitted in the premises.

15. Reasoning will not discover any new facts or enable us to predict a coming event. No new fact can ever be discovered by logic, but we can obtain new meanings of old truths. New truths must come to us either by testimony or by observation through the senses. To get a new view of an old truth, to learn new names for old things, and to ascertain the correct names for ideas which are opposite to or are inconsistent with the ideas contained in the premises is sometimes of as great value as it is to learn a new fact.

16. When a speaker puts forth a proposition, the proposition has a *prima facie* meaning. This is the common meaning that almost everyone who understands the meanings of the words used in the proposition, would put upon it, but most propositions, if they are simple ones, have at least four meanings and if they are complex in their structure, they may have many more. It is a defect in syllogistic logic, or the old logic, as I shall usually call it, that it only attempts to give one new meaning out of the numerous ones which two premises furnish. A system which only gives a part of the truth when it ought to give the whole truth, is not much better than no system.

17. In the system which I am about to describe we are enabled to draw with one operation every possible meaning which is consistent with the premises, no matter how complex the premises are, or how many there are of them. Of course, it will be understood by the reader, that before the mind can reason, it must know the meaning of the various words used in framing the premises. I do not doubt that animals can reason without language, but it would be almost impossible for human beings to reason with each other without employing language. Logic is concerned in the first place with language, and in the second place with the ideas which the terms used stand for.

18. Intellectual thought is usually divided into sensations, perceptions, ideas, conceptions and notions, but I shall use the term ideas as a general name to stand for all these different kinds of thought. Ideas also represent things. Things themselves are not in thought, only the ideas which represent the

things perceived by the mind are in thought. An idea may be simple, complex, or compound, but whichever it may be, it is always a whole, a unit.

19. The fundamental theory of this new system of logic is that every idea has its opposite, and that the two are inseparably joined together in thought. It is impossible to separate them, they have one common life. If you destroy one, you necessarily destroy the other, if you posit one, you necessarily posit the other.

20. We may describe the idea, and its opposite as positive and negative. These are probably the best terms we have. I do not like the term negative in this connection, because it usually implies either negation or the absence of some quality, but the truth is, that a negative idea is just as positive really as a positive idea. The terms positive and negative are purely relative to each other. We might call the positive, negative, and the negative, positive, and it would make no difference in the results. The fundamental thought is, that they are simply opposites.

21. When a term which is used as the name of an idea has the prefix "not" attached to it, we will call that a negative term, and the idea which it represents, a negative idea. When the term used is without the prefix "not," we will call that a positive term and its idea a positive idea. The reasoning process, in order to be correct, must deal with a positive and its negative, or with a negative and its positive at the same time. Unless this is done, the results of the reasoning process will be incorrect. The whole art of the reasoning process proper consists in positing a negative term with every positive, and a positive with every negative, and of then making every possible combination, or proposition, which can be made out of these positive and negative terms, and of then selecting the combinations, or propositions, which are consistent with the premises, by eliminating those which are inconsistent. This must necessarily and infallibly give us every possible proposition which is consistent with the premises.

22. It is axiomatic, that if we have two kinds of propositions, namely, those which are consistent with the premises, and those which are inconsistent, and we remove one kind, the other kind must remain. If I hold a number of white balls and a number of black balls in my hand, and take away the black balls, the white balls must remain. This process of combining the expressed and implied terms of a proposition, and of eliminating the inconsistent propositions thus obtained, is the only way by which the mind can learn all the true meanings which are hidden, or latent, in any proposition.

23. The old logic, which is based upon the mutual relations of two classes, can never yield satisfactory results, because it is based upon a wrong theory. According to the old logic, we must either reason from the whole to the parts, or from the parts to the whole. The first was called deductive reasoning and the second inductive. This is an exceedingly narrow and imperfect view of the reasoning powers of the mind, as we shall show further on.

24. A little reflection will convince the reader that the results which can be obtained by our combining and eliminating system must be as accurate as those which can be obtained in the solution of arithmetical problems by the processes of addition, subtraction, multiplication and division. There is a pronounced analogy between our logical method and the arithmetical method. With both systems it is necessary to have correct data. The computation may be correct, but if the items of an account are wrongly stated, a correct computation alone will not give correct results. Likewise in logic, our premises must be true, or our conclusions will be false. In arithmetic the data must be definite or the problem cannot be solved. If I were to ask a scholar what some apples, at some cents apiece, would cost, he could not tell me. And similarly in logic, if our premises are indefinite, we can arrive at no definite conclusion.

25. The great superiority which mathematical reasoning has hitherto enjoyed over logical reasoning is principally due to the fact that mathematical problems are usually stated in definite and unambiguous terms. Archbishop Whately quaint-

ly says: "It is a wise remark of Dr. Barrow, that 'Confusion is the mother of iniquity.' "

26. Logic is especially useful in solving the problems of the so called theoretical sciences, such as Jurisprudence, Law, Political Economy, Politics, Metaphysics, Philosophy and Theology. In the past it has suffered considerably at the hands of its over-zealous friends who have made the most extravagant claims for its usefulness in every department of knowledge.

27. It is worth while to remark here that logic has little to do with Grammar and Grammar has still less to do with logic. In pursuing its own methods, logic must be independent of Grammar, and if it is necessary for us to violate the rules of Grammar, in order to obtain logical precision, we should do so without any hesitancy. In order to reason logically, we must use logical terms, logical signs, and logical laws. Grammar must take care of itself, and it must allow logic to do the same for itself.

CHAPTER II.

DEFINITIONS.

28. Logic being a technical science, the terms used are not always employed in their common acceptation, and it is necessary, therefore, in order to guard against mistaken meanings, that we should define the terms we employ. And when we have once put a definition upon a term we should not change it, but adhere strictly to the meaning given. When no definition is given for a word it will be understood that the word is used in its popular and customary sense. When we define a word we usually state in a single word, or a phrase, the particular circumstance of resemblance or of difference which the word has to other objects.

29. Definitions are divided into nominal and real. A nominal definition is used to explain the meaning of the word used. For instance, the meaning of the word decalogue would be, the ten commandments. A real definition is used to explain the nature of the object, as for instance, silver can be defined as a white, heavy, fusible metal, used largely to make coined money. In the exact sciences, where words are used strictly, the nominal and real definitions are usually the same. For instance, the definition of a circle or of a triangle would include the qualities which are implied by the name.

30. Accidental definitions enumerate properties which can be considered as belonging accidentally, but not necessarily, to the word, as for instance, George Washington was born in Virginia, was the first President and was called the Father of his country.

31. Definitions should neither be too broad nor too narrow. They should be full enough to convey a clear meaning of the word. And, again, a definition should be plainer and clearer than the word which is defined, otherwise the definition would be useless. It should be brief, and yet not so brief as to be

obscure, nor should it be so long as to become prolix. Tautological definitions are not elegant so far as their phraseology is concerned, but they are most accurate in their nature, and when one wishes to be exceedingly precise in his definitions a tautological definition will be the most exact one that he can use.

32. Logical definition grows out of logical division. In a logical division we divide the genus into its different species and the species into the different individuals; thus animal would be a genus; horses, dogs, etc., would be species, and then we could divide the species into the parts or the individuals which compose the species.

33. Physical division differs from logical division in this: That physical division separates an object into different parts. Thus, a man could be divided into head, body, limbs, organs, etc. In logical division we should divide the whole so that each part would be less than the thing divided, and all the parts into which the thing is divided should exactly equal the whole, and none of the members should be contained in one another. In other words, we should not be guilty of the fault which is called cross-division. For instance, if we divided books into English, French, quarto, octavo, etc., we should violate the rules of logical division.

34. Classification differs from division in this respect: That when we want to classify objects we separate them according to some difference, and we continue the separating process until a difference is no longer wanted, or until it cannot be found. Cuvier's system of classification was as follows: Individual, species, kind, family, order, class.

35. I like Hobbe's definition of the word name. He said, "A name is a word or set of words serving as a mark to raise in our minds a given idea, and also to indicate to others what idea is before the mind of the speaker." In this work I shall use the word name in a general sense to cover appellations, patronymics, titles, designations and descriptions. So that a name may be a single word or it may include a number of words. For instance, when I say "Grover Cleveland is the

President of the United States," the President of the United States is another name for the same person. "The British Museum is the largest collection of books in the world in one building." In this sentence, the largest collection of books in the world in one building is a name for the same thing that the British Museum is a name for.

36. Adjectives are names, thus: "George is wise;" wise is a name for George. Adjectival names are almost always indefinite, but a sentence which is indefinite, because the second member of it is an adjective, can be made definite by repeating the subject of the sentence after the adjective. Thus the indefinite sentence, "this man is wise" becomes definite in "this man is this wise man." This sentence read backward, "this wise man is this man," is equally definite.

37. A definite sentence will have the same meaning when read backward that it has when read forward. In logic, which should be an exact science, indefinite propositions should always be converted into definite ones, by repeating the subject in the predicate, or conversely before we proceed to reason from them.

38. A name, it seems to me, always indicates the existence of the idea or thing spoken of. Existence does not necessarily mean material existence. It may mean ideal or mythical existence, but it must mean some kind of existence.

39. There is a difference between singular and general names. Singular names apply to individuals; New York, the Alleghanies, the British Museum, designate each a particular thing, but general names apply to objects which belong to a class; man, animal, book, house, are examples of general names. General names are said to be connotative because they imply that the members of the class have certain attributes in common.

40. A general name, like an adjectival name, is always indefinite, and when the predicate of a sentence is a general name, before it can be treated logically it must be made definite; thus "man is an animal" is an indefinite sentence, because when it is read backward, "an animal is a man," we know as a

matter of fact that it is not true. But we can convert it into a definite sentence by making it tautological; thus, "man is an animal man." This is a true sentence when read forward or backward.

41. One of the mistakes of the old logic is that it attempts to reason with general terms without rendering them definite. According to the old logic "men are mortal beings" would be a good logical proposition, but a little examination will show us that it is not a definite proposition, because if it were definite it would be just as true a proposition, when read backward, as when read forward. But when we read it backward, "mortal beings are men," we know, as a matter of fact, that many mortal beings, such as horses, dogs, etc., are not men. But the sentence, "men are mortal beings," can be made into a definite and true sentence; thus, "men are mortal men." It is now in the proper shape for an exact logic to deal with. One of the principal safeguards against drawing false conclusions is the conversion of indefinite propositions into definite ones by repeating the subject in the predicate or conversely. According to Sir William Hamilton, a general name is a concept, but I prefer to use the term concept to represent abstract terms, such as justice, whiteness, etc.

42. There is a class of singular names called individual names. Individual names are non-connotative; they serve to mark a particular person or thing without implying any qualities. Thomas, George, John, etc., are examples.

43. A name is called a term when it is used as the subject or predicate of a sentence. A term is also called a predicate. There are five kinds of predicates in the old logic, viz: Genus, species, difference, property, accident. In the sentence, "man is an animal," man is the species and animal is the genus. In "man is a rational animal," rational is the difference. In "this man is tall and ignorant," tall and ignorant are accidents. In "the magnet has polarity," polarity is a property. The same word may be used to exhibit each of the five classes of predicates; thus, red in relation to a gown is an accident, to blood

it is a property, to a rose it is a difference, to pink it is a genus, to color it is a species.

44. Another class of names is called relatives. A relative name always implies a correlative. Parent and child, husband and wife, father and mother are examples. Positive and negative also express relativity. We might call parent positive and child negative, or vice versa. Logically, it would make no difference. We could let A stand for either one and not-A stand for the other. It must always be borne in mind that in logic the term "negative" does not necessarily mean negation or the absence of a quality. It does, however, imply a difference. When I say, "A horse is not a giraffe," I mean, that a horse is different from a giraffe. "Not a giraffe" is a name or description applied to a horse. If I should say "a giraffe is not a horse," then, "not a horse" would be a name applied to a giraffe.

45. Negative names, if indefinite, when used as predicates, must be converted, in the manner heretofore pointed out, into definite names before they are proper subjects for logical treatment. Some logicians call negative terms infinite terms.

46. A word which can be used by itself for a name is called a categorematic word and a word which must be used in connection with others to make a name is called a syncategorematic word, e. g., prepositions.

47. A name which can be applied to a single whole that is made up of a number of individuals is called a collective name. Army, congress, supreme court, are examples.

48. When the mind perceives an abstract idea, it is said to apprehend it. This act of the mind is very similar to the act of the mind when it perceives a material object. The old logic says that judgment is the act of comparing the perceptions or apprehensions of the mind and of deciding whether they agree or disagree in their qualities. I think this is a mistaken idea. I believe that a judgment is the act of the mind in giving a name to one of its ideas in addition to the name by which the idea is commonly known. But before we can determine what

a judgment is we must define the word "is." When I say "this thing is salt," I mean "this thing has the name of salt;" when I say, "this thing is chloride of sodium," I mean that "this thing has the name of chloride of sodium." So we see that "is" logically means "has the name of." Now when I say that, "salt is chloride of sodium," I express a judgment of the mind. The sentence, "salt is chloride of sodium," means that the same identical thing, has two different names, viz., salt and chloride of sodium. So that a judgment is the giving of two names to one thing.

49. A judgment expressed in words becomes a proposition, and a series of propositions becomes a discourse. A discourse can be analyzed into propositions, and propositions can be analyzed into names, so that in a sense the science of logic is the science of names. The complete interpretation of all the implied meanings of names used in propositions is the art of logical reasoning.

50. A term is a word or any combination of words used as a name for a subject or a predicate of a sentence. A single word used as a name is called a simple term.

51. Where two or more names are joined together to make a subject or a predicate we have a complex term. In the sentence, A is B or C, B or C is a complex term. This kind of a proposition is called a disjunctive proposition. The elements of a disjunctive term are called alternates.

52. Positive and negative terms, such as A and not-A are called opposites in our system, because the same thing cannot have the name of A and the name of not-A at the same time.

53. Terms which are incompatible with each other are called contraries; thus, white and black are contraries, but white and not-white are opposites. Another name for opposites is inconsistents. This means that they cannot stand together, that is, that they cannot both be applied to the same thing.

54. The old logic divides propositions into universal and particular. When the predication is made of the whole of a

subject it is universal, and when it is made of a part of a subject it is particular. This is called the doctrine of quantity. This division, it seems to me, is not a sound one. The subject of a proposition must be an idea, and that idea must be a whole, a unit. If we subdivide an idea into its parts then each part becomes a whole or unit idea. Whenever the mind takes an idea for a subject it must take it as a whole. There is no such thing as taking a part of an idea, unless that part is taken as a whole. It makes no difference how far we may carry the process of dividing and subdividing an idea, whatever part we may take we must speak of the whole of that part. The idea that the mind can apply a predicate to a part of an idea is one of the strangest misconceptions of the old logic and one which is fatal to any correct logical system.

55. The old logic also divides propositions into affirmative and negative. Affirmative means that the predicate is affirmed of the subject, and negative means in the old logic that the predicate is denied of the subject. This is called the doctrine of quality. It seems to me this is another error. The copula of a proposition is, always, is; the negative particle "not" does not belong to the copula, but it belongs to the predicate; it is a part of the name of the predicate. A proposition, no matter whether it contains the word "not" or does not contain it, is always affirmative; it affirms that the subject and predicate are two names for one thing. Now names may be affirmative or they may be negative, but the proposition is always affirmative.

56. The doctrine of quantity and the doctrine of quality has given rise in the old logic to four forms of propositions. The first form is called the universal affirmative, and its formula is all S is P. In the formula S stands for subject and P stands for predicate. It means all S is some P. In this form it is an indefinite proposition, and before using it it should be changed to the form of all S is PS. This makes a logical proposition out of it, and we can read it backward, all PS is S, which is equally true with the proposition that all S is PS.

57. The second form is called the particular affirmative, and its formula is some **S** is **P**, meaning some **S**'s are some **P**'s. It is said by the old logic that the subject some **S** means that the subject is not taken altogether, and that the word some is indefinite. Now it may be true that in grammar the word some is called an indefinite adjective, but because it is indefinite in grammar that does not make it indefinite in logic; the two sciences are distinct and must be kept distinct. When I say "some people are patriotic," I am not taking a part of my subject,—my subject is "some people," and I am taking the whole of my subject. I mean by the word "some people," all of the people that I have in my mind. True, I may not know how many of them there are, but that is a different question. When I say "the American people are patriotic" I do not know how many Americans there are in this case any more than I know how many people there are when I say "some people," but no one will pretend that "the American people" is an indefinite subject; and I contend that "some people" is not indefinite in logic any more than the phrase "the American people" is indefinite. When I say "some people" are patriotic it is perfectly clear that by the word "some" I mean "patriotic" people, so that the proposition means patriotic people are patriotic. In this last case the term "patriotic" being an adjective is indefinite; it can be converted into a definite predicate by combining the subject with it, thus giving us the tautologous proposition patriotic people are patriotic people, and nothing can be truer than that is.

58. The third form is called the universal negative, and its formula is No **S** is **P**, or properly, No **S**'s are **P**'s. Its logical effect is to affirm that the name **S** goes with the name Not-**P**.

59. The fourth form is called the particular negative, and the formula is "Some **S** is not **P**." As I have said before, "some **S**" means the whole of the class which we have in our mind.

60. These four forms have four symbols, A, I, E, O, which are taken from the Latin words *affirmo* and *nego*; *affirmo* means I affirm, and *nego* means I deny. A and I are the first two vowels of the word *affirmo*, and stand respectively for the universal

affirmative and the particular affirmative, and E and O are the vowels in *nego* and stand respectively for the universal negative and the particular negative. The old logic is based on these four forms of propositions. Yet we find on examination that the distinction between universal propositions and particular propositions has no basis in logic; in reasoning we must take the whole of our subject. Neither do we find that the division of propositions into affirmative and negative has any sound basis. Every proposition is affirmative and must be so in the very nature of things. This view will be more fully developed later on.

61. Some subjects are called Indesignate, for instance: Comets are subject to the law of gravitation. Dr. Keynes says that indesignates are generally taken as universal. This is correct when we know as a matter of fact that all comets, or whatever the subject may be, are intended. But, unless we are perfectly sure that an indesignate proposition is intended to be taken universally we should throw it into the proper logical form so that there can be no doubt about the matter.

62. Another theory of the old logic is called the division of terms into distributed and undistributed. This division is arbitrary and technical. The word distributed corresponds to the word universal as applied to propositions, and the word undistributed corresponds to the word particular. Distributed means that the subject or the predicate is undivided, that is, that it is taken as a whole. It implies that the subject or predicate which is said to be distributed, is a definite term.

63. The word undistributed implies that a part of the subject or the predicate is taken and that the term is indefinite. An undistributed term usually has as a part of its name one of the words "few, some, many or most." These words are said in the old logic to indicate that only a part of the term is taken.

64. This division of terms into distributed and undistributed has, it seems to me, no more foundation in logic than the division of propositions into universal and particular has.

65. We have seen that the subject and the predicate of propositions are two names for the same thing. Now an idea with

an indefinite name, like a wheelbarrow with only one handle, is not good for much, and an idea with two indefinite names is as useless in logic as a wheelbarrow without any handles; it is just as bad to have an indefinite predicate as it is to have an indefinite subject. The old logic places a good deal of importance on the meanings of the terms "subject" and "predicate." But, as we have seen, the subject is a name, and the predicate is a name and the only real difference between them is that one comes first and the other comes last; both being names of the same thing, it is a matter of no importance, from a logical standpoint, which comes first. We may say indifferently salt is chloride of sodium or chloride of sodium is salt; both mean the same thing, and in logic one form is as good as the other. In a proposition the subject affirms that it is the name of an idea; the predicate does the same. Strictly speaking the predicate is not a predication about the subject; it is a predication of the idea which it represents. The subject likewise by its very existence predicates that it is the name of the same idea for which the predicate is a name.

66. In the universal affirmative proposition A, the subject, only is said to be distributed; the predicate is indefinite and therefore undistributed. An example, "All men are mortal," shows this. The term "mortal" is an adjective and indefinite. It applies to many other kinds of beings than men; it is therefore not a definite name for men. The proposition, therefore, is not strictly true. If it were strictly true we could say "All mortals are men," but this we cannot do because we know as a matter of fact that horses, dogs, etc., are mortal and yet they are not men.

67. A logical proposition must have a logical form. When it is in a logical form it will say just what it means. A model form of a proposition is "I am I." "Salt is salt," "A quintillion is a quintillion." The mind intuitively perceives the truth of these propositions. I may not know what a quintillion is, but still I know that a quintillion is a quintillion. Every true proposition can ultimately be reduced to the formula "A is A,"

and every false proposition to the formula "A is not-A." And hereby we can solve the old problem of "What is truth?" Truth is the giving of the right name to the right thing.

68. The universal affirmative can be easily converted into a logical proposition by repeating the subject after the predicate, thus: "All men are mortal" becomes logically "All men are mortal men." The mind immediately perceives the truth of this proposition. But in the form in which the old logic puts it we must reject it as being indefinite and unfit for logical treatment.

69. The particular affirmative proposition I does not distribute the subject. An example is: "Some men are tall," and we know that this means "Some men are some tall beings." Both terms are indefinite and unless we can reform the proposition and make both terms definite we shall have to reject it. But we can reform it. When reformed it becomes "Tall men are tall men." This is a true proposition.

70. The universal negative proposition E distributes both subject and predicate. An example is "No men are immortal."

71. The particular negative O distributes the predicate, but not the subject. "Some men are not tall" is an example. When we ask who the "some men" designated by the subject are we know that the "some men" are the same men who are called in the predicate "not tall," so that our subject really means "Men who are not tall," and when we ask what does the word "tall" in the predicate refer to, the answer must be that it refers to the same men described by the subject. So that the proposition really means "Men who are not tall are not-tall men." In this form it is strictly true and can be read backward or forward indifferently.

72. Dr. Keynes in his work on Formal Logic divides propositions into several other classes. He says "Copulative propositions are complex propositions which can be analyzed into a conjunction of two or more affirmative propositions having the same subject, and remotive propositions are those which can be analyzed into two or more negative propositions having the same subject."

73. Exceptive propositions are propositions in which the subject is limited by some such word as ‘unless’ or ‘except.’

74. “Exclusive propositions contain some such word as ‘only’ or ‘alone,’ whereby the predicate is limited to the ‘subject.’”

75. It is sometimes difficult to translate into a logical form the words “unless” and “except,” especially when the proposition which contains those words is taken out of its connection. The meaning of a proposition is frequently contained in the proposition and its context taken together. By taking a proposition with its context we can almost always determine the logical force of the proposition. If I say “I am going to town unless it rains,” that may mean I am going to town if it does not rain, or if it rains I am not going to town, or I am going to town or it will rain. The word “except” usually has the meaning of “not.” If I say “A is B except when it is C,” that may mean A is B when A is not C, or it may mean A is B when B is not C.

76. In common speech language is generally elliptical and indefinite. We seem to be unwilling to take the time or trouble to express ourselves with precision. We are contented with giving in as brief terms as possible a general idea of our thoughts. But when it becomes necessary to reason, then our thoughts and our language must be clear, definite and exact.

77. Propositions are also divided into real, verbal, and formal. A proposition which contains more information than a mere name would give is said to be real. For instance, “The angles of any triangle are together equal to three angles.”

78. A verbal proposition gives no information except that which is contained in the meaning of the term. Examples are: “Salt is chloride of sodium;” “Charles Egbert Craddock is Miss Murfrey.”

79. A formal proposition is one in which signs are used to stand for the terms, and the proposition intuitively appears to be true. Examples are: “A is A;” “All A is either B or not B.” This example has been given as a formal proposition: “If all A is B and all B is C, then all A is C.” But this example is not

satisfactory. Suppose we substitute a concrete example constructed in a parallel manner and test it: "If this rose is red, and red is a color, then this rose is a color;" "If this finger ring is gold, and gold is heavy, then this finger ring is heavy;" "If this man is one, and one is a number, then this man is a number." These arguments are not logical. But if we make the premises definite we shall be saved from the error of drawing conclusions which are not warranted by the premises. Thus if we say, "This rose is a red rose" we will not take the next step and say "A red rose is a color." If we say "This finger ring is a gold finger ring" we will pause before we say "This gold finger ring is heavy." If we say "This man is one man" we are not likely to say that one man is a number.

80. Real propositions, again, are divided by the old logic into true and false. This is another of the useless refinements of the old system. It is extra-logical. Logic has nothing to do with the truth or falsity of propositions. The problem of logic is, given any proposition, what are all the implied meanings of that proposition? The other sciences must determine whether propositions do or do not contain matters of fact.

81. When different objects resemble one another in some particular, though differing from each other in a hundred other particulars, the mind can perceive this partial resemblance of objects, and give a general name to the resemblance. This is called generalization. For instance, some animals are called quadrupeds because they agree in having four legs. The objects to which a general name is applied are called a class, and the objects are said to be contained in the class. It seems to me that this is an erroneous idea. When I try to realize in my mind the idea of a horse being contained in a quadruped, I find it impossible to do so. How is it contained in a quadruped? The truth is that the quality of being a quadruped is in the horse. When I say "Man is an animal," what I really mean is that the quality of animality is in a man, and not the ridiculous notion that a man is in an animal.

82. These general terms logically have the nature of adjectives. When I say "George is wise" I do not mean that George

is contained in wise, but I mean that the quality of being wise is in George. Consequently whenever we meet with a proposition which ends with a general term we must treat it as an adjective and make the proposition definite by adding the subject to the predicate, and then our propositions will be definite and true. Thus "A horse is a quadruped-horse;" "Man is an animal-man;" "George is wise George." When we do this, it will be impossible to draw any false conclusions from a premise rendered definite in this way.

83. Abstract names, such as "whiteness," "benevolence," "kindness," "friendship," are a result of the generalizing process. They are names of qualities without containing any reference to the objects which possess the qualities.

84. The names of things, that is material things, are called concrete names. The distinction seems to be that an abstract name is the name of a quality, and a concrete name is the name of a thing possessing qualities. This is extra-logical, and a concrete name may be used as an abstract name, and conversely.

85. When we consider a name in regard to the objects to which it is applied, we consider its extension. When we consider it with regard to its qualities we consider its intension. This also is an extra-logical distinction.

86. According to J. S. Mill, names which are considered with regard to their extension and intension are connotative. According to Sir Wm. Hamilton, denotation corresponds with extension, and connotation with intension.

87. Kant subdivided propositions into four divisions, and each division into three subdivisions. The divisions are, Quantity, Quality, Relation and Modality. Quantity is divided into singular, "This is a horse;" particular, "Some horses are black;" universal, "All horses are quadrupeds." Quality is divided into affirmative, "All men are mortal;" negative, "No man is immortal; infinite, all men are not immortal." Relation is divided into categorical, "Man is mortal;" hypothetical, "If it does not rain I shall go to town;" disjunctive, "Washington was born in Virginia or Pennsylvania." Modality is divided into problematic, "John may be rich;" assertoric, "John is rich;" appodeictic, "Body must have weight."

88. Very few of these fine-drawn distinctions are of any logical importance. In regard to quantity in a true logic, all propositions must be universal in both subject and predicate, in other words, the proposition must be definite. In regard to quality all propositions must affirm something, that is, be affirmative, or they would not be propositions. A negative name does not make a negative proposition. In relation to modality the divisions into problematic, assertoric and apodeictic are of no use at all in logic. The divisions of relation, however, are important. A categorical proposition asserts without any condition that a certain thing or idea has two names. But a hypothetical proposition indicates a certain amount of doubt as to whether a name given to an idea is the correct name or not. "If" is the word which usually implies the doubt. A hypothetical proposition, though, can be converted into a categorical proposition by the use of the words "the case of." Take the proposition "If Cæsar was a tyrant he deserved death," and we can say "The case of Cæsar being a tyrant is a case of Cæsar deserving death," this is a categorical proposition. Very frequently we can treat a hypothetical proposition as a categorical proposition by supplying the implied premise. Thus, Cæsar was a tyrant; tyrants deserve death (the implied premise), therefore Cæsar deserved death." Nearly all hypothetical arguments are cases of a categorical argument with a suppressed or implied premise. When we supply the implied premise there is no difficulty in dealing with hypothetical propositions.

89. In a disjunctive proposition one or both of the terms has two names which are separated by the word "or." There has been a great deal of discussion by the old logicians as to the logical force of the word "or," some contending that "or" is not exclusive in its meaning; that it may mean one or the other or both; others contending that it is exclusive and means one or the other and not both. I contend that "or" should be exclusive, and at the same time it may mean both when that meaning is expressed. By "exclusive" I mean that of the three forms one or the other or both, it can only have one of them; it cannot

have two. Ordinarily there is no doubt that it means one or the other and not both. "Washington was born in Virginia or Pennsylvania," one or the other can be true, but not both. "Columbus discovered America in 1492 or 1592." One or the other may be true, but not both. In this sentence, "A gem is rare or beautiful or both," either one of the alternatives may be true. But if it was not expressly stated that a gem was both rare and beautiful, it seems to me that the fair implication of the words "rare or beautiful" is that it means one and not the other. In this system we shall always construe "or" to mean one or the other and not both, unless it is expressly stated or manifestly implied in the context that it may mean both.

CHAPTER III.

THE LAWS OF THOUGHT.

90. The question which we are now to consider is this: Can there be an infallible logic or system of reasoning? If we can find certain indisputable facts on which to base our system and then can find certain infallible laws by which its operations shall always be conducted, then it seems to me we can have an infallible logic. Are there such facts and such laws?

91. The first fact is this: That all knowledge is a knowledge of differences between things. If there were no differences between things there could be no knowledge of them. When I make a mark like this ——— I could not perceive the mark unless it differed from the paper on which it is made. If the mark were exactly like the paper I could not perceive it. If I look out of the window at a tree the necessary condition for my perception of the tree is that it shall differ from other things. If a person prick me and I could not perceive any difference in my feelings I should have no knowledge of the fact that I had been pricked. A universal sensation of sameness is the same as no sensation at all. This doctrine is called the Law of Relativity. It lies at the foundation of all thought and all knowledge. Its truth is beyond dispute. The mind intuitively perceives that it must be true.

92. The next fact to which we shall call attention is this: That when, for instance, a line is perceived by the mind, the mind must perceive whether the line has the quality of straightness or not-straightness. The mind cannot think of straightness without thinking of not-straightness. An object cannot be thought of as being high without other objects are thought of as being not-high. A thing cannot be thought of as being dead without other objects being thought of as being not-dead. In other words, every name necessarily implies that it has an opposite and that when one of these is posited in thought

the other must also be posited. They cannot be separated in thought. If the mind has one it must have the other. This doctrine I propose to call the Law of Opposites. This doctrine is a necessary corollary of the doctrine of Relativity. The doctrine of Relativity says there must be a difference in order to have knowledge, and the doctrine of Opposites says that we must have two names to express differences.

93. The third law is called the Law of Identity. As usually expressed, "whatever is, is," it means that a thing is equal to itself. In logic it means that whatever name a thing has, it has that name. "John is John," means that if John has the name of John, he has the name of John. This is so plain that no amount of illustration can make it any plainer. It is an intuitive truth. The mind immediately perceives that it must be true. It is a fundamental truth. All truths cannot be proved, or else there would never be any end to demonstration. When in a course of reasoning we arrive at intuitive truths we can go no further.

94. The formula for the Law of Identity is "A is A." A, the subject, stands for the name of some thing. A, the predicate, stands for the name of the same thing. Both subject and predicate refer to the same identity. When we say "A is A" there is only one thing that the mind has in view. Now if by observation or testimony the mind learns that the name "B" is equivalent to the name "A," then the mind can substitute B for one of the A's and say "A is B," or "B is A." Both propositions have exactly the same meaning. If the mind is given the two terms A and B and it has no further information respecting them than the fact that they are names, it cannot tell whether A is B or not-B. It knows that it must be one or the other, but that is the extent of its power. The case of "A is A" is different; its truth is intuitively perceived. If the proposition "A is B" is given to the mind and it is a true proposition, then B must be equal to A, and then the mind can substitute A for B and the proposition is reduced to the identical proposition "A is A," and it is impossible for the mind to doubt the truth of "A is A."

95. We must either suppose that the Law of Identity is certain, or else that there is no certainty whatever. And if a proposition can be reduced to this form, everyone must admit either that the proposition is true, or else that the Law of Identity is false.

96. The next law is called the Law of Contradiction. Usually it means a thing cannot both be and not be. In logic it means that a thing cannot have a name and at the same time not have it. For instance, a line cannot, at the same time, have the name of straight and not-straight. A body cannot have, at the same time, the name of living and not-living. An object cannot, at the same time, have the name of high and not-high. It cannot be here and not-here. In other words, inconsistent, or opposite, names cannot be applied to the same object. Consistency demands that when we give a name to an object we must stand by that name. We cannot say that a line is straight and that it is not-straight; any such course would be dishonest. Of course different parts of an object may have different qualities, and if we are speaking of the parts then we may say that one part is straight, of a line, for instance, and another part is not straight. One part of an object may be white and another part may be black. What the Law of Contradiction affirms is that the same part, the same thing, cannot at the same time and place have contradictory names.

The formula for the Law of Contradiction is " $Aa = O$." It means the non-existence of an object said to possess opposite qualities. A capital letter and its small letter are opposites in this work. Hence every proposition which affirms the presence of opposite qualities is necessarily false. " AB and Ab " stand for opposite propositions. " ABC and ABC " are inconsistent, and, in fact, any two propositions, no matter how many terms they contain, if there is in one of them a term which is opposite to a term in the other, is inconsistent with that other and one of the propositions must be false. Thus " $ABCDEF$ and $ABCdEF$ " are inconsistent. One affirms D and the other affirms d, i. e., not-D, and both cannot be true.

97. The next law is called the Law of the Excluded Middle. The Law of the Excluded Middle means, given any object, we can say that it either does or does not possess any given quality. Thus: "Sugar is sweet or it is not sweet;" "A man is living or he is not living;" "A dog is here or he is not here." It means that there are only two logical alternatives; that there is no middle between having and not-having, being and not-being. Hence the arbitrary designation, Law of the Excluded Middle.

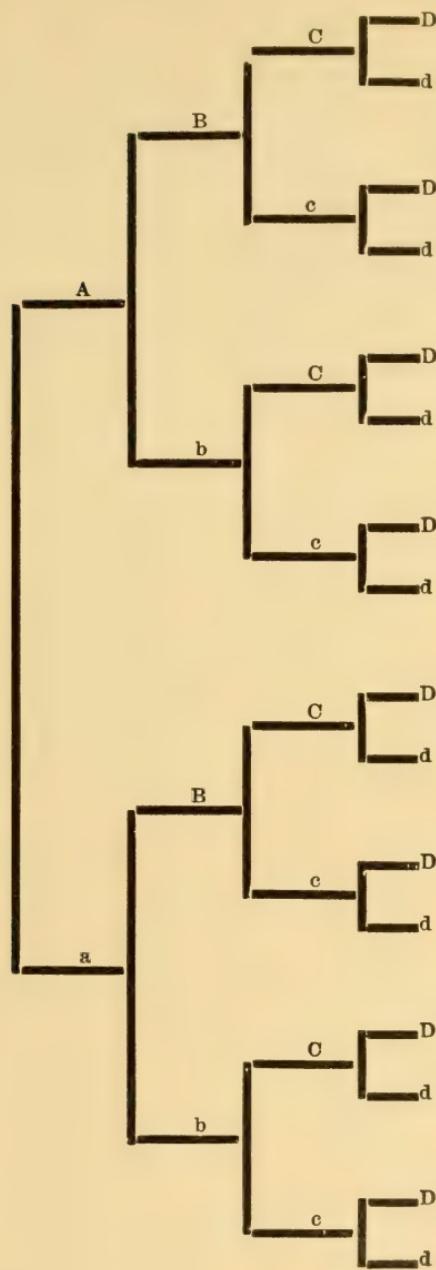
98. In logic it means that a thing must either have a given name or else have its opposite, that is, it must have the name of "sweet or not-sweet," "living or not-living," and so on. The formula for the Law of the Excluded Middle is "A is B or it is not-B." For the term "not-B" let us substitute the smaller letter b; this will be briefer, more convenient and quite as explicit. So now the formula will read "A is B or A is b," and hereafter capital letters will stand for positive terms and small letters will stand for negative terms. The small letters a, b, c, etc., are to be read and pronounced "not-A," "not-B," "not-C," etc.

99. The Law of the Excluded Middle is a corollary of the Law of Contradiction. The Law of Contradiction affirms that of two contradictory propositions both cannot be true, and the Law of the Excluded Middle, that of two contradictory propositions both cannot be false.

100. Another law is called the Law of Logical Division. It affirms that anything or any class can be divided into two parts which shall be mutually exclusive, and which, taken collectively, shall be equal to the whole. Thus we can divide A into B and b, and B into C and c, and b into C and c, etc. To take a concrete example: Bodies can be divided into living bodies and not-living bodies. Living bodies can be divided into animals and not-animals. Not-living bodies can be divided into minerals and not-minerals. Animals can be divided into men and not-men. A man can be divided into head and not-head; head can be divided into face and not-face. Face can be divided

into cheeks and not-cheeks, etc. This process is called dichotomy. The two divisions into which a thing is divided are together equal to the whole thing and at the same time each excludes the other.

AN ILLUSTRATION OF DICHOTOMY.



DICHOOTOMOUS TABLES.

102. Another law, which I propose to call the Law of Elimination, means that if we have two different kinds of things and we remove one kind the other will remain. This also is axiomatic. If I have two kinds of numbers before me, viz.: odd and even, and I remove all of the even numbers, the odd numbers will remain. If from a flock composed of white and black sheep I take away all the white ones, the black ones must remain, and if I have two kinds of propositions, those which are consistent with the premises and those which are inconsistent with the premises, and I eliminate all those which are inconsistent with the premises, those which are consistent with the premises must remain.

103. The process of making all the imaginable propositions which can be made out of the terms of given propositions and of then eliminating the inconsistent propositions, is the reasoning process for non-numerical propositions.

104. Suppose we have the proposition "Salt is chloride of sodium," the two terms are "salt" and "chloride of sodium," and if we posit these two terms, then by the Law of Opposites we must also posit "not-salt" and "not-chloride of sodium." These four terms which we now have can be combined into four propositions:

- 1, "Salt is chloride of sodium;"
- 2, "Salt is not-chloride of sodium;"
- 3, "Not-salt is chloride of sodium;"
- 4, "Not-salt is not-chloride of sodium."

These four propositions can be read either way, forward or backward. Now, if "Salt is chloride of sodium," then the combination which says "Salt is not-chloride of sodium" cannot be true by the Law of Contradiction, which says that a thing cannot be and not be at the same time, and if "Chloride of sodium is salt,"—which is No. 1 read backward—then the proposition which says that "Chloride of sodium is not-salt" is inconsistent, and by the Law of Contradiction must be eliminated. We have now left No. 4 which says that "Not-salt is not-chloride of sodium." This is not inconsistent with the

proposition that "Salt is chloride of sodium," or with the proposition that "Chloride of sodium is salt." As it is a consistent proposition we cannot eliminate it. Therefore from the proposition that "Salt is chloride of sodium," we have found a new proposition, viz.: that what is not-salt is not-chloride of sodium. This is an equivalent proposition to the proposition that "Salt is chloride of sodium." For if we take this conclusion for our premise, "Not-salt is not-chloride of sodium," we can get for our conclusion "Salt is chloride of sodium," which is our original premise.

105. Thus, if we posit the terms "not-salt" and "not-chloride of sodium," we must also posit the terms "salt" and "chloride of sodium." These four terms give us four propositions, viz.:

- 1, "Not-salt is not-chloride of sodium;"
- 2, "Not-salt is chloride of sodium;"
- 3, "Salt is not-chloride of sodium;"
- 4, "Salt is chloride of sodium."

Now if No. 1 is true, "Not-salt is not-chloride of sodium," then No. 2 "Not-salt is chloride of sodium" is inconsistent and must be eliminated. Now if No. 1 is true when read backward "Not-chloride of sodium is not-salt," then No. 3, which is "Not-chloride of sodium is salt" when read backward, is inconsistent and must be eliminated. No. 4, "Salt is chloride of sodium," remains, and as that is not inconsistent with No. 1, "Not-salt is not-chloride of sodium," it stands. And thus from the proposition "Not-salt is not chloride of sodium," we have returned to the proposition with which we started, "Salt is chloride of sodium."

If anyone should fail to see that the proposition "Salt is chloride of sodium" can be read backward as well as forward, it can be easily demonstrated by using the Law of the Excluded Middle, thus: "Chloride of sodium is either salt or it is not-salt." If we suppose that it is not salt, then since by our premise "Salt is chloride of sodium," salt would be not-salt, which is impossible according to the Law of Contradiction, therefore

chloride of sodium must be salt. And, again: if not-salt is not-chloride of sodium, then by the Law of the Excluded Middle not-chloride of sodium must be either not-salt or salt. But if we suppose not chloride of sodium to be salt, then since not-salt is not-chloride of sodium, not-salt would be salt,—which is absurd. Therefore not-chloride of sodium must be not-salt.

106. The Law of Combinations in logic is a different law from the Law of Permutations in mathematics. The Law of Combinations in logic pays attention only to the fact that terms are positive or negative, and does not pay attention to the order in which they are read. In logic, as in algebra, it makes no difference whether we say x is y , or y is x . But the Law of Permutations is concerned with the order in which terms can be read. The Law of Combinations doubles the number of combinations with each additional letter. Its formula is $2 \times 2 \times 2 \times 2$, etc. Two letters make four combinations, three letters make eight combinations, four letters make sixteen combinations, five letters make thirty-two combinations. The formula of permutations is, $2 \times 3 \times 4 \times 5$, etc. Thus, two letters can be read in two ways, three letters in six ways, four letters in twenty-four ways, and five letters in a hundred and twenty ways.

107. Suppose we have the two letters A and B, we can make four combinations as follows: AB, Ab, aB and ab. If we have three letters ABC, we can make the following combinations: ABC, ABC, AbC, Abc, aBC, aBc, abC and abc. The method pursued in making these combinations is simply changing one letter at a time. This will give us every possible combination. It is more convenient to commence to change the last letter first and proceed backward until every letter has been changed. These combinations are to be considered as propositions which can be read in any order we choose. Thus, suppose a boy had three christian names, Andrew, Benjamin and Charles. We could say Andrew is Benjamin, or Charles is Andrew, or Benjamin is Andrew Charles,—in short,

we could read the names in as many different ways as there are permutations and combinations, and each and every one of the propositions thus derived will be the equivalent of each and every one of the others. We can choose any one of these equivalent expressions which suits our convenience.

CHAPTER IV.

INFERENCE.

108. Inference is of two kinds, immediate and mediate. Immediate Inference is where one proposition being given to the mind, the mind immediately perceives the truth of another proposition. Mediate Inference is where two or more propositions are given to the mind and from them the mind infers other propositions. This is the view of Inference which is taken by the old logic. It is doubtful to me whether there are any immediate inferences except those which we call Axioms and Laws of Thought.

109. My view of Inference is that it is the result of a process by which we make all the possible new propositions which can be made out of the terms of the given propositions, and then eliminate those propositions which are inconsistent with the premises. The consistent propositions remain.

110. According to the old logic, from a proposition in the form of "All S is P," by Immediate Inference we can get "Some P is not-S." When we try the experiment of asking persons who are not skilled logicians whether they can immediately infer "Some P is not-S" from "All S is P," the experiment proves that the great majority of men cannot see the Immediate Inference. I do not think myself that there is any such Immediate Inference. In the first place "All S is P," which means "All S is some P," is not a true proposition. "Some P" is not a definite and synonymous name for "All S." To take a concrete example: "All men are animals." It is easy to prove that animals is not a true name for men, because when we use the Law of the Excluded Middle and say "Animals are men or not-men" we come to a stop. We must first convert the proposition "All men are animals" into a definite proposition, "All men are animal-men," and then we can immediately infer that

"Animal-men are men." But if anyone should hesitate, it can be proved by the Law of the Excluded Middle, thus: "Animal-men are men or not-men." If we suppose animal-men to be not-men, then since men are animal-men, men would be not-men,—which is absurd according to the Law of Contradiction, therefore "Animal-men are men."

111. Whenever we have a proposition whose subject and predicate are tautologous or synonymous, we can get a new proposition by reading it backward. This is a case of Immediate Inference.

112. Some of the old logicians think that we can have Immediate Inference by qualifying both the subject and predicate by the same term, thus: "A negro is a fellow creature," therefore "A suffering negro is a suffering fellow creature." But when we construct parallel examples we see that this method is illogical. Thus: "An elephant is an animal." Therefore "A small elephant is a small animal;" "A cricketer is a man," therefore "A poor cricketer is a poor man." By converting the above indefinite propositions into true propositions, such as "An elephant is an animal-elephant;" "A cricketer is a man-cricketer," then we can say, "A small elephant is a small animal-elephant," "A poor cricketer is a poor man-cricketer," and our inferences will be true.

CHAPTER V.

SIGNS.

113. In complicated reasoning we require the help of signs. Of course it would be possible to write out all the propositions which can possibly be made out of the terms of given propositions, at full length. But it would be a very laborious and tedious task. Use of appropriate signs will save us a great deal of time and trouble. The success of the mathematical sciences is due largely to the fact that they have an appropriate language of signs. We shall need signs to stand for terms, for the copula, for "or," and for the Universe of Discourse.

114. The sign for the copula which I shall use is two short parallel lines, thus: ==. I use this sign because it has been heretofore generally used to stand for "is," but at the same time I wish to say that it must not be confounded with the similar sign used in mathematics. In logic it means "has the name of;" in mathematics it means "is equal to." These two meanings are distinct.

115. For the word "or" I shall use a perpendicular line, thus: |. Of course this is arbitrary.

116. The letters of the alphabet are probably the best signs we can use to stand for terms. We shall need two kinds of letters, one to represent positive terms and one to represent negative terms. We shall use the capital letters, A, B, C, etc., to stand for the positive terms, and the small letters, a, b, c, etc., to stand for the negative terms. Thus, if A stands for man, a will stand for not-man; if B stands for straight, b will stand for not-straight; if C stands for Charles, c will stand for not-Charles.

117. We do not need any sign for a conjunction. The juxtaposition of two letters will enable us to supply the conjunc-

tion which will best suit our convenience. Thus, AB can be read "A and B," when we wish to indicate that they are to be taken together. If we have several letters like ABCD we can read them "A, B, C and D." Our letters will stand for the names of ideas. When the names are either adjectives or common nouns we must convert them into definite and synonymous terms before representing them by signs. When converted we can represent them by either one or two signs, or even more, as best suits our convenience. Thus: if A stands for man, we can use BA to stand for animal-man. Take the proposition "The British Museum is the largest collection of books in the world." If A stands for the British Museum, we can let B stand for "the largest collection of books in the world."

118. Usually it will only be necessary to use a single letter to stand for a subject or a predicate, except in those cases where we have indefinite subjects or predicates or compound subjects or compound predicates, or both. Where we have a compound subject or a compound predicate, each elementary term which goes to make up the compound term can be, and generally should be, represented by a separate letter. With these four kinds of signs, viz.: capital letters, small letters, the copula and the sign for "or," we can represent all necessary logical distinctions.

119. The copula, though necessary from a grammatical point of view, is not necessary from a logical standpoint. When the baby says "papa, man," it utters a true proposition, and in our system of logic after having once formally stated a proposition we shall dispense with the use of the sign=for the copula, and simply write the letters one after the other. Thus "AB" will mean "A is B, and B is A." When we have more than two letters we can insert the copula wherever we please. Thus, when we have "ABC," by inserting the copula wherever we please, we can get a great variety of propositions, but, of course, they will only be equivalent propositions, because "ABC" means simply that a certain object has three names, viz.: A, B and C.

120. In our system a letter stands for a name, whether that name be a designation, description, or proper name. The name stands for an idea, a thought; the idea must be definite, either an individual or a class or a collection, or two or more things taken together as one thing, or an indefinite number of things taken together as one thing. In other words, the name must represent either a thing or a bundle, no matter how many things are contained in the bundle.

121. In our system we pay no attention to many of the old distinctions of the syllogistic system, such as denotation and connotation, essential and accidental, exclusion and inclusion, etc.

122. The reason why we have no signs to indicate a denial is because denial is no part of logic.

123. We are not obliged to read our letters in the order ABCD, etc. Of course we are obliged to put one letter before another in speech, but in logic they all stand on an equality and it is just as logical to read them DCBA as it is to read them ABCD.

124. The proposition "A is B or C" is written " $A = Bc | bC$." By the law of Opposites, whenever we posit a thought we are also obliged to posit its negative, and as the word "or" is exclusive the proposition "A is B or C" really means A is B and not-C or C and not-B, that is, it is one or the other, but not both, and in order to express this meaning clearly and definitely by our signs we are obliged to express it $A = Bc | Cb$. To put it in another light: in stating alternatives by means of signs we must give the full and complete symbolic description of each alternative. Thus: A is B or C or D is stated thus: $A = Abcd | ACbd | ADbc$; this gives us a perfect symbolical description of each alternative. In stating alternatives it is better practice to repeat the subject in the predicate unless the contrary is indicated. This is necessary when we do not know whether in the proposition A is B or C or D, that either B or C or D is synonymous with A.

125. Whenever we make a proposition we always have in

view what is technically called the Universe of Discourse, that is, our field is a more or less limited one, and our remarks are made with reference to the subject which we have under consideration; thus our field of discourse may be business or law or logic or everything which is conceivable by the mind, but whatever our Universe of Discourse may be, our propositions are to be taken with reference to it. Now we need a sign to represent this Universe of Discourse. The sign which I have chosen is a square. I prefer a square because it can be divided and subdivided into smaller sections indefinitely, at least the only limits are physical limits. Now when I want to represent **A** and **a** I make a square, thus:

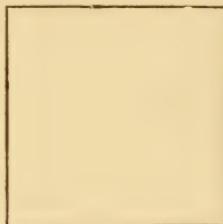


Fig. 1.

This square represents the Universe of Discourse. Now draw a perpendicular line through the center of it which will divide it into two parts, and over one part write "**A**" and over the other part write "**a**," thus:

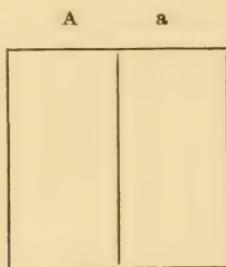


Fig. 2.

Now we have represented **A** and **a** visually by means of a diagram. Now if we want to divide our Universe of Discourse still further into **B** and **b** we will draw a line horizontally

through the center of our square and mark the upper part B and the lower part b, thus:

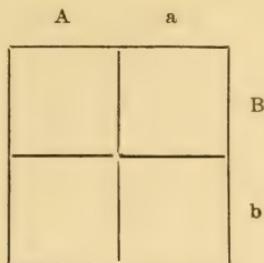


Fig. 3.

126. The sections which run from right to left we will, for the sake of convenience, call rows, and the sections which run up and down we will call files. Thus we have A files and a files and B rows and b rows. Each of our sections thus obtained will represent a conceivable idea, and its name will be the letters which would meet in that section. Thus, the section where A and B would meet will be the AB section and it means that an imaginable idea is described by the names A and B, or to put it in the form of a proposition, "A is B, or B is A." The file section below AB is the Ab section; it means A is b, or b is A. The row section to the right of AB is the aB section and it means a is B, or B is a, whichever way we choose to put it. The remaining section is the ab section, and it means a is b, or b is a. If we wish to do so we can repeat a letter and read our propositions thus: A is BA, B is AB; a is ba, etc. It will be understood that the file letters A and a which stand at the head of the file columns run all the way down, that is they are to be repeated in each section under them, and the row letters B and b which stand at the right of the row sections run all the way across and are to be repeated in each of those sections. Our diagram will now have this appearance:

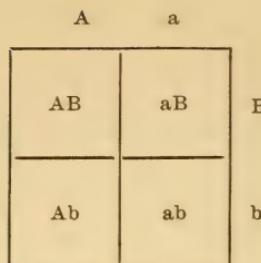


Fig. 4.

127. Now this Diagram represents that our Universe of Discourse is divided into four sections and each section represents a conceivable idea. Given the terms A and B we must posit their opposites a and b, and our diagram now represents every conceivable combination which can be made with our terms. It is a practical illustration of the process of dichotomy. The two terms A and B produce the four combinations and eight propositions represented in our diagram.

128. I call this diagram a Reasoning Frame, because I believe that with its aid the process of reasoning can be represented visually, the combinations can be made mechanically and the elimination of inconsistent propositions from the Frame can also be done in a mechanical manner, and then the propositions which are consistent with the premises will automatically remain.

129. I believe that the brain is a thinking machine, and this system represents the mechanical nature of the brain's activity in the reasoning process.

130. Suppose our proposition is "A is B." Now we know by the Law of the Excluded Middle that B is A or a. If we suppose B to be a, then since A is B, A would be a, which is impossible by the Law of Contradiction. Therefore B is A.

Now if A is B, then the combination Ab, which means that A is b, is inconsistent with the proposition A is B, and we eliminate it from our Reasoning Frame by making a figure 1 in that section. The sign 1 indicates that we have eliminated an inconsistent proposition. Now if B is A then the combination Ba, which means that B is a, is an inconsistent combination and we

eliminate it by making a figure 2 in the aB section. The section ab , which means a is b , is not inconsistent with the proposition A is B , and we cannot eliminate it. It therefore automatically remains.

131. By our process we have now ascertained that the proposition that A is B is consistent with three other propositions, viz.: B is A , a is b , and b is a . We may start with either one of them as a premise and we will get the other three as conclusions. Our diagram, after performing the above operation, will have this appearance:

A	a	
AB	2 aB	B
1	ab	b

Fig. 5.

132. Suppose to make this operation clearer we take a concrete example. Take the proposition, "Salt is chloride of sodium." This is a proposition in which the terms are synonymous and we can read it backward, viz.: Chloride of sodium is salt.

Let A stand for salt

B stand for chloride of sodium, then

a will stand for not-salt

b will stand for not-chloride of sodium.

The proposition can be stated, thus:

$$A = B.$$

Next we make our square, thus:

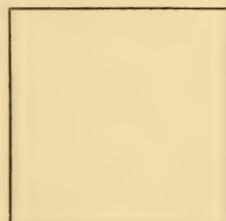


Fig. 6.

Then we divide it into A and a, thus:

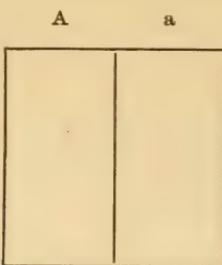


Fig. 7.

Next we divide it into B and b, thus:

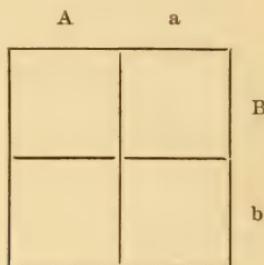


Fig. 8.

Then we mark our sections, thus:

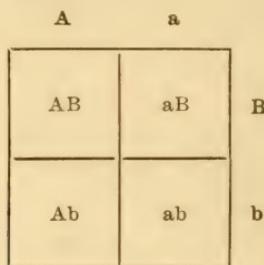


Fig. 9.

Now if A is B, Ab is an inconsistent proposition and we eliminate it by making a figure 1. Again, if B is A, Ba (aB) is an inconsistent proposition and we eliminate it by making a figure 2. But the combination ab is not inconsistent with our premise, which is A is B, and we cannot eliminate it. It therefore remains, and all that remains for us to do is to translate it into concrete terms.

Thus, "What is not-salt is not-chloride of sodium; what is not-chloride of sodium is not-salt. Our diagram has this appearance after having performed these operations:

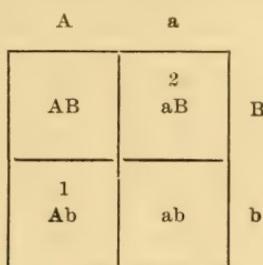


Fig. 10.

The propositions which we struck out on account of their inconsistency with the premise could be translated, Salt is not-chloride of sodium, and Chloride of sodium is not-salt. If our premise is true then by the Law of Contradiction these two propositions, Salt is not-chloride of sodium, and Chloride of sodium is not-salt, cannot be true.

133. Next let us take the proposition "Chloride of sodium is salt."

Let $B = \text{chloride of sodium}$,

$A = \text{salt} \bullet$

$b = \text{not-chloride of sodium}$

$a = \text{not-salt}$

Then the premise will be stated thus:

$$B = A.$$

Then draw a square, divide it into two parts by a perpendicular line and over the left hand part write A and over the right hand part write a ; then divide it again by a horizontal line into equal parts, and at the right hand of the upper part write B , and at the right hand of the lower part write b . Our square is now divided into four sections, each section represents an imaginable idea. These are all the imaginable ideas which can be obtained from two terms, A and B . Next name each section with the file letter over it and with the row letter to the right of it. Our sections will then be named AB , Ab ,

aB , ab . These combinations mean that each imaginable idea has two names, and these two names will make two imaginable propositions. Thus AB means A is B and B is A . Our square will have this appearance after the operation.

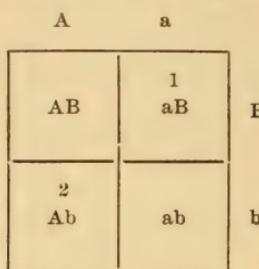


Fig. 11.

In removing the inconsistent propositions from our square or Reasoning Frame it is not necessary for us to pay attention to our concrete proposition, "Chloride of sodium is salt;" it is only necessary to pay attention to our abstract signs which represent that proposition, viz.: "B is A."

Now if B is A it follows by the Law of the Excluded Middle that A is B , because A is B or b . Now if we suppose A to be b , then since B is A by the premise, B would be b , which is impossible by the Law of Contradiction, therefore A is B .

Now if B is A , then the combination Ba , which means that Chloride of sodium is not-salt, is inconsistent and we eliminate it by placing a figure 1 in the Ba section. Again, if A is B , the combination Ab , which means that Salt is not chloride of sodium, is inconsistent and we eliminate it by a figure 2 in the Ab section. The section marked ab remains, and it is not inconsistent with AB . Its translation into concrete terms is, What is not-salt is not chloride of sodium, and What is not-chloride of sodium is not-salt, and of course both of these propositions are true if the premise "Chloride of sodium is salt" is true, since we have used nothing but infallible laws of thought to deduce them. The system is infallible no matter whether the premises are so or not.

134. According to the old logic, no inference could be drawn from two negatives. This is another of its mistakes. Let us

take the proposition "What is not-salt is not-chloride of sodium."

Let $a = \text{not-salt}$

$b = \text{not-chloride of sodium}$

$A = \text{salt.}$

$B = \text{chloride of sodium}$

Make a square, divide into two equal parts by a perpendicular line, then into two equal parts by a horizontal line; over the files write respectively A and a , to the right of the rows write respectively B and b ; mark each section with the letters which would meet in that section. Our square will have this appearance after the operation:

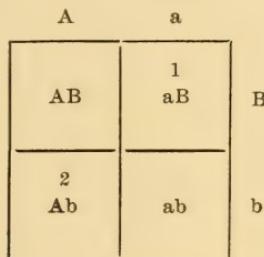


Fig. 12.

Our concrete proposition, "What is not-salt is not-chloride of sodium," can be stated thus, $a = b$.

Now if a is b it follows by the Law of the Excluded Middle that b is a , because b is either A or a . But if we suppose b to be A , then, since a is b , a would be A , which is impossible by the Law of Contradiction,—therefore b is a .

Now if a is b , the proposition a is B , aB , is inconsistent and we eliminate it with a figure 1. If b is a then the combination bA is inconsistent and we eliminate it with a figure 2. The combination AB remains. It is not inconsistent with the premise, ab , and it can be translated "Salt is chloride of sodium."

135. Thus, from the proposition a is b we have deduced the affirmative proposition A is B and this shows that in a correct logic you can reason as accurately and as easily with negative terms as you can reason with positive terms.

136. Next let us take the proposition, "What is not-chloride of sodium is not-salt."

Let $b = \text{not-chloride of sodium}$

$a = \text{not-salt}$

$B = \text{chloride of sodium}$

$A = \text{salt}$

Our premise can be stated thus:

$$b = a.$$

Make an AB diagram.

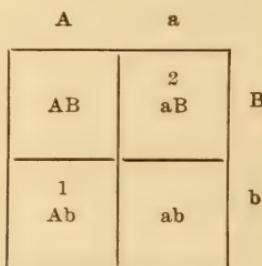


Fig. 13.

If $b = a$, then a will equal b , because by the law of the Excluded Middle $a = B$ | (or) b . But if we suppose a to equal B , then, since b is a , b would be B , which is impossible by the Law of Contradiction. Therefore a is b .

Now if b is a , then the combination bA is inconsistent and we mark it with a figure 1. Again, if a is b , then the combination aB is inconsistent and we mark it with a figure 2. The combination AB is not inconsistent with the premise, and it automatically remains in the Reasoning Frame, and it can be translated, "Salt is chloride of sodium or Chloride of sodium is salt."

137. We have now shown that given any simple categorical proposition of the AB type with synonymous terms there are four ways of stating it, viz.: AB , BA , ab , and ba . This furnishes us with an easy method of testing the truth of simple propositions, for if any one of the four equivalent ways in which the proposition can be stated, is untrue, then the premise is untrue. A simple proposition when first stated may appear to be true, but if we convert it into its equivalent forms, if it is

not true, one of the equivalent forms will usually show us that the proposition is false.

138. This diagram for two terms (and the examples which we have given of its proper use), is the key to our system. It can be extended to any number of terms. All that we will have to do is to divide our Universe of Discourse into as many sections as there are imaginable combinations of the terms of our premises, and then eliminate the inconsistent combinations.

139. The process of combining the terms so as to make all the imaginable combinations, can be done in a purely mechanical manner. The making of the Reasoning Frame is also mechanical. The eliminating of the inconsistent combinations is a mechanical process. When we have done this, our conclusions, and the other consistent combinations, automatically appear.

140. No matter how many letters there may be in any section, they always indicate that they are the names of one idea.

141. If we start with two terms we shall have, by the Law of Combinations, four combinations, and every additional letter will double the number of combinations, three terms make eight combinations; four terms make sixteen, and so on. In our Reasoning Frame we must have a section for each combination. It follows necessarily that each section will have a different combination of names from every other section.

142. Let us make a square and bisect it with a vertical line, and mark one section A and the other a, thus:

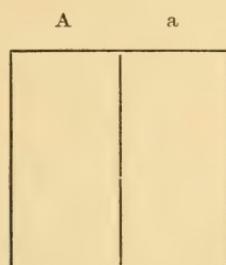


Fig. 14.

We have now an illustration of the Law of Relativity. We perceive that A differs from a, and that a differs from A. The diagram also illustrates the Law of Opposites, that if we have A we must also have a, and if we have a we must also have A. We can make a hypothetical proposition out of it, thus: If A, then a, and if a, then A.

143. The Law of Identity is also exemplified. It says A is A, and a is a, and it means if a thought has the name of A, then it has the name of A, and if an idea has the name of a then it has the name of a.

144. The Law of Contradiction can also be found represented in the diagram, which says that the same thing cannot be A and a at the same time and place. It means that the same thing cannot have two opposite names.

145. The Law of the Excluded Middle is also illustrated. It says that anything is either A, or it is a, and it means that anything either has a given name, or it has the opposite name.

146. Now let us make a square and bisect it with a vertical line and then bisect it with a horizontal line, and over the files write A and a, and against the rows mark B and b, respectively, and mark each section with the letters that would meet in it, thus:

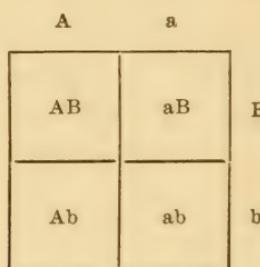


Fig. 15.

147. By the Law of Relativity we perceive each section is different from every other section.

148. By the Law of Opposites, for every A there is a and for every B there is b, and conversely.

149. By the Law of Identity we can read AB is AB, aB is aB, and so on.

150. By the Law of Contradiction we perceive that A cannot be AB and Ab at the same time and place, and so on.

151. We can also read the Law of the Excluded Middle, for A is either AB or Ab, and a is either aB or ab, and so on.

152. Thus we have illustrated in the Reasoning Frame for two terms all the laws which are necessary to use in the most complicated logical reasoning. Of course a Reasoning Frame is not logic, but it is rather a foundation for logic and it enables us by its visual demonstration of logical processes to more easily comprehend a rather abstruse science, but still, one that is not so abstruse as arithmetic or algebra.

153. Suppose we have the proposition "Death is not life," and we wish to know what inferences can be drawn from this premise.

Let A = Death,

b = not-life,

a = not-death,

B = life.

Make an AB diagram, as heretofore described, thus:

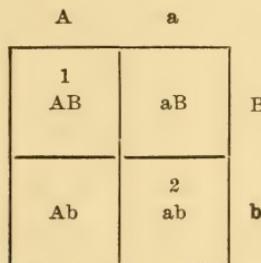


Fig. 16.

Our premise can be stated, thus:

A is b

Then if A is b the combination AB, which says that A is B, is inconsistent and we eliminate it by making a figure 1 in that section.

Again, if A is b, b is A by the Law of the Excluded Middle, for b is either A or a. But if we suppose b to be a, then, since

A is b by the premise, A would be a, which is impossible by the Law of Contradiction, therefore b is A.

Now, if b is A; then the combination ba, which means that b is a, is inconsistent, and we eliminate it by making a figure 2 in that section. The combination aB is not inconsistent with the premise Ab; it automatically remains in the Reasoning Frame and all we have to do is to translate it into its proper concrete terms, viz.: "Not-death is life," or "Life is not-death."

154. Supposing now that we take the proposition "Life is not-death." In stating propositions we take capital letters to stand for positive terms, and we take negative letters, i. e., small letters, to stand for negative terms. The object in doing this is to prevent us from becoming confused in the use of our symbols. We could if we chose, let a capital letter stand for a negative term, but the probability is that in so doing we should fail to remember that our positive letter was standing for a negative term, and thus make a mistake in manipulating our symbols. To return to our premise.

Let B = life,

a = not-death.

b = not-life.

A = death.

The premise can be stated thus:

B is a

Make an AB diagram as heretofore described, thus:

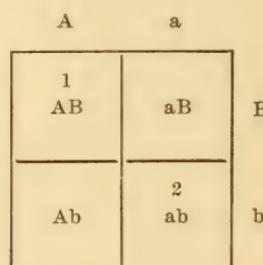


Fig. 17.

Now if B is a, then the combination BA, which means that

B is A, is inconsistent, and we eliminate it by making a figure 1 in that section.

If B is a, then a is B by the Law of the Excluded Middle, because a is either B or b, and if we suppose a to be b, then, since B is a, by the premise, B would be b, which is impossible by the Law of Contradiction, therefore a is B.

If a is B, then the combination ab, which means a is b, is inconsistent, and we eliminate it by making a figure 2 in that section. The combination bA is not inconsistent with the premise, Ba, and it therefore automatically remains in the Reasoning Frame and it can be translated "Not-life is death."

155. There is a shorter method of working these examples than the one which we have been pursuing. If the reader will look back over the examples which we have worked he will see that when our premise was the combination AB, then the combination in the same file, viz.: Ab was inconsistent, and the combination in the same row, viz.: aB was inconsistent. Again, when our premise was Ab the combination in the same file, viz.: AB was inconsistent, and the combination in the same row, viz.: ab was inconsistent. Again, when our premise was aB, then the combination in the same file, viz.: ab was inconsistent, and the combination in the same row, viz.: AB was inconsistent. Again, when our premise was ab, then the combination in the same file, viz.: aB, was inconsistent, and the combination in the same row, viz.: bA, was inconsistent. From these examples we can see that the combinations in the same row and in the same file with our premise, are always inconsistent in the case of propositions of the AB type, where the terms are synonymous; and the inconsistent propositions can be eliminated by this rule without stopping to reason out the matter.

Suppose our premise is "Not-death is life."

Let a = not-death.

B = life,

A = death,

b = not-life.

Our premise can be stated, thus:

$$a = B$$

Make an AB diagram as hereinbefore directed, thus:

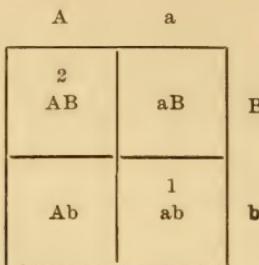


Fig. 18.

Now as aB is our premise, by the rule laid down in the last preceding section, we can eliminate the ab section (1), because it is in the same file, and the AB section because it is in the same row (2); the Ab combination remains and its translation is "Death is not-life."

156. If our premise is "Not-life is death,"

Let $b = \text{not-life}$,

$A = \text{death}$,

$B = \text{life}$,

$a = \text{not-death}$.

Our premise can be stated, thus:

$$b = A$$

Make an AB diagram, thus:

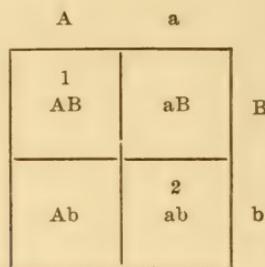


Fig. 19.

Now as our premise is the bA combination, then the combination in the same file, viz.: AB , is inconsistent, and we elimi-

nate it, and the combination ab in the same row is inconsistent and we eliminate it, as shown in the diagram above. We thus see that there are four equivalent ways of stating a proposition having synonymous terms which can be reduced to the Ab form, viz.: Ab, Ba, bA and aB. By taking any one of these for a premise we get the others. If any one of the equivalent propositions should as a matter of fact be false, that would prove that our premise was not true.

157. By our system reasoning is reduced to the repetition of a few uniform operations of analyzing our proposition into terms, of representing the terms by appropriate signs, of making all the possible logical combinations and of eliminating the contradictory propositions. The most complicated questions can be solved in this routine manner when we understand the meaning of the premises. Of course if we do not understand the meaning of the terms used in the premises we cannot reason about what we do not comprehend. We must know what the terms mean before we can state them by means of signs.

158. Most all logicians of the old school have had great difficulty in interpreting negative terms and of deducing the consequences which follow from them. De Morgan asks how many persons would be able to say confidently and off-hand whether either, and if so which, of these two statements is true? (1) The English who do not take snuff are included in the Europeans who do not take tobacco. (2) The English who do not take tobacco are included in the Europeans who do not take snuff. (Snuff-takers of course are included in tobacco takers.) Or, again, Who are the non-ancestors of all the non-descendants of A. B?

159. Prof. Venn in his excellent work on Symbolic Logic, p. 24, speaking of logical diagrams, says:

"A way of interpreting and arranging propositions which may be substituted for both the preceding (for the purpose of an extended symbolic logic), is perhaps best described as implying the occupation or non-occupation of compartments." His idea seems to be that when a section is eliminated it means

that it is unoccupied. When we eliminate a combination we do so because there is no idea which has the names which are symbolized by the combination, and when we let a combination stand because it is consistent with the premise we do so on the theory that the premise being true there is a thought which has the names which are symbolized by that combination.

160. Boole discovered an algebraical system of logical inference which worked in a mechanical manner. His system is a great improvement on the old logic, but it is a very difficult system to master, and it takes much longer to reach the conclusions which we can reach by our system in a few minutes.

161. The Eulerian system of representing propositions by means of circles is well known to readers of the old logic. It is a very imperfect scheme, however, and is of little practical use.

162. Kant and De Morgan both suggested the use of a square and a circle, one to represent the subject and the other to represent the predicate. R. G. Latham and a Mr. Leechman use a square, circle and triangle all in one figure.

163. Bolzano used parallelograms. All of these diagrams were used to represent the inclusion and exclusion of classes according to the old logic.

164. The best diagrams that I have read of are those made by Dr. Marquand of Johns Hopkins University. He makes the squares in the same way that I do. His system of lettering the sections is different.

165. When I discovered this method in March, 1895, I was not aware of the progress that had been made by other logicians in solving logical problems with the aid of diagrams. Since then I have looked the matter up as far as I could and have been surprised at the extent to which diagrams have heretofore been used. Some logicians object strenuously to the use of diagrams. But those who believe in a mechanical system to represent the thinking process, will have no sympathy with the views of these writers.

166. Plouquet also made use of squares which he claimed

to have invented prior to Euler's system, and Krause was one of the first logicians to use diagrams.

167. I have gathered these facts from Prof. Venn's work on Symbolic Logic. Prof. Venn uses ellipses which intersect each other, for the purpose of illustrating the reasoning process. His system is a good one, but in comprehensiveness and facility of manipulation it is not equal to the system of squares.

168. Our system is able to detect any inconsistency in the premises. When there is a contradiction in the premises it will manifest itself by the disappearance from the Reasoning Frame of one or more of the letters which we use as signs of the terms which represent the premises. Suppose we had for one premise "Salt is chloride of sodium" and for another premise "Salt is not-chloride of sodium."

Let $A = \text{salt}$,

$B = \text{chloride of sodium}$,

$a = \text{not-salt}$,

$b = \text{not-chloride of sodium}$.

Our premises can be stated, thus:

$$(1) A = B$$

$$(2) A = b$$

Now make an AB diagram, thus:

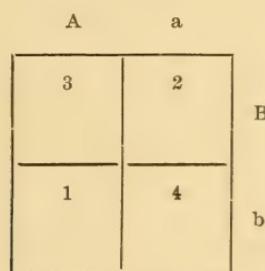


Fig. 20.

Now since A is B , the combination Ab in the same file is inconsistent and we eliminate it by making a figure 1. And the combination a is B in the same row is inconsistent and we eliminate it by making a figure 2. And, secondly, since A is

b, the combination AB in the same file is inconsistent and we eliminate it by making a figure 3, and the combination ab in the same row is inconsistent and we eliminate it by making a figure 4. Thus we see that on account of using contradictory premises every letter term has disappeared from the Reasoning Frame.

169. The fact that we eliminated the combination Ab because it was inconsistent with the combination AB, does not prevent us from using it as a premise for the purpose of ascertaining whether there are any combinations in the Reasoning Frame which are inconsistent with it.

170. Whenever in the working of propositions one or more of the letters used disappears from the frame we must stop right there. The Frame tells us that our premises are inconsistent and that no inferences can be deduced from them.

171. So far we have been considering simple propositions of the kind which are often used in definitions. They are very important and occur with great frequency in everyday life, and yet the old logic failed to recognize their importance or to provide a place for them. Aristotle went so far as to say that singulars cannot be predicated of other terms. This means that an idea could not have two singular names. Such, for instance, as "salt" and "chloride of sodium." Of course any system founded on such mistaken views of logic must be full of errors.

EXAMPLES FOR PRACTICE.

172. What inferences can be drawn from the following premises? (1) The British Museum is the largest collection of books in one building in the world. (2) The capital of the United States is Washington, D. C. (3) Matter is not spirit. (4) What is not fit to teach is not proper to learn.

CHAPTER VI.

INDEFINITE PROPOSITIONS.

173. Indefinite Propositions are those in which the predicate term is not limited to the subject term. They do not clearly and definitely point to the subject. When I say, "George is wise," the term "wise" applies to a great many individuals besides George; it is not limited to him, nor does it point him out as the person thought of.

If "George is wise" is a logical proposition, then "wise" is George, by the Law of the Excluded Middle, for "wise" is either "George" or "not-George," but if we suppose "wise" to be "not-George," then since "George is wise" by the premise, George would be not-George. But this is impossible by the Law of Contradiction, therefore "wise is George" is true, if our premise "George is wise" is a true proposition.

But we know as a matter of fact that "wise is George" is not a true proposition and therefore we must reject all propositions in the form of "George is wise" unless we can reform them and make them true propositions. This we can easily do by adding the subject to the predicate, thus: "George is wise George." The correctness of this form is immediately perceived by the mind.

174. A parallel case is where the predicate is a common term. Thus in the proposition "Iron is a metal," "metal" is a common term, and applies to a good many objects besides iron. The proposition "Iron is a metal" is not a logical proposition, because if it were logical we could read it backward and say "Metal is iron." We know by the Law of the Excluded Middle that metal is either iron or not iron; if metal is not iron, then since iron is metal, iron would be not iron, which is impossible. Therefore metal is iron. But we know as a matter of fact that the proposition "Metal is iron" is not true, and therefore the proposition "Iron is metal" is not true in that form. But if we

add the subject to the predicate and say that "Iron is metal-iron" we have a true proposition and one that can be read either way.

175. The reason why propositions must be made so that they can be read either way is because the subject and predicate are both names for the one thought; they must be equally definite, and being names for the one idea they must be capable of transposition.

176. The old logic calls this kind of propositions universal affirmative propositions. It says the meaning is that one class is included in a higher class. This is without any foundation in fact. If there is any inclusion about it, the implied meaning is that the quality indicated by the predicate is included in the subject, which is exactly the reverse of the idea of the old logic.

177. Boole in his system used the letter V to stand for the word "some." Thus, "Iron is metal" would be stated by him $A=VB$, meaning, Iron is some metal. Liebnitz, Lambert, and Jevons would state it in the form of

$$A = AB$$

meaning "iron is iron-metal." This form is correct. Let us take the proposition "Iron is metal" and solve it by means of our diagrammatic system. The proposition means "iron is metallic iron."

Let A = iron,

B = metal,

then our proposition can be stated,

$$A = BA$$

Make an AB Reasoning Frame in the usual manner, thus:

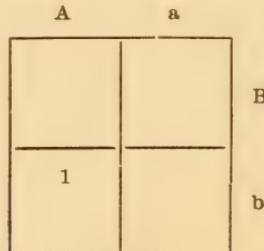


Fig. 21.

The premise A is BA is represented in the AB combination. As we know by the Law of Identity that A is A and B is B, we can repeat these letters as often as necessary without altering the logical effect of our proposition. Now if A is BA, then by the Law of Contradiction the combination Ab, which means that A is b, is inconsistent and we make a figure 1 in that section. There is no other combination in the Frame which is inconsistent with the premise.

If we read our premise backward, BA is A, there is no other combination in the Frame which is inconsistent with it.

We thus have the following combinations left in the Reasoning Frame as consistent: (1) AB, (2) aB, (3) ab. We can read them as follows: A is B, which means "Iron is metallic-iron;" b is a, which means "what is not-metal is not-iron." Now in the case of B we have two combinations in which B appears, viz: BA and Ba, so that we cannot say that B is A alone, or that B is a alone.

178. When a letter occurs in two combinations, like BA and Ba, we must read it thus: B is either BA | Ba, (the sign | means "or"), and we can translate it thus: "Metal is either metallic-iron or metallic not-iron," or in popular language "Metal is either iron or not-iron." Of course we know by the Law of the Excluded Middle that metal is either iron or not-iron, so that this information given to us in the Reasoning Frame about B, is of no especial value. It is, however, a true definition of B and it saves us from drawing any wrong inferences in regard to B.

The letter a also appears in two combinations, aB and ab. Of course we cannot say that a is B alone, or that a is b alone, but we must read it a is either aB, | ab, which means that what is not-iron is either metal or not-metal.

179. The only conclusions which we can regard of special value are those which are not in the disjunctive form, such as, b is a, in the present case.

180. The old logic says that from the universal affirmative

proposition all A is B, we can infer the particular proposition "Some A is B." This is extra logical.

The old logic in order to sustain its position that some A is an inference from all A, says that "some" means "some and it may be all." This is an arbitrary meaning.

181. The universal negative proposition in the old logic is symbolized thus: "No A is B." It means that the object A has the name not-B.

182. In reading our results where the subject is repeated in the predicate it is optional with us whether to translate it into concrete words or not.

183. The particular affirmative proposition of the old logic has the word "some" in the subject, and the predicate term is generally undistributed, that is the whole of it is not taken, it is therefore indefinite. "Some men are wise" is a sample of the particular indefinite class. Now when we try to realize this proposition in our minds and ask what does the word "Some" in the subject mean, the answer must be that it means the wise men who are referred to in the predicate. "Some men" therefore means in this case some men who are wise or some wise men. And if we ask what the adjective "wise" in the predicate qualifies, the answer must be that it qualifies the "Some men" referred to in the subject. Therefore the proposition "Some men are wise" really means "wise men are wise men."

If we let A = men

B = wise

then the proposition could be stated,

$$AB = AB$$

184. In such absurd propositions as "Some horses are dogs," the proposition "Some horses are dogs" means really "dog-horses are dog-horses." The proposition is true although we cannot realize the idea of some dog-horses. No one can deny the proposition that "dog-horses are dog-horses" even if he has not the slightest idea what the term "dog-horse" means. As we have said before, logic has nothing to do with the truth of our premises, its sole function is, given the premises to

deduce all the latent and implied meanings contained in them.

185. It seems strange that the old logic should have given such a prominent place to particular affirmative propositions in which the subject is qualified by the word "some," and should have rejected the equally valid propositions in which the subject is qualified by the adjectives "few," "many," or "most." There is no reason in this arbitrary exclusion of "few," "many" and "most." To my mind, "few," "many" and "most," are just as good words as "some." If "few, many and most" are to be rejected, certainly "some" should accompany them.

186. Particular negative propositions are admitted by the old logic as valid logical propositions and are given a prominent place in that system. "Some elements are not metals" may be taken as an example of this type of proposition. When we come to ask ourselves what do we mean by the term "Some elements" described in the premise, the answer must be that we mean by "Some elements" the not-metals referred to in the predicate. The subject of the proposition therefore really means "Elements which are not metals," or to put it in other words, "elementary not-metals." Now it is clear that the term "not-metals" used in the predicate refers to the "Some elements" used in the subject. To make our proposition definite, therefore we must put it in this form: "Elementary not-metals are elementary not-metals. If we let

$$A = \text{elements}$$

$$b = \text{not-metals}$$

the proposition can be stated in the form of

$$Ab = Ab$$

CHAPTER VII.

SIMPLE CATEGORICAL PROPOSITIONS INVOLVING THREE TERMS.

187. We have seen that the two terms A and B will yield four combinations, viz.: AB, Ab, aB and ab. Now when we add a third term C, we can divide each of these combinations into C and c. This will give us the following combinations:

- (1) ABC
- (2) ABc
- (3) AbC
- (4) Abc
- (5) aBC
- (6) aBc
- (7) abC
- (8) abc

Thus three terms give us eight combinations, which we have obtained by dividing each of the four combinations of AB, Ab, aB, and ab, into two divisions, the division which is C, and the division which is c.

188. There are two ways in which we can make a diagram containing eight sections. A square which is divided into two sections by four sections will of course give us eight sections.

Our square may have its two sections run perpendicularly and its four sections run horizontally, or it may have its four sections run perpendicularly, and its two sections run horizontally. There is no logical difference between the two plans, but I have found the latter plan a little more convenient in practice. Let us make one of that kind first. Make a square, thus:

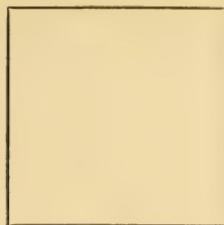


Fig. 22.

Then divide it into four perpendicular sections by three equidistant vertical lines, thus:

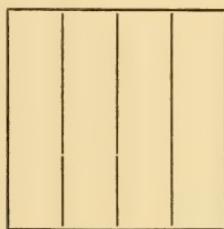


Fig. 23

Then bisect these four sections by a horizontal line, thus:

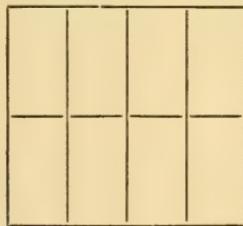


Fig. 24.

We have now eight sections, representing our eight combinations of the ABC class.

189. In order to letter these combinations we write over the first or left-hand file-section the letters AB; next, change the letter B into b and write over the second file Ab; next, change the letter A and write over the third file aB; next, change the letter B and write over the fourth file ab. It will be understood that these file letters run all the way down the

files. Now at the right hand of the upper row of sections write C, and at the right hand of the lower row of sections write c.

The row-letters C and c run all the way across, and in whatever sections the file-letters and the row-letters meet we write those letters in that section, thus:

AB	A _b	aB	ab	
ABC	AbC	aBC	abC	C
ABc	A _{bc}	aEc	abc	c

Fig. 25.

190. In making an ABC Frame the other way, we make a square, then we bisect it by a vertical line; over one section we write A and over the other section we write a; then we divide the square into four rows by drawing three equi-distant horizontal lines, and at the right of the top row we write the letters BC; at the right of the second row we write the letters Bc, at the right of the third row we write the letters bC, and at the right of the fourth row we write the letters bc.

191. In this process we have changed a letter at a time, and we letter each section with the letters which meet in that section. Our diagram has this appearance:

A	a	•
ABC	aBC	BC
ABc	aEc	Bc
AbC	abC	bC
A _{bc}	abc	bc

Fig. 26.

192. Of course by this system every section has a different

combination. If we change one letter at a time it will be impossible for us to have two combinations which are alike. These two diagrams are of equal logical value, but I think that the first one given explains the theory of logical division more clearly than the other, and in practice I have found it more convenient.

193. There is no iron rule about lettering our sections. Any plan will do which will give us a different combination for each section. Instead of commencing with AB, we might have commenced with ab, and then proceeded to change a letter at a time. Or, instead of commencing with BC in the second example, we might have commenced with bc. In fact we can commence with any two letters we like, provided that we change a letter at a time, and letter each section with the letters which meet in that section. Neither is it absolutely necessary that we make a square, a rectangle will do as well. The important matter is that we get our eight sections.

194. Prof. Jevons says that we can read the combinations in an ABC Frame by changing the order and by taking either one, two, or three terms at a time in two hundred and fifty-five different ways, or, in other words, there are two hundred and fifty-five imaginable propositions in the Reasoning Frame for three terms. In a frame for four terms—ABCD—there are sixty-five thousand five hundred and thirty-five conceivable propositions, and in a frame for five terms—ABCDE—there are four billion two hundred and ninety-four million nine hundred and sixty-seven thousand two hundred and ninety-five different readings. These different readings may be called alternative readings; which of them will eventually remain will depend upon the premises. An alternative predicate leaves the subject in doubt. It means that the subject is one or more of the alternatives; this, or the other, or both, or neither, but it does not say positively which it is.

195. It will be noticed in the ABC Frame that there are four A's; viz.: ABC, AbC, Abc, ABe, and we can say that A is either ABC | ABe | AbC | Abc. Likewise, each of the other

letters B, C, a, b, c, has four combinations, and we can read them alternately as we did in the case of A, thus, c is either cBA | cbA | cBa | cba.

196. Again, if we like, we can drop in reading any letter and its opposite we choose, thus in the case of c we can drop the A's and a's and then our definition is c is either cB | cb. Or, again, in reading we can drop the A's, B's, a's and b's and then we can say c is c.

197. We can read the letters in any order we please. If a boy has three names, Andrew, Benjamin and Charles, so far as logic is concerned it does not make the slightest difference in what order we use them.

198. And again, we can repeat any letter or any combination of letters as often as we please, because we know by the Law of Identity that A is always A, and AB is always AB, etc., and the repetition of a letter or a combination of letters is always allowable.

199. Suppose we get a conclusion which reads:

ABC. Now, in reading we can drop the B and say A is C, and C is A, or we can drop the A and say B is C and C is B. To take a concrete example,

Let A = man,

B = rational-animal,

C = reasoning-living-being.

ABC can then be read "Man is a rational-animal and a reasoning-living-being," or, we can drop the "rational-animal" and say "Man is a reasoning-living-being, and a reasoning-living-being is a man." Or we can drop the term "man" and say "A rational-animal is a reasoning-living-being, and, a reasoning-living-being is a rational-animal." What we have in ABC is three names for the one thought; we can use all three of the names, or any two of them, or any one of them just as we choose.

200. In the syllogism what is called the "middle term" is always dropped in the conclusion. Therefore the syllogism

does not give us all the information which it can give.

201. Let us take the following propositions and ascertain what inferences can be deduced from them:

(1) A man is a rational animal.

(2) A rational animal is a reasonable living being.

Let $A = \text{man}$,

$B = \text{rational animal}$,

$C = \text{reasoning-living-being}$.

We can assume

(3) A rational animal is a man.

(4) A reasoning-living-being is a rational animal.

Our premises can then be stated, thus:

(1) $A = B$

(2) $B = A$

(3) $B = C$

(4) $C = B$

Should any one doubt that a rational-animal is a man we can prove it by using the Law of the Excluded Middle, thus: A rational animal is a man or not-a-man. If a rational animal is not-a-man then, since a man is a rational-animal, a man would be not-a-man, which is impossible. Therefore, a rational-animal is a man. And similarly with the proposition a reasoning-living being is a rational animal.

Now make an ABC Reasoning Frame by drawing a rectangle, then by dividing it into four files, and then by bisecting the four files so as to make two rows. Then letter the files, rows and sections, as in the first example:

AB	Ab	aB	ab	
ABC	AbC	aBC	abC	C
ABc	Abc	aBc	abc	c

Fig. 27.

Our first premise is A is B. Of course all the combinations of A with b will be inconsistent by the Law of Contradiction, because if the proposition A is B is true, the proposition A is b, cannot be true at the same time. Therefore, we eliminate the combinations AbC and Abc by making a figure 1 in those sections.

Our second premise is that B is A, and therefore, all the combinations of B with a will be inconsistent, because if B is A, B is a is contradictory, therefore, we eliminate the combinations aBC and aBc by making a figure 2 in those sections.

Our third premise is that B is C; now any combination of B with c will be inconsistent because it implies that B = c, hence, we eliminate the combinations ABC and aBc by making a figure 3 in those sections.

Our last premise is that C is B, and of course any combination that says that C is b is an inconsistent combination, hence, we eliminate the combinations Cba, and CbA, by making a figure 4 in those sections.

We have now remaining in the Frame two combinations ABC and abc. ABC contains our original premises; abc automatically remains in the Frame. We can read ABC in any order we please and we can use any number of letters we choose. We can say A is BC, or AB is C, or BC is A, or CB is A, or CA is B, etc. Similarly with abc. To use concrete terms, we can say, "What is not-a-man is not-a-rational-animal and is not-a-reasoning-living-being," or, we can say, "What is not-a-man and what is not-a-rational-animal is not-a-reasoning living-being." or, we can say, "What-is-not-a-rational-animal and what is not-a-reasoning-living-being is not-a-man," or, we can say, "What is not-a-reasoning-living-being and what is not-a-man is not-a-rational-animal."

Thus, with one operation we have exhausted all the information contained in our premises and have brought to light a great many hidden and latent meanings which were contained in them.

202. Every proposition contains latent meanings, and the

problem of logic is to discover them. The beauty of this system is that it reduces the process of reasoning to a routine process of a few mechanical operations; viz.: the drawing of the squares, the lettering of the sections, the stating of the propositions and the eliminating of the inconsistent combinations. Of course, after we have eliminated all the inconsistent propositions, the consistent ones must remain.

203. In eliminating the inconsistent combinations we pay attention only to our signs. It is unnecessary to try to realize the concrete meaning of our signs during the eliminating process. From the premises in this case, the old logic would draw only one conclusion; viz.: A is C, "A man is a reasoning-living-being." So that in its ability to draw every possible conclusion which can be drawn from the premises by one operation, our system is greatly superior to the old logic.

204. From the premises A is B, B is A, B is C and C is B, we derive the conclusions which can be read in the combination abc. By the old logic this process is called deduction, and it is said to be a very different process from induction.

205. In induction the problem is—given the conclusions to obtain the premises. Now let us take some of the conclusions contained in abc and use them as premises and ascertain what the result will be by our system. Let us take these conclusions:

- (1) "What is not-a-man is not-a-rational-animal."
- (2) "What is not-a-rational-animal is not-a-reasoning-living-being."

We can read these propositions backward, because the predicates are synonymous with the subject, and thus obtain two more conclusions; viz.:

- (3) "What is not-a-rational-animal is not-a-man."
- (4) "What is not-a-reasoning-living-being is not-a-rational-animal."

It is always necessary to make our propositions read backward as well as forward.

Let a = not-man,

$b = \text{not-rational-animal}$,
 $c = \text{not-reasoning-living-being}$.

Our premises can be stated symbolically thus:

- (1) $a = b$
- (2) $b = a$
- (3) $b = c$
- (4) $c = b$

Make an ABC diagram and letter it, thus:

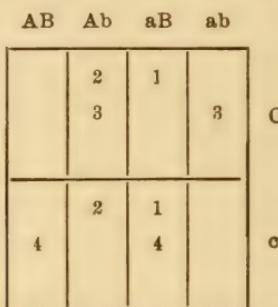


Fig. 28.

Now, as a is b , then every combination containing aB will be inconsistent by the Law of Contradiction; therefore, the combinations aBc , and aBC , are inconsistent with premise No. 1, and we make a figure 1 in those sections, which indicates that they are inconsistent with premise No. 1.

We put figures in the sections eliminated to show which premises those sections are inconsistent with.

Now, as b is a , the combinations containing bA will be inconsistent by the Law of Contradiction, therefore, the combinations bAC and bAc are inconsistent and we eliminate them by making the figure 2 in those sections.

And, again, as b is c , any combination having bC , will be inconsistent, therefore, the combinations bCA and bCa are inconsistent and we eliminate them by making a figure 3 in those sections.

And, lastly, c is b . This means that c can combine with b only, and that any combination of c with B is inconsistent, therefore, the combinations cBA and cBa are inconsistent and

we eliminate them by making a figure 4 in those sections.

The following combinations automatically remain in the diagram; viz: abc and ABC; abc is our premises, and from these premises we derive the conclusions contained in the combination ABC.

From among the various readings of this combination we can take the following:

- (1) A is B
- (2) B is C
- (3) A is C

which we can translate thus:

- (1) "A man is a rational animal."
- (2) "A rational animal is a reasoning living being."
- (3) "A man is a reasoning living being."

206. We thus see that our system works equally well backward and forward, and that, so to speak, it proves itself. From the original premises in ABC we derive the conclusions in abc, and from the conclusions in abc we derive the original premises in ABC. It also proves that deduction and induction are merely reverse operations, and that in a true logic we can reason from the conclusions to the premises just as easily and just as accurately as we can reason from premises to conclusions.

207. According to the old logic propositions containing negative terms were called, incorrectly, negative propositions, and it was a rule of the old logic that from two negative propositions no conclusions could be deduced. According to the old logic the propositions, "What is not a man is not a rational animal," and "What is not a rational animal is not a reasoning living being," would be negative propositions, and, therefore, no conclusions could be deduced from them. But the example which we have just worked demonstrates that this is another of the mistakes, as I think, of the old logic.

208. Let us take these propositions:

- (1) "What is not fit to learn is not proper to teach."

(2) "What is not proper to teach ought not to be printed," and ascertain what conclusions we can derive from them. As these are synonymous propositions we can read them backward and thus get two more propositions, viz.:

- (3) "What is not proper to teach is not fit to learn."
- (4) "What ought not to be printed is not proper to teach."

Let $a =$ what is not-fit-to-learn,

$b =$ what is not-proper-to-teach,

$c =$ what ought not-to-be-printed.

Always let negative signs stand for negative terms. Our propositions can be stated, thus:

- (1) $a = b$
- (2) $b = c$
- (3) $b = a$
- (4) $c = b$

Make an ABC Reasoning Frame by dividing a square into four files and the four files into two rows, and then letter the files, rows and sections, thus:

AB	Ab	aB	ab
2	1	2	
3			
\BC	AbC	aBC	abC
4	3	1	
ABe	Abc	aBc	abc

Fig. 29.

Then, as a is b , any combination of a with B will be inconsistent. We therefore eliminate the combinations aBC and aBe by making a figure 1 in those sections.

Next, as b is c , every combination containing b and C will be inconsistent and we eliminate it by making a figure 2 in those sections. This will cause us to eliminate the sections AbC and abc .

Again, as b is a , any combination of b with A will be incon-

sistent. The combinations AbC and Abc are therefore inconsistent and we eliminate them by making a figure 3 in those sections.

And, lastly, c is b. Any combinations, therefore, which say that c is B are inconsistent and we eliminate them by making a figure 4 in those sections. The combinations ABC and abc are inconsistent. The combinations abc and ABC automatically remain in the Reasoning Frame. The combination abc contains the premises, and the combination ABC contains the conclusions.

We can read ABC in a variety of ways; among others, we can read A is B, B is C, and A is C, thus:

- (1) "What is fit to learn is proper to teach."
- (2) "What is proper to teach ought to be printed," and
- (3) "What is fit to learn ought to be printed."

By a previous example we have already learned that if we take these conclusions, A is B, and B is C, for premises, we shall get as conclusions the inferences contained in abc.

209. Let us take these propositions (from Jevons):

- (1) London is the capital of England.
- (2) London is the most populous city in the world.

We can read these propositions backward and obtain two more propositions.

- (3) The capital of England is London.
- (4) The most populous city in the world is London.

We should always bear in mind that it is necessary to state all the *prima facie* meanings of our propositions.

Let A = London,

B = the capital of England,

C = the most populous city in the world.

Our premises can then be stated, thus:

- (1) A = B
- (2) A = C
- (3) B = A
- (4) C = A

Next make an ABC diagram and letter it as hereinbefore described.

AB	Ab,	aB	aB	
	1	3		
		4	4	C
2	1	3		c

Fig. 30.

Now, as A is B, any combination of A with b, which means A is b, is an inconsistent combination. The combinations AbC and Abc are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, as A is C, all the combinations containing Ac are inconsistent combinations, and we eliminate them by making a figure 2 in those sections. The combinations ABe and Abc are inconsistent.

Again, as B is A, the combinations containing Ba are inconsistent, and we eliminate them by making a figure 3 in those sections. The combinations aBC and aBc are inconsistent.

And, lastly, as C is A, the combinations containing Ca are inconsistent, and we eliminate them by making a figure 4 in those sections. The combinations aBC and abc are inconsistent.

The following combinations automatically remain, viz.: ABC and abc. The combination ABC contains our premises; the combination abc contains our conclusions. The conclusion which the old system would draw is, "The capital of England is the most populous city in the world." This is contained in the combination ABC.

But besides the conclusion which the old logic would draw we have all the conclusions contained in the combination abc, and we can read,

- (1) "What is not London is not the capital of England."

(2) "What is not London is not the most populous city in the world."

(3) "What is not the capital of England is not the most populous city in the world."

(4) "What is not the most populous city in the world is not London and is not the capital of England," etc.

210. Let us take these propositions (from Jevons):

(1) The substance of least density is hydrogen.

(2) The substance of least atomic weight is hydrogen.

As these are synonymous propositions we can read them backward, and obtain two more propositions, thus:

(3) Hydrogen is substance of least density.

(4) Hydrogen is substance of least atomic weight.

Let A = substance of least density,

B = substance of least atomic weight,

C = hydrogen.

Our propositions can be stated, thus:

$$(1) A = C$$

$$(2) B = C$$

$$(3) C = A$$

$$(4) C = B$$

Make an ABC diagram and letter it as usual, thus:

AB	Ab	aB	ab	
	4	3	3	
			4	C
1	1	2		
2				c

Fig. 31.

The following combinations are inconsistent with A is C and we eliminate them by making a figure 1 in those sections; viz: ABC, Abc.

The following combinations are inconsistent with B is C, and we eliminate them by making a figure 2 in those sections; viz.: ABe and aBe.

The following combinations are inconsistent with C is A, and we eliminate them by making a figure 3 in those sections; viz.: aBC and abC.

Lastly, as C is B, every C which is combined with b is inconsistent and we eliminate it by making a figure 4 in those sections. The combinations AbC and abC are inconsistent.

The following combinations remain: ABC and abc. In ABC we have our original premises, and of course we have several other readings besides; for instance, we can read A is B, B is A, and translate thus:

- (1) "The substance of least density is the substance of least atomic weight."
- (2) "The substance of least atomic weight is the substance of least density."

In abc we can read,

- (1) "What is not the substance of least density is not the substance of least atomic weight."
- (2) "What is not the substance of least atomic weight is not hydrogen."
- (3) "What is not hydrogen is not the substance of least density, etc.

211. Let us take two of the conclusions obtained in the last example for premises.

- (1) What is not the substance of least density is not the substance of least atomic weight.
- (2) What is not the substance of least atomic weight is not hydrogen.

Let a = what is not substance of least density,

b = what is not substance of least atomic weight,

c = not hydrogen.

212. As the subject and predicate are synonymous terms we can read the propositions backward and get two additional premises. The reason why we wish to get additional premises

is this: In a Reasoning Frame the various combinations represent alternative propositions, and every premise given us enables us to strike out some of these alternative propositions, and the more alternative propositions we can strike out, the more definite will be the propositions which remain.

213. A proposition in the alternative does not give us that definite information of a subject which we desire. Our object is to get rid of as many alternatives as we can, so that the information which remains in the Frame shall be positive and definite, and every additional premise which enables us to strike out one or more of our alternative combinations is a help to this end.

Our premises can be stated symbolically as follows:

- (1) $a = b$
- (2) $b = c$
- (3) $b = a$
- (4) $c = b$.

Now, if a is b , then every combination of aB , which means that a is B , is inconsistent by the Law of Contradiction. The combinations aBC and aBc are inconsistent and we eliminate them by making a figure 1 in those sections:

AB	Ab	aB	ab
2	1	2	C
3			
4	3	1	c
	4	4	

Fig. 32.

And, if b is c , then every combination of bC is inconsistent. The combinations AbC and abC are inconsistent and we eliminate them by making a figure 2 in those sections.

And if b is a , then the combinations bA are inconsistent and we eliminate them by making a figure 3 in the sections marked AaC and Abc .

And if c is b, then the combinations ABC and aBc are inconsistent and we eliminate them by making a figure 4 in those sections.

The combinations abc and ABC automatically remain in the Frame; abc contains the premises and ABC contains the conclusions. In the combination abc we can find readings which were not stated in our premises, and these readings can properly be called conclusions. For instance, we can read a is c and c is a, which being translated read,

- (1) "What is not-the-substance-of-least-density is not-hydrogen."
- (2) "What is not-hydrogen is not-the-substance-of-least-density."

In addition to these conclusions, we have also the conclusions contained in the combination ABC, and from that combination we can read,

- (1) "Hydrogen is the substance of least density."
- (2) "Hydrogen is the substance of least atomic weight."
- (3) "The substance of least atomic weight is the substance of least density," etc.

This example demonstrates again the fact that we can reason from premises to conclusions and from conclusions to premises by our system with equal ease and accuracy.

214. Let us now take some inconsistent propositions and ascertain what the result will be.

- (1) The Queen of England is not the Empress of India.
- (2) Queen Victoria is the Queen of England.
- (3) The Empress of India is Queen Victoria.

Let A = Queen of England,

b = not-the-Empress of India,

C = Queen Victoria,

B = Empress of India,

The second and third propositions are synonymous and can be read backward; thus giving us two more propositions; viz.:

- (4) The Queen of England is Queen Victoria.

(5) Queen Victoria is the Empress of India.

174. Make a square and divide it into eight sections and letter them as heretofore:

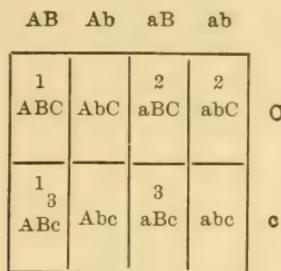


Fig. 33.

Our premises can be stated as follows:

- (1) $A = Ab$
- (2) $C = A$
- (3) $B = C$
- (4) $A = C$
- (5) $C = B$

Now, if A is Ab , then the combinations which contain AB are inconsistent. The combinations ABC and ABc are inconsistent and we eliminate them by making a figure 1 in those sections.

And if C is A , then the combinations containing Ca are inconsistent. We therefore eliminate the combinations aBC and abC by making a figure 2 in those sections.

Again, if B is C , then the combinations containing Bc are inconsistent. We therefore eliminate the combinations ABc and aBc by making a figure 3 in those sections.

By looking at our diagram, we now see that the following combinations have been eliminated, viz: ABC , aBC , ABc and aBc . There are no other B 's in the Frame; all the B 's having been eliminated, the diagram tells us that our premises are inconsistent and that it is of no use for us to eliminate any more combinations. The diagram says that our propositions have

affirmed that there was an Empress of India, and there was not an Empress of India.

215. Having discovered an inconsistency in our premises, it is of no use for us to proceed in the working of the problem set before us, viz.: to find the conclusions which could be deduced from the given premises.

216. The Law of Opposites told us that we could not have an idea without its opposite; that if we have the idea of straightness, for instance, we must have the idea of not-straightness; if we have the idea of light, we must have the idea of not-light, and that if we posit one, we posit the other. And, consequently, if we eliminate one, we must eliminate the other. Now, as we had no B's left in our Reasoning Frame, we must eliminate all the b's, because we cannot have one without the other. If we eliminate the b combinations which are left in the Frame, viz.: AbC, Abc and abc, there is nothing left in the Frame for us to reason about.

And, again, as C = B by our premises, if there is not any B, then there can not be any C. And, as A = C by our premises, if there is not any C, then there can not be any A.

217. Whenever the premises are consistent and our eliminations have been correctly performed, every letter both positive and negative, will remain in the Reasoning Frame and appear in the conclusions. The absence of a single letter tells us that our premises are inconsistent.

218. Let us take another example of inconsistent propositions:

- (1) Grover Cleveland is President of the United States.
- (2) The President of the United States is Commander in Chief of the Army of the United States.
- (3) The Commander in Chief of the Army of the United States is not Grover Cleveland.

In (1) and (2) the subjects and predicates are synonymous and we can read them backward and get two additional propositions, viz.:

- (4) The President of the United States is Grover Cleveland.

(5) The Commander in Chief of the Army of the United States is the President of the United States.

We can make (3) a synonymous proposition by adding the subject to the predicate, and it will then read "The Commander in Chief of the Army of the United States is the Commander in Chief of the Army of the United States not Grover Cleveland." In doing this we shall have to disregard the laws of rhetoric, but, as we have said before, the science of logic is independent of other sciences, and in solving its problems it must be allowed to make its own rules and have its own methods. Having converted (3) into a synonymous proposition, we can read it backward.

Let $A =$ Grover Cleveland,

$B =$ The President of the United States,

$C =$ Commander in Chief of the Army of the United States,

$a =$ not-Grover Cleveland.

Our premises can be stated as follows:

- (1) $A = B$
- (2) $B = C$
- (3) $C = aC$
- (4) $B = A$
- (5) $C = B$

Draw a rectangle and divide it into eight sections and letter them as heretofore:

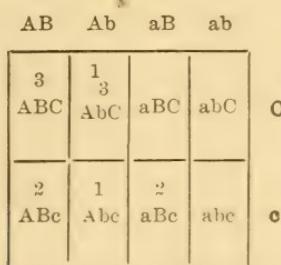


Fig. 34.

Now, if A is B , then the combinations which contain Ab are inconsistent. To put it in other words, wherever we have an

A, we must find B with it; if we find an A with b we must eliminate the combinations. We therefore eliminate the combinations AbC, and Abc by making a figure 1 in those sections.

Again, if B is C, then every combination Bc is inconsistent. We therefore eliminate the combinations ABC and aBc by making a figure 2 in those sections.

Again, if C is aC, then any combination containing AC is an inconsistent combination. We therefore eliminate the combinations ABC and AbC by making a figure 3 in those sections.

Now, all our A's have disappeared from the Reasoning Frame. This tells us that our premises are inconsistent and it is useless for us to proceed.

EXAMPLES FOR PRACTICE.

219. What inferences can be deduced from the following pairs of premises:

- (1) $aB = aBC$
 $ab = abc$
- (2) $aB = aBc$
 $ab = abC$
- (3) $A = ABC$
 $c = cab$
- (4) $C = CA$
 $c = cab$
- (5) $C = Ca$
 $c = cAB$
- (6) $aB = aBC$
 $A = Abc$

CHAPTER VIII.

INDUCTION.

220. We have already learned that in logic a proposition is two or more names, titles, designations or descriptions of the same idea, and that we can represent these names by letters. And that a square represents our Universe of Discourse or Field of Thought, and that the combinations contained in the squares represent all the possible propositions which can be made from the terms of our premises.

221. In inductive logic, so called, the problem is to find premises which will produce the conclusions which are given to us. In the last chapter we have seen that, given the premises contained in ABC we can get the conclusions contained in abc; and given the premises in abc we can get the conclusions contained in ABC.

222. The problem which we are now to consider is—given as conclusions ABC and abc, what premises will produce all the conclusions contained in ABC and abc. From ABC we can get propositions reading A is B, A is C, etc. These propositions are called definitions. A is B is a definition of A, B is C is a definition of B, b is c is a definition of b. A complete definition of A in ABC would be, A is BC, and a complete definition of a in abc is, a is bc.

223. We can always define a letter-term contained in only one conclusion by saying that it is the other letters contained in the conclusion.

Now, if A is BC, it is clear that all the other combinations of A, viz.: ABc, AbC and Abc, are inconsistent combinations. If A is BC, there is no other definition of A in the ABC Reasoning Frame. This is just as true as the axiom that a thing cannot be in two places at the same time.

Similarly, if the definition of a is, a is bc, then the other com-

binations of a are inconsistent. Thus: aBC, aBc and abC are inconsistent. To illustrate this concretely, let us suppose that

A = man,

B = rational-animal,

C = reasoning-living-being,

a = not-man,

b = not-rational-animal,

c = not-reasoning-living-being.

With these terms we can make a number of propositions which can be stated symbolically, thus:

$$(1) A = B$$

$$(2) B = C$$

$$(3) A = C$$

$$(4) b = a$$

$$(5) c = b$$

$$(6) c = a$$

These can be translated as follows:

- (1) Man is a rational animal.
- (2) A rational animal is a living reasoning being.
- (3) Man is a reasoning living being.
- (4) What is not-a-rational-animal is not-a-man.
- (5) What is not-a-reasoning-living-being is not-a-rational-animal.
- (6) What is not-a-reasoning-living-being is not-a-man.

Of course, we could get several other propositions from these premises, but the above are sufficient. The problem is, What premises will produce all these conclusions? Now, from these conclusions let us get a definition of the positive term man and of its negative not-man. A complete definition of the positive term man will be; A man is a rational animal and a reasoning living being. And a complete definition of the term not-man, is What is not-a-man is not-a-rational-animal and not-a-reasoning-living-being. We can state them symbolically thus:

$$(1) A = BC$$

$$(2) a = bc$$

Make a rectangle, dividing it into eight sections, and letter as heretofore:

AB	Ab	aB	ab	
ABC	1 AbC	2 aBC	2 abC	C
ABc	1 Abc	2 aBc	abc	c

Fig. 35.

Now, if A is BC, then the combinations ABC, AbC, Abc, are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if a is bc, then the combinations abC, aBC and aBc, are inconsistent, and we eliminate them by making a figure 2 in those sections.

The conclusions contained in ABC and abc automatically remain in the Frame. And, thus, from the definitions of A and a we have obtained all the conclusions which are contained in those combinations. Again, if we take these definitions,

- (1) A rational animal is a man and a reasoning living being,
 - (2) What is not-a-rational-animal is not-a-man and is not-a-reasoning-living-being,
- we shall get the same results. We can state these definitions symbolically, thus:

$$(1) B = AC$$

$$(2) b = ac$$

Make a rectangle, divide it into eight sections and letter as heretofore:

AB	Ab	aB	ab	
ABC	2 AbC	1 aBC	2 abC	C
ABc	1 Abc	2 aBc	1 abc	c

Fig. 36.

If B is AC, then the combinations aBc, BA_c, and aBC are inconsistent and we eliminate them by making a figure 1 in those sections.

And if b is ac, then the combinations abC, AbC and Abc are inconsistent and we eliminate them by making a figure 2 in those sections.

The combinations ABC and abc automatically remain in the Frame, and thus from the complete definition of the positive term, "rational animal," and the complete definition of its negative, "not-rational-animal," we have obtained all the conclusions which can be read in the combinations ABC and abc.

224. From this we can deduce the general rule: That the complete definitions of a positive term and its negative which can be obtained from the conclusions, will give us all the premises.

225. There are other ways of obtaining the premises for given conclusions, but the above is the simplest and easiest method. Another way to obtain the premises for given conclusions is, when the conclusions are categoricals, to get a definition of every one of the positive letters, or a definition of every one of the negative letters, and then to use one or the other set of definitions for premises. Thus from the conclusion ABC we can get the definitions:

- (1) A = ABC
- (2) B = BAC
- (3) C = CAB

Make an ABC diagram and letter it as heretofore:

AB	Ab	aB	ab
ABC	$\frac{1}{3}$ AbC	$\frac{2}{3}$ aBC	$\frac{3}{3}$ abC
ABc	1 Abc	2 aBc	ab abc
•	•	•	•

Fig. 37.

Now if A is BC we must eliminate the combinations ABC, AB_c, Abc. Make a figure 1 in those sections.

Now, if B is AC, we must eliminate the combinations ABC, aBC and aBc. Make a figure 2 in those sections.

Now, if C is AB, then the combinations AbC, abC, aBC are inconsistent and we eliminate them by making a figure 3 in those sections.

The combinations ABC and abc automatically remain. Thus, from the complete definitions of all the positive terms we have all the conclusions which can be read in the combinations ABC and abc.

226. The reader will notice that, in this case, we have not been reading our propositions backward. The reason for our not doing so is because it would be a useless proceeding. That is, by reading these definitions backward, we cannot eliminate any more propositions than we can by simply reading them forward. Let us illustrate this rule further by taking definitions of the negative terms.

Let (1) $a = abc$

(2) $b = bac$

(3) $c = cab$

Make an ABC diagram and letter it as heretofore:

AB	Ab	aB	ab	
ABC	2	1	1 2	C
ABc	3	2	1	
Abc	3	3	abc	c
aBc	abc	aBc	abc	

Fig. 38.

Now, if a is bc , then the combinations abC , aBC , aBc , are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if b is ac , then the combinations abC , AbC , Abc , are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if c is ab , then the combinations ABc , Abc , aBc are inconsistent, and we eliminate them by making a figure 3 in those sections.

The combinations ABC and abc automatically remain, and thus from the complete definitions of the negative terms used in the given conclusions, we can get premises which will produce the given conclusions.

227. The old logic did not profess to have any method for obtaining the premises for given conclusions. So far as I know, Prof. Jevons was the first logician to undertake to solve the problem. Ordinarily, he is a very clear writer on the old logic, and his text book on logic is deservedly popular. But in treating of Induction he has made mistakes. We have seen that the Inductive problem is quite as easy of solution as the Deductive problem. But in Jevon's *Principles of Science*, p. 121, we read, "It must be allowed that Inductive investigations are of a higher degree of difficulty and complexity than any questions of Deduction; and it is this fact, no doubt, which led some logicians, such as Francis Bacon, Locke, and J. S. Mill to erroneous opinions concerning the exclusive importance of Induction. In Induction all is inverted. The truths to be ascertained are more general than the data from which they are drawn. The process by which they are reached is analytical and consists in separating the complex combinations in which natural phenomena are presented to us and determining the relations of separate qualities.

Given events obeying certain unknown laws, we have to discover the laws obeyed. Instead of the comparatively easy task of finding what effects will follow from a given law, the effects are now given and the law is required."

228. The reader will have observed that in our system it is quite easy to state our conclusions, *i. e.*, effects, symbolically and from those symbolical statements of conclusions, *i. e.*, effects, to obtain definitions either of a positive term and its negative, or of each of the positive terms or of each of the negative terms, and by using these definitions as premises to obtain the given conclusions, *i. e.*, effects.

229. Again, Prof. Jevons says, "Differentiation, the direct process (*i. e.*, Deduction), is always capable of performance by fixed rules, but as these rules produce considerable variety of results, the inverse process of Integration (*i. e.*, Induction), presents immense difficulties, and in an infinite majority of cases surpasses the present resources of mathematicians. There are no infallible and general rules for its accomplishment and it must be done by trial, guess-work, or by remembering the results of Differentiation and using them as a guide." The Inverse process does not present "immense difficulties." It is quite as easy, as we have shown, as Deduction. The rules for finding premises for conclusions are as "general and infallible" as the rules for finding conclusions from premises. Guess-work has no place in any logical system.

230. On p. 125, Principles of Science, he says to the reader, "To test the facility with which he can solve this Inductive problem, let him casually strike out any of the combinations of the fourth column of the logical alphabet and say what laws the remaining combinations obey. Observing that every one of the letter-terms and their negatives ought to appear, in order to avoid self contradiction in the premises."

231. We have not reached in this work combinations containing four terms, but we can try the experiment equally well with combinations involving three terms. Let us make an ABC diagram, as usual, thus:

\mathbf{AB}	\mathbf{Ab}	\mathbf{aB}	\mathbf{ab}	
X		X	X	C
ABC	AbC	aBC	abC	
	X		X	c
ABC	Abc	aBe	abc	

Fig. 39.

And let us casually strike out these five combinations, ABC, Abc, aBC, abC, abc. Make an X in those sections. The combinations which remain are ABc, AbC, aBe. These represent the given conclusions. The problem is, What premises will produce these conclusions? This is a very easy task. All we have to do is to obtain from these three conclusions definitions of A and a, or of B and b, or of C and c, and any one of these pairs will furnish the required answer.

The reader will notice that there are two combinations in which A occurs, viz.: ABc and AbC. In this case we cannot say that A is either one of these combinations alone, but it is one or the other. The definition of A, therefore, is,

$$A = ABc \mid AbC$$

There is only one combination containing a; the definition of a, therefore, is,

$$a = aBc$$

Now, if A is either ABc or AbC, then the combinations ABC and Abc are inconsistent and we eliminate them.

And if a is aBc, then the combinations aBC, abC, abc, are inconsistent and we eliminate them. The original conclusions ABc, AbC and aBc automatically remain.

Similarly, if we take the definitions of B and b. The definition of B is,

$$B = ABc \mid aBc$$

The definition of b is,

$$b = AbC$$

Now, if B is either ABC or aBc, then the combinations ABC and aBC are inconsistent and we eliminate them.

And if b is AbC, then the combinations Abc, abC and abc are inconsistent and we eliminate them.

And again we have the given conclusions, ABC, AbC and aBc.

232. We have seen early in this chapter that where the given conclusions were ABC and abc, that by getting definitions of each of the positive terms, or of each of the negative terms, and by using either set of definitions for premises, we could obtain the given conclusions. This method succeeds where we have no alternative definitions. But when we have alternative definitions in the given conclusions, then we must either use the process of taking the definitions of a letter and its negative or we must, so to speak, feel our way in getting the required premises. Thus: the definition of a in the case before us is,

$$a = aBc$$

$$b = AbC$$

Make an ABC diagram and letter it as usual, thus:

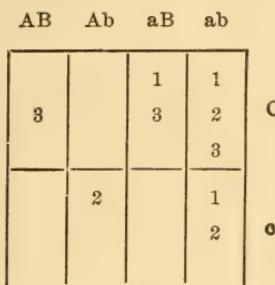


Fig. 40.

Hereafter it will be unnecessary to letter the sections, because, by this time, the reader will understand that each section is supposed to be lettered with the letters which would meet in that section.

Now, if a is aBc, then the combinations aBC, abC and abc are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if b is AbC, then the combinations abC, abc, Abc, are inconsistent, and we eliminate them by making a figure 2 in those sections.

But now the reader will perceive that a definition of c would not cause us to strike out the combination ABC. The definition of c is,

$$c = ABC \mid aBC$$

and the only combinations which are inconsistent with this definition are Abc and abc, and these have already been eliminated.

If, however, we take the definition of C, which is,

$$C = AbC$$

then the combinations ABC, aBC, abC, are inconsistent, and we eliminate them by making a figure 3 in those sections. So that the definitions of a, and b, and C are the premises required.

This tentative method of finding premises for given conclusions can be followed in all cases.

233. Let us make another ABC diagram and letter it as usual, thus:

AB	Ab	aB	ab	
	X 5 1 3		X 1 5	o
X 4 2 3		X 2 4		o

Fig. 41.

Now let us strike out four sections, viz.: ABC, AbC, aBc and abC. In this case the definitions of C and c are,

$$C = ABC \mid aBC$$

$$c = Abc \mid abc$$

If C is ABC or aBC, then the combinations AbC and abC are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if c is $A\bar{B}c$ or $a\bar{B}c$, then the combinations $A\bar{B}c$ and $a\bar{B}c$ are inconsistent and we eliminate them by making a figure 2 in those sections.

Thus, from these premises we have obtained the given conclusions ABC , aBC , $A\bar{b}c$, abc .

Let us try the tentative method of obtaining definitions with this problem.

The definition of A is,

$$A = ABC \mid A\bar{B}c$$

Then the combinations $A\bar{B}C$ and $A\bar{B}c$ are inconsistent and we eliminate them by making a figure 3 in those sections.

The definition of B is,

$$B = ABC \mid aBC$$

Then the combinations $A\bar{B}c$ and $a\bar{B}c$ are inconsistent and we eliminate them by making a figure 4 in those sections.

The definition of C is,

$$C = ABC \mid a\bar{B}c$$

Then the combinations $A\bar{B}C$ and abC are inconsistent, and we eliminate them by making a figure 5 in those sections.

The given conclusions remain. Thus we see that in this case the definitions of ABC , which we obtained from the given conclusions, have furnished us the required premises.

234. Let us make another ABC diagram and letter it as usual and strike out three combinations, thus:

AB	Ab	aB	ab	
X	X			c
2	1			
•	X			c
	1			

Fig. 42.

Let us strike out AbC , $a\bar{B}c$, $a\bar{B}C$. The definitions of B and b are,

$$B = ABC \mid ABC$$

$$b = Abc \mid abc \mid abC$$

If B is ABC or A \bar{B} c, then the combinations aBC and aBc are inconsistent, and we eliminate them by making a figure 1 in those sections.

And if b is either Abc or abC or abc , then the combination AbC is inconsistent and we eliminate it by making a figure 2 in that section. The sections which remain without a figure are the given conclusions. And the definitions of B and b have been the premises which have yielded the given conclusions.

235. We obtained our given conclusions in the first place by striking out casually three combinations, viz.: AbC, aBC, aBc. After striking out these combinations, the combinations which remained represented the given conclusions for which we were to find the premises. Our rule told us that the definitions of a term and its negative will always yield us premises which will produce the given conclusions. We try this process and we find that it causes us, in every case, to strike out again the same combinations which we casually struck out in the first place. We indicate the re-striking out process by the figures which we put in the sections. The conclusions which remain are exactly the same as those obtained by the casual striking-out process.

Casually striking out, means striking out by chance or without design.

236. Let us make another ABC diagram and strike out, casually, two combinations.

	AB	Ab	aB	ab	
X					C
1					
X					C
2					

Fig. 43.

We will strike out ABC and Abc. The conclusions which remain are AbC, aBC, abC, ABc, aBc, abc.

The definitions of B and b are,

$$B = ABC \mid aBC \mid aBc$$

$$b = AbC \mid abC \mid abc$$

If $B = ABc \mid aBC \mid aBc$, then ABC is inconsistent, and we eliminate it by making a figure 1 in that section.

If b is AbC or abC or abc, then Abc is inconsistent, and we eliminate it by making a figure 2 in that section.

Thus we see that the definitions of B and b which we obtained from the given conclusions caused us to strike out exactly the same combinations which we struck out casually in order to obtain the given conclusions.

We could just as well have taken the definitions of A and a, or of C and c, and either of them would have produced the given conclusions.

237. Let us now take a concrete example. Suppose that these propositions are given to us as conclusions, and we are required to find what premises will produce them.

- (1) The powers delegated to the United States by the Constitution are not reserved to the States and are not reserved to the people.
 - (2) The powers reserved to the States are not reserved to the people.
 - (3) The powers not reserved to the States and not reserved to the people are delegated to the United States.
- (1) Let A = powers delegated to the United States,
 B = the powers reserved to the States,
 C = the powers reserved to the people,
 a = the powers not-delegated-to-the-United States,
 b = the powers not-reserved-to-the-States,
 c = the powers not-reserved-to-the-people.

The propositions can be stated, thus:

$$(1) A = Abc$$

$$(2) B = Bc$$

$$(3) bc = Abc$$

Then make an ABC diagram, and write down the file-letters and the row-letters.

Remember that a figure in a section indicates that the combination in that section is inconsistent with the proposition having the same number.

AB	Ab	aB	ab
1	1		
2		2	
1			3
			c
			o

Fig. 44.

Now, if A is Abc, then the combinations ABC, ABc, AbC, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if B is Bc, then the combinations ABC and aBC are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if bc is Abc, then the combination abc is inconsistent, and we eliminate it by making a figure 3 in that section.

The following combinations remain as consistent propositions, viz.: Abc, aBc, abC, and from them we can get the following definitions:

- (1) A = Abc
- (2) B = Bac
- (3) C = Cab
- (4) a = aBc | abC
- (5) b = baC | bAc
- (6) c = cAb | caB

These propositions may be translated into concrete terms, as follows:

- (1) The powers delegated to the United States are not reserved to the States and are not reserved to the people.

- (2) The powers reserved to the States are not delegated to the United States and are not reserved to the people.
- (3) The powers reserved to the people are not delegated to the United States and are not reserved to the States.
- (4) The powers not delegated to the United States are either reserved to the States and not reserved to the people, or they are not reserved to the States and are reserved to the people.
- (5) The powers not reserved to the States are either not delegated to the United States and are reserved to the people, or they are delegated to the United States and not reserved to the people.
- (6) The powers not reserved to the people are either delegated to the United States and not reserved to the States, or they are not delegated to the United States and are reserved to the States.

It will be seen that we have obtained from the conclusions which were given to us, twice as many propositions as we started with, and out of these six propositions thus obtained, we can get three sets of premises which will produce the original propositions; we can take the definitions of A and a, or B and b, or C and c.

238. Of course in any given case it would be impossible for us to say which set of premises were the ones which the person who proposed the conclusions took for the purpose of obtaining those conclusions. We know that we have produced all the premises and that among the premises thus produced he must have taken two or more to obtain the conclusions originally given to us.

239. Just as Deduction gives us a great many more conclusions than we need, so Induction gives us more premises than we need, and we have to make a selection, but the selection which we make may not be the selection that some one else would make. It is enough, however, that we have produced all the possible premises. This example proves again, that there is no real difference between Deduction and Induction. In

either case we obtain a number of propositions which are consistent with the propositions which were given to us.

240. A proposition may be either a premise or a conclusion. If we take it as a premise, then we call the consistent propositions obtained from it, conclusions; if we take it as a conclusion, then we call the consistent propositions obtained from it, premises.

241. In reasoning, all that we can do is to obtain other propositions. Or, in other words, given any proposition, reason enables us to discover all the different ways in which we can say the same thing.

242. We cannot make any progress, we cannot discover any new facts; we can turn a proposition over and over again, exhibit it in a great many new lights, and from each new position get a new proposition, but all the propositions which we obtain, describe either the same idea or its opposite, and it follows that, in all cases, the descriptions of an idea and of its opposite are equivalent, that is, from the one we can always obtain the other.

243. If we start with a proposition like "Salt is chloride of sodium," we know that both these terms describe the same thought and are equivalent. We know that from the proposition "Salt is chloride of sodium," we can obtain the equivalent proposition that not-salt is not-chloride of sodium, and the proposition not-salt is not-chloride of sodium will produce the proposition that salt is chloride of sodium, therefore, these propositions are equivalent.

244. But the one proposition describes an idea and the other describes its opposite and it follows that a proposition which describes an idea is equivalent to a proposition which describes the opposite, and vice versa.

245. When we say that the problem of logic is the problem of discovering all the hidden and latent meaning of propositions, we really mean that it is the problem of discovering in how many different ways we can state the proposition.

246. The problem of logic is, given an idea and its opposite and a description of either of them, either by name, title, desig-

nation or description, to find all the consistent descriptions of the idea and of its opposite that it is possible to discover.

247. Our Universe of Discourse is always limited to one idea and its opposite, though of course we may change from one Universe of Discourse to another as often as we please, but a new Universe of Discourse implies a new idea, or field of thought.

248. Let us return to our example. The conclusions given us were,

- (1) The powers delegated to the United States by the Constitution are not reserved to the States and are not reserved to the people.
- (2) The powers reserved to the States are not reserved to the people.
- (3) The powers not reserved to the States and not reserved to the people are delegated to the United States.

We stated these propositions symbolically and analyzed them into their elements, and by our diagram showed every possible combination which could be made with the terms of these propositions. We then eliminated all the inconsistent propositions and from the consistent propositions which remained we selected six. These six contained definitions of all the terms in the original proposition. We then said that the definition of any one of these terms and its opposite would produce the original conclusions. Let us prove this proposition.

We will take these two propositions:

- (1) The powers delegated to the United States are not reserved to the States and are not reserved to the people.
- (2) The powers not delegated to the United States are either reserved to the States and not reserved to the people, or they are not reserved to the States and are reserved to the people.

Let A = powers delegated to the United States,

b = the powers not-reserved to the States,

c = the powers not-reserved to the people,

a = the powers not-delegated to the United States,

B = the powers reserved to the States,
 C = the powers reserved to the people.

In the two definitions taken, we have a definition of the powers delegated to the United States and a definition of the opposite idea,—the powers not-delegated to the United States. The definitions can be stated as,

- (1) $A = Abc$
- (2) $a = aBc \mid abC$

Now, if A is Abc, then every combination of A which contains B or C or both, is inconsistent and must be eliminated; we therefore eliminate the combinations ABC, AbC, ABe.

Make an ABC diagram and eliminate those combinations by making a figure 1 in those sections, thus:

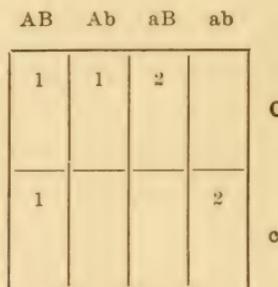


Fig. 45.

Again, if a is aBc or abC, then the combinations aBC and abc are inconsistent and we eliminate them by making a figure 2 in those sections.

The combinations Abc, aBc and abC automatically remain, and from them we can get the following definitions:

- (1) $A = Abc$
- (2) $B = Bac$
- (3) $bc = Abc$

Which may be translated,

- (1) The powers delegated to the United States are not reserved to the States and are not reserved to the people.

- (2) The powers reserved to the States are not delegated to the United States and are not reserved to the people.
 (3) The powers not reserved to the States and not reserved to the people are delegated to the United States.

By omitting from the second definition the words "are not delegated to the United States" our premises have produced the original conclusions. Of course when a definition contains more than is necessary for us to use, we may omit in our reading the part we do not need.

249. Next, let us take the definitions of the powers reserved to the States, and the powers which are not reserved to the States.

- (1) The powers reserved to the States are not delegated to the United States and are not reserved to the people.

It may be stated thus:

$$(1) B = Bac$$

- (2) The powers not reserved to the States are either not delegated to the United States and are reserved to the people, or they are delegated to the United States and not reserved to the people.

It may be stated thus:

$$(2) b = baC \mid bAc$$

Make an ABC diagram as usual:

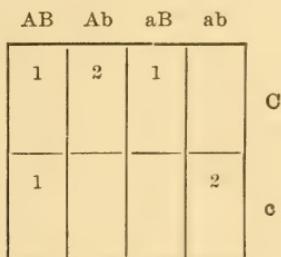


Fig. 46.

Now, if B is Bac, then the combinations of B which contain A or C or both, are inconsistent and must be eliminated. We therefore eliminate ABC and aBC and ABC, by making a figure 1 in those sections.

Again, if b is $baC \mid bAc$, then the combinations AbC and abc are inconsistent, and we eliminate them by making a figure 2 in those sections. The combinations Abc , aBc , abC , automatically remain, and as they are identical with the definitions obtained from the definitions of "The powers delegated to the United States and not-delegated to the United States," they may be translated in the same manner that we translated them in the last preceding section.

250. Now let us take the definitions of The powers reserved to the people, and of The powers not-reserved to the people.

(1) The powers reserved to the people are not delegated to the United States, and are not reserved to the States.

It may be stated thus:

$$(1) C = Cab$$

(2) The powers not-reserved to the people are either delegated to the United States and are not-reserved to the States, or, they are not delegated to the United States and are reserved to the States.

It may be stated thus:

$$(2) c = cAb \mid caB$$

Make an ABC diagram, as usual:

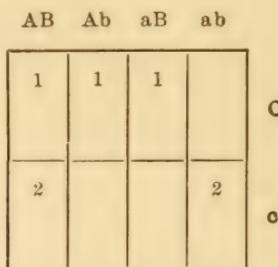


Fig. 47.

Now if C is Cab , then any combination of C with A or B , or both, is inconsistent. We therefore eliminate ABC , AbC , aBC by making a figure 1 in those sections.

Again, if c is cAb or caB , then the combinations Abc and abc are inconsistent, and we eliminate them by making a figure 2 in those sections.

As in the other cases, the combinations $A\bar{B}c$, $a\bar{B}c$, $ab\bar{C}$, remain, and from them we can get our original conclusions.

Thus:

- (1) $A = Abc$
 - (2) $B = Bac$
 - (3) $bc = Abc$

And they may be translated:

(1) The powers delegated to the United States by the Constitution are not reserved to the States and are not reserved to the people.

(2) The powers reserved to the States are not reserved to the people.

(3) The powers not reserved to the States and not reserved to the people are delegated to the United States.

Thus, we have proven that, given any conclusions, by obtaining from those conclusions the definitions of any term and its opposite, which are contained in the conclusions, and by using these definitions as premises, we can always obtain the original conclusions.

251. Now let us try the tentative plan of obtaining premises for the given conclusions. Let us take the same conclusions as in the last section. The definition "The powers delegated to the United States are not reserved to the States and are not reserved to the people," may be stated thus:

- $$(1) A = Abc$$

Make an ABC diagram:

AB	Ab	aB	ab
1	1	2	
2			
1			
2			
3			3

Fig. 48.

Now if A is Abc, then the combinations ABC, AB_c and A_bC are inconsistent and we eliminate them by making a figure 1 in those sections.

(2) The definition "The powers reserved to the States are not delegated to the United States and are not reserved to the people," may be stated thus:

$$(2) B = Bac$$

Now if B is Bac, then the combinations ABC, AB_c, aBC, are inconsistent, and we eliminate them by making a figure 2 in those sections.

Now, in order to obtain the given conclusions, we must get rid of the combination abc. Of course a definition of C would not enable us to get rid of c, because a definition of c would not be inconsistent with a definition of C. We must, therefore, get a definition of c.

(1) "The powers not reserved to the people are either delegated to the United States and not reserved to the States, or they are not delegated to the United States and are reserved to the States."

It may be stated thus:

$$(3) c = cAb \mid caB$$

If c is cAb or caB, then the combinations AB_c and abc are inconsistent, and we eliminate them by making a figure 3 in those sections.

Our original conclusions again remain. They are:

$$(1) A = Abc$$

$$(2) B = Bac$$

$$(3) bc = Abc$$

Thus by the tentative process we have found that the definitions of the powers delegated to the United States, and of the powers reserved to the States, and of the powers not reserved to the people, have given us the original conclusions.

252. Prof. Jevons, in his Principles of Science, p. 125, speaking of the problem of Induction, that is, of finding premises for given conclusions, says: "The only modes of discov-

ery consist either in exhaustively trying a great number of supposed laws, a process which is exhaustive in more senses than one, or else in carefully contemplating the effects and endeavoring to remember cases in which like effects followed from unknown laws."

253. Dr. Keynes, in his work on Formal Logic, shows that the Inductive problem may be solved in the way which I have given.

254. Again, on p. 137 of Principles of Science, Jevons says: "Now, we may make selection from eight things in two hundred and fifty-six ways; so that we have no less than two hundred and fifty-six different cases to treat, and the complete solution is at least fifty times as troublesome as with two terms." It may take more time to solve a problem involving three terms than it takes to solve a problem involving only two terms, but it is not much more troublesome, it merely takes a little more time.

255. Again, he says on p. 141: "The above investigations are complete as regards the possible logical relations of two or three terms. But when we attempt to apply the same kind of method to the relations of four or more terms, the labor becomes impracticably great. Four terms give sixteen combinations compatible with the laws of thought, and the number of possible selections of combinations is no less than 65,536. Some years of continuous labor would be required to ascertain the types of laws which may govern the combinations of only four things, and but a small part of such laws would be exemplified or capable of practical application in science. The purely logical inverse problem whereby we pass from combinations to their laws, is solved in the preceding pages, as far as it is likely to be for a long time to come, and it is almost impossible that it should ever be carried more than a single step further."

256. The Inductive problems containing four or more terms are not much more troublesome to solve than those containing two or three terms. Instead of "some years," it can only

take a few minute's time and labor to find the premises which will produce the conclusions containing four or more terms.

257. Miss Jones in her *Elements of Logic*, p. 60, points out very clearly a fallacy in Inductive reasoning made by some of the old logicians. She says: "Sunday, Monday.....and Saturday are all (*omnes*) twenty-four hours in length; Sunday, Monday.....and Saturday are all (*euneti*) the days of the week."

Therefore, all (*omnes*) the days of the week are twenty-four hours in length. It may be remarked that this syllogism is incorrect in form, the minor terms being taken collectively in its premise, distributively in the conclusion. I do not remember to have seen this inaccuracy noted.

Mansel (Mansel's Aldrich, 4th Ed., p. 221), Whately (Logic, 9th Ed., p. 152), and Jevons (Elementary Lessons, 7th Ed., pp. 214 and 215) among others, offer as instances of perfect or Aristotelian Induction, arguments exactly corresponding in form to the one I have given, without any remark upon their formal incorrectness."

258. Induction, as it is treated by the old logic, does not seem to me to be logical at all. According to the old logic, we state the instances which have come under our observation, and from these instances we infer a general law. From "Some A is B," we infer all A is B, according to the old logic. But this is not inference; this is making an unwarranted jump. Of course it may be true that all A is B, but it is not a logical inference from some A is B. The author of the article on Logic in the *Encyclopedia Britannica*, says: "Induction makes clear only, and does not prove." He is speaking of the old logic.

259. There are two other problems in reasoning which our system enables us to solve very easily. One is, given any proposition, how can we prove it to be true? The other is, given any proposition, how can we prove it to be false? Let us take this example: Suppose we are given the proposition, "Iron is metal-element."

Let A = iron,
 B = metal,
 C = element.

As the predicate is undistributed, that is, it does not mean that all metallic elements are iron, we can state it thus:

$$A = ABC.$$

Make an ABC diagram:

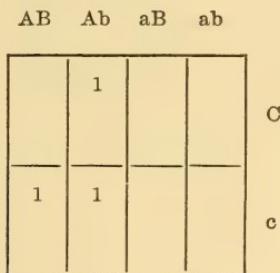


Fig. 49.

Now, if ABC is true, this would cause us to eliminate the other three combinations of A, viz.: ABc, AbC, Abc. These may be translated,

- (1) Iron is metal, not-element.
- (2) Iron is not-metal element.
- (3) Iron is not-metal and not-element.

Now, we can prove conclusively that A is ABC, by proving that ABc and AbC and Abc are false. There are only four combinations of A; if one is true the other three must be false; proving that the three are false, proves that the other one is true.

In order to prove that ABC is false, it is only necessary for us to prove that either ABc or AbC or Abc is true. In this case the truth of any one of the propositions which ABC would cause us to eliminate, proves the falsity of ABC. Suppose this proposition is given us: A is either ABC or Abc. This would cause us to eliminate the other two combinations of A, viz.: AbC and ABc. Now, we can prove that A is ABC, or Abc by proving that AbC and ABc are false. Again, we can prove the

falsity of the statement that A is ABC or Abc by proving the truth of either of the statements that A is AbC or that A is ABC.

260. This leads us to consider what propositions are equivalent to each other. Propositions which are equivalent to each other will cause us to strike out and save exactly the same combinations from our diagram.

261. Prof. Jevons in his *Principles of Science*, p. 116, says: "In the following list each proposition or group of propositions is exactly equivalent in meaning to the corresponding one in the other column, and the truth of this statement may be tested by working out the combinations of the alphabet."

$A = Ab$	$B = aB$
$A = b$	$a = B$
$A = BC$	$a = b c$
$A = AB AC$	$b = ab AbC$
$A B = C D$	$ab = cd$
$A c = B d$	$aC = bD$
$A = ABC AbC$	$\begin{cases} A = AB AC \\ AB = ABC \end{cases}$
$A = B \}$	$\begin{cases} A = B \\ B = C \end{cases}$
$A = AB \}$	$\begin{cases} A = AC \\ B = AC aBC. \end{cases}$
$B = BC \}$	

The first two examples are correct. Let us try the third.
Make an ABC diagram:

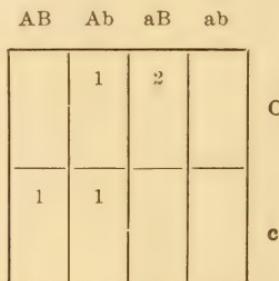


Fig. 50.

Now, If A equals BC; AbC, Abc and ABC are inconsistent and we eliminate them by making a figure 1 in those sections.

And if BC equals A, reading the proposition backward, then aBC is inconsistent and we eliminate it by making a figure 2 in that section. Now make an ABC diagram:

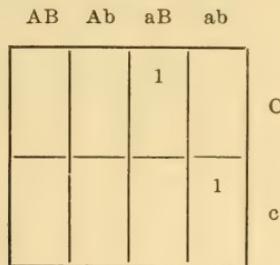


Fig. 51.

The proposition a equals b or c , may have two different meanings, depending upon whether we consider "or" as meaning one or the other and not both, or as meaning one or the other or both. Taking the first meaning it should be stated,

$$a = bC \mid Cb$$

Taking the second meaning it should be stated,

$$a = bC \mid Bc \mid bc$$

If the first meaning is correct, the combinations ABC and abc are inconsistent and we eliminate them by making a figure 1 in those sections. Make another ABC diagram:

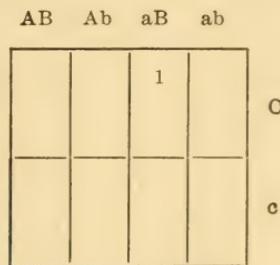


Fig. 52.

If the second meaning is correct, it will cause us to eliminate aABC and we eliminate it by making a figure 1 in that section.

The appearance of the three diagrams shows that Jevons' statement in this case is incorrect. We shall now pass on to the last example given by Jevons.

262. The last example reads,

$$\left. \begin{array}{l} A = AB, \\ B = BC \end{array} \right\} \text{the second column reads } \left\{ \begin{array}{l} A = AC, \\ B = A + aBC, \end{array} \right.$$

If A is AB, AbC and Abc are inconsistent and we eliminate them by making a figure 1 in those sections:

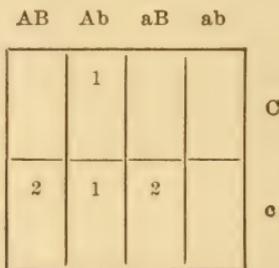


Fig. 53.

Again, if B is BC, the combinations ABC and aBc are inconsistent and we eliminate them by making a figure 2 in those sections. Now make another ABC diagram:

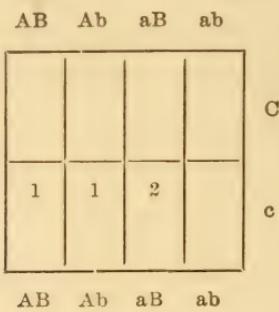


Fig. 54.

Now, if A is AC, then the combinations ABC and Abc are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if B is A or aBC, then the combination aBc is incon-

sistent and we eliminate it by making a figure 2 in that section. The appearance of the two diagrams again shows a mistake on Jevons' part.

263. Let us take these two examples which were incorrectly solved by Prof. Jevons, and show the correct solution by our system. Make an ABC diagram:

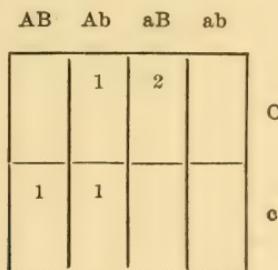


Fig. 55.

Now, if A is BC, we eliminate the combinations ABC, AbC, Abc, as inconsistent, by making a figure 1 in those sections.

And if BC is A, then aBC is inconsistent and we eliminate it by making a figure 2 in that section.

Now, as we showed before in treating of the Inductive problem, if from the conclusions which remain as consistent, we get the definition of any term and its opposite, these definitions will cause us to strike out the same combinations which, when struck out before, gave us the conclusions.

Now, in this case, the conclusions which remain are ABC, abC, abc and abc. The definitions of B and b are,

$$(2) \quad b = abC \mid abc$$

$$(1) \quad B = ABC \mid aBc$$

The definitions of C and c are,

$$(3) \quad C = ABC \mid abC$$

$$(4) \quad C = aBc \mid abc$$

The definitions of A and a are,

$$(5) \quad A = ABC$$

$$(6) \quad a = aBc \mid abC \mid abc$$

Either pair of definitions will cause us to strike out exactly the same combinations that the proposition A is BC will cause us to strike out, therefore, either pair of definitions is the exact equivalent of the proposition A is BC. By the tentative process, already explained, we could get other equivalents for the given proposition but none of them can possibly resemble the equivalent which Jevons gives. We will work out the pairs of definitions given above, on the diagram:

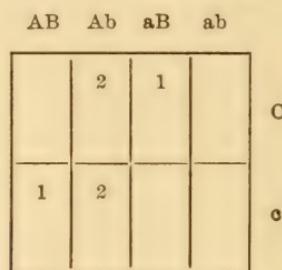


Fig. 56.

If B is ABC or aBc, then the combinations aBC and ABC are inconsistent and we eliminate them by making a figure 1 in those sections.

If b is abC or abc, then the combinations AbC and Abc are inconsistent and we eliminate them by making a figure 2 in those sections.

And now we have struck out exactly the combinations which the proposition A is BC caused us to eliminate.

Again, if C is ABC or abC, then the combinations AbC and aBC are inconsistent and we eliminate them by making a figure 1 in those sections.

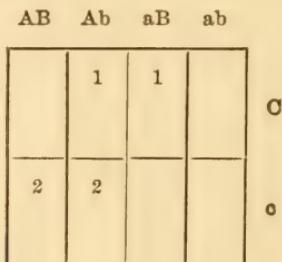


Fig. 57.

If c is aBc or abc , then the combinations ABc , and Abc are inconsistent and we eliminate them by making a figure 2 in those sections. And again, we have struck out the same sections which the original proposition A is BC caused us to eliminate.

Now make another ABC diagram:

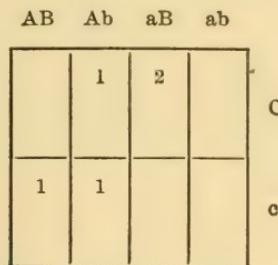


Fig. 58.

If A is ABC , then the combinations ABc , AbC and Abc are inconsistent and we eliminate them by making a figure 1 in those sections.

If c is aBc , or abC , or abc , then the combination aBC is inconsistent and we eliminate it by making a figure 2 in that section.

Again we have struck out the same sections which the original proposition, A is BC , caused us to eliminate. This proves that either pair of definitions given is equivalent to the original proposition.

264. We will now take Jevons' last example. Make an ABC diagram:

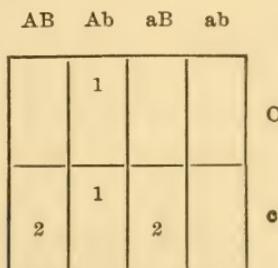


Fig. 59.

If A is AB, then the combinations AbC, ABC are inconsistent and we eliminate them by making a figure 1 in those sections.

If B is BC, then the combinations ABC, abC are inconsistent and we eliminate them by making a figure 2 in those sections.

The propositions which remain in the diagram are ABC, aBC, abC, abc. If from these propositions which remain in the diagram, we get the definitions of any term and its opposite, those definitions will cause us to strike out exactly the same combinations which the original proposition caused us to strike out. They will, therefore, be the true equivalents of the original propositions. The definitions which we can obtain are:

$$(1) \quad A = ABC$$

$$(2) \quad a = aBC \mid abC \mid abc$$

If A is ABC, then the combinations ABC, AbC, Abc are inconsistent and we eliminate them by making a figure 1 in those sections:

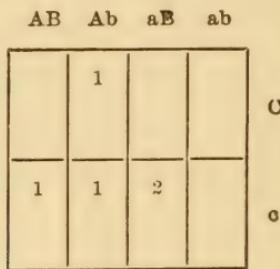


Fig. 60.

If a is aBC, or abC, or abc, then the combination ABC is inconsistent and we eliminate it by making a figure 2 in that section.

Thus, the definitions of A and a have caused us to strike out the same combinations which the propositions A is AB and B is BC caused us to eliminate.

Now make another ABC diagram:

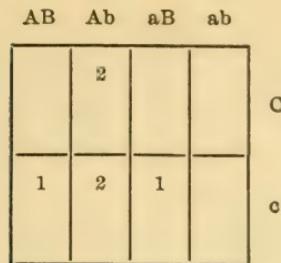


Fig. 61.

The definitions of B and b are,

$$(3) \quad B = ABC \mid aBC$$

$$(4) \quad b = abC \mid abc$$

If B is ABC, or aBC, then the combinations AB_c, aB_c, are inconsistent and we eliminate them by making a figure 1 in those sections.

If b is abC, or abc, then the combinations AbC, Abc are inconsistent and we eliminate them by making a figure 2 in those sections.

Again we have struck out the same combinations which the original propositions A is AB and B is BC caused us to strike out.

The definitions of C and c are,

$$(5) \quad C = CAB \mid CaB \mid Cab$$

$$(6) \quad c = abc$$

Make another ABC diagram:

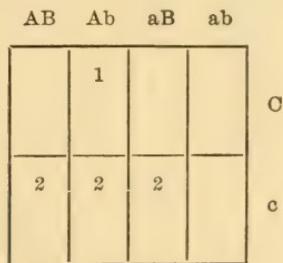


Fig. 62.

If C is CAB or CaB or Cab, then the combination AbC is

inconsistent and we eliminate it by making a figure 1 in that section.

Again, if c is abc , then the combinations ABC , Abc , aBc , are inconsistent and we eliminate them by making a figure 2 in those sections.

And again we have struck out the same combinations which the original propositions caused us to strike out. This proves that either pair of definitions is the logical equivalent of the original propositions given to us, viz: **A is AB and B is BC.**

EXERCISES.

265. What conclusions can be drawn from the following premises?

- (1) $A = AB$
 $C = Ca$
- (2) $a = cab$
 $C = Ca$
- (3) $C = Cb$
 $c = caB$
- (4) $C = CB$
 $c = abc$
- (5) $C = CAB$
 $c = ca$
- (6) $B = AC$
 $a = bc$

(7) What premises will produce the conclusions

- ABC
- Abc
- aBc
- abC

CHAPTER IX.

A CHAIN OF REASONING.

266. Thus far we have acted on the hypothesis that the propositions which were given to us were true, but it is possible that a premise may be false.

If a proposition is true, then all the propositions which are inferences from it are true, and all the propositions which are inconsistent with it are false.

267. If a proposition is false, then all the propositions which are inferences from it are false, and at least one of the propositions, which are inconsistent with it, is true.

268. We can thus reason from the truth of a proposition to the truth or falsity of other propositions, and from the falsity of a proposition we can reason to the falsity of other propositions and to the truth of at least one proposition. Let us illustrate this by a concrete example.

269. Let us take this example: The powers which are delegated to the United States are not reserved to the States and are not reserved to the people.

Let $A =$ the powers delegated to the United States,

$b =$ the powers not reserved to the States,

$c =$ the powers not reserved to the people.

We take it that the predicate is distributed, that is, that the powers not reserved to the States and not reserved to the people are delegated to the United States, so that we state the proposition both ways,

$$(1) \ A = bc$$

$$(2) \ bc = A$$

270. It is proper to observe that in stating propositions we drop the conjunctions, prepositions and other connecting words, and when we come to read our symbolic propositions we supply whatever conjunctions, prepositions and other con-

necting words may be necessary to make readable English. Our symbols represent only logical elements, and it sometimes takes considerable ingenuity to translate our symbols into good English.

Now, if the proposition A is bc is true, then the other combinations containing A are false, viz.: ABC , ABc , AbC , and if the proposition bc is A is true, then the combination abc is false.

Make an ABC diagram:

AB	Ab	aB	ab	
1	1			C
1			2	c

Fig. 63.

This diagram will assist the reader to follow this demonstration. We can translate the letters in the preceding paragraph thus:

If the proposition,

(1) The powers which are delegated to the United States are not reserved to the States and are not reserved to the people, ($A = bc$) be true, then the propositions,

(2) The powers which are delegated to the United States are reserved to the States and are reserved to the people ($A = BC$) and the proposition,

(3) The powers which are delegated to the United States are reserved to the States and are not reserved to the people ($A = Bc$), and the proposition,

(4) The powers which are delegated to the United States are not reserved to the States and are reserved to the people ($A = bC$), are false; and if the proposition,

(5) The powers which are not reserved to the States and which are not reserved to the people are delegated to the United States ($bc = A$), be true, then the proposition,

(6) The powers which are not reserved to the States and which are not reserved to the people, are not delegated to the United States ($bc = a$), is false.

It will be necessary for us to translate the symbolic propositions which follow. They are all to be interpreted similarly to the preceding, and it will save us time and labor to work with symbols only, in the first instance.

(1) If the proposition A is ABC is false, then one of the combinations ABC, AbC, Abc is true.

(2) If the proposition A is ABC is true, then the combinations ABC, AbC, Abc are false.

(3) If the proposition A is AbC is false, then one of the combinations ABC, ABC, Abc is true.

(4) If the proposition A is Abc is false, then one of the combinations ABC, ABC, AbC is true.

(5) If the proposition A is ABC is true, then the combinations ABC, AbC, Abc are false.

(6) If the proposition A is ABC is false, then one of the combinations ABC, AbC, Abc is true.

(7) If the proposition A is AbC is true, then the combinations ABC, ABC, Abc are false. The translations are as follows:

If the proposition,

(1) The powers delegated to the United States are not reserved to the States and are not reserved to the people ($A = bc$), is false, then one of the combinations,

(a) The powers which are delegated to the United States are reserved to the States and not to the people ($A = Bc$),

or,

(b) Are not reserved to the States and are reserved to the people ($A = bC$),

or,

(c) Are not reserved to the States and not reserved to the people ($A = bc$),
is true.

If the proposition,

(2) The powers which are delegated to the United States are reserved to the States and not to the people ($A = Bc$), is true, then all the combinations,

(a) The powers which are delegated to the United States are reserved to the States and to the people ($A = BC$),
and,

(b) Are not reserved to the States but are reserved to the people ($A = bC$),
and,

(c) Are not reserved to the States and not reserved to the people ($A = bc$),
are false.

If the proposition,

(3) The powers which are delegated to the United States are not reserved to the States, but are reserved to the people ($A = bC$),

is false, then one of the combinations,

(a) The powers which are delegated to the United States are reserved to the States and to the people ($A = BC$),

or,

(b) Are reserved to the States and not to the people ($A = Bc$),

or,

(c) Are not reserved to the States and not reserved to the people ($A = bc$),

is true.

If the proposition,

(4) The powers which are delegated to the United States are not reserved to the States and not reserved to the people ($A = bc$),

is false, then one of the combinations,

(a) The powers which are delegated to the United States are reserved to the States and to the people ($A = BC$),

or,

(b) Are reserved to the States and not to the people ($A = Bc$),

or,

(c) Are not reserved to the States but to the people ($A = bC$), is true.

If the proposition,

(5) The powers which are delegated to the United States are reserved to the States and to the people ($A = BC$), is true, then all the combinations,

(a) The powers which are delegated to the United States are reserved to the States and not to the people ($A = Bc$), and,

(b) Are not reserved to the people but are reserved to the States ($A = bC$),

(c) Are not reserved to the States and not to the people ($A = bc$),

are false.

If the proposition,

(6) The powers which are delegated to the United States are reserved to the States and not to the people ($A = Bc$), is false, then one of the combinations,

(a) The powers which are delegated to the United States are reserved to the States and to the people ($A = BC$),

or,

(b) Are not reserved to the States but are reserved to the people ($A = bC$),

or,

(c) Are not reserved to the States and not to the people ($A = bc$),

is true.

If the proposition,

(7) The powers which are delegated to the United States are not reserved to the States but are reserved to the people ($A = bC$),

is true, then all the combinations,

(a) The powers which are delegated to the United States are reserved to the States and to the people ($A = BC$),

and,

(b) Are reserved to the States and not to the people (A = Bc),

and,

(c) Are not reserved to the States and not to the people (A = bc),

are false.

We have now considered the propositions in which the subject is,

(A) "The powers which are delegated to the United States," as to their truth or falsity, and we have found that there were eight different combinations. So that these propositions which have the proposition,

"The powers which are delegated to the United States," for their subject, have made quite a chain of reasoning. There are eight links in the chain. But we could go on and consider next, propositions which have,

(a) "The powers which are not delegated to the United States,"
for their subject.

Then we can take up in order the propositions which have for their subjects respectively,

(B) "The powers which are reserved to the States,"
and,

(b) "The powers which are not reserved to the States,"
and then the propositions which have for their subject,

(C) "The powers which are reserved to the people,"
and,

(c) "The powers which are not reserved to the people,"
and then the propositions which have for their subjects,
respectively,

(AB) "The powers which are delegated to the United States
and are reserved to the States."

(Ab) "The powers which are delegated to the United States
and are not reserved to the States,"

(aB) "The powers which are not delegated to the United States
and are reserved to the States."

(ab) "The powers which are not delegated to the United States and not reserved to the States," and then the propositions which have for their subjects, respectively,

(BC) "The powers which are reserved to the States and to the people."

(Bc) "The powers which are reserved to the States but not to the people."

(bC) "The powers which are not reserved to the States and are reserved to the people."

(bc) "The powers which are not reserved to the States and not to the people,"

and then the propositions which have for their subjects, respectively,

(AC) "The powers which are delegated to the United States and are reserved to the people."

(Ac) "The powers which are delegated to the United States but not reserved to the people."

(aC) "The powers which are not delegated to the United States but are reserved to the people."

(ac) "The powers which are not delegated to the United States and not reserved to the people."

271. Now, as there are eighteen different subjects, which we may call eighteen chains of reasoning, and as there are eight links in each chain, when the chains are joined we have a chain with 144 links.

272. We suggest to the reader that if he wishes to obtain an exhaustive view of all questions which are contained in a single proposition involving three terms, that he pursue the same course with the other subjects that we have pursued with the subject, "The powers which are delegated to the United States."

273. It is clear that we can also reason from the truth of a proposition to the truth of other propositions, and from the falsity of a proposition to the falsity of other propositions. With the case in hand we can obtain just as many subjects

and just as many propositions, in reasoning from truth to truth and falsity to falsity, as we did in reasoning from truth to falsity and from falsity to truth. This will give us another chain with 144 links in it, which added to the other chain would make a chain of 288 links. As there are four combinations in each link, the total number of combinations would be 1152. These combinations, consisting each of three terms, can be read in various ways, according to the Law of Permutations. I have not figured it out, but I am under the impression that our system would give us over 25,000 different readings of the given example. It shows the immense variety which can be obtained from a very few terms.

274. In reasoning from the truth of a proposition to the truth of other propositions, of course all the propositions which are inferences from the given proposition, will be true, and in reasoning from the falsity of a proposition to the falsity of other propositions, all the propositions which are inferences from the given proposition, will be false.

CHAPTER X.

TERMS.

275. When the subject or predicate of a proposition can be represented by a single letter, we call the subject or predicate a single term, but when it is necessary to represent either of them by two or more letters, viz.: AB, BC, etc., we call the combination a complex term.

276. Single terms can be combined conjunctively or disjunctively. In the proposition A is BC, the subject A is a single term, the predicate BC is a conjunctive term and it means B and C. In the proposition A is B or C, the predicate is a disjunctive term and it means A is B without C, or C without B and its symbolic expression is, A is Bc or Cb.

277. The use of negative letters is necessary to the full logical expression of a disjunctive term, but in reading a disjunctive term it is usual to omit the negative terms, when the use of positive terms will express the meaning. Similarly, when in order to state a proposition logically, it is necessary to repeat the letter which represents the subject in the predicate, it is not necessary, in reading, to repeat the letter which represents the subject.

278. It has been suggested that the letters in the conjunctive term, e. g., AB, be called determinants. A and B are the determinants in the conjunctive term AB.

279. It has also been proposed that the single letters in a disjunctive term be called the alternants. Thus, in the proposition A is B or C, B and C are the alternants, but this is not strictly correct. The proper expression of A is B or C is, A = Bc | bC, so that strictly the alternants are Bc and bC.

280. In order to fully express the logical meaning of an alternant, we must use both positive and negative determinants.

281. The order of stating and reading the determinants in a conjunctive or disjunctive term is a matter of indifference. We can state and read them forward or backward or in any order we please.

282. In this system we understand by the expression A is B or C, that A cannot be both B and C or neither.

283. When we say A or B is C, we mean, in this system, that A without B, or B without A, is C, and its symbolic expression is, $Ab \text{ or } aB \text{ is } C$. It excludes the idea that both A and B or neither, can be C.

284. The question has been asked what is the contradictory of AB?

Make an AB diagram.

A	a	
	1	B
1	1	b

Fig. 64.

285. I do not think that terms have contradictories; terms have opposites, but propositions have contradictories; the question should be what is the opposite of AB.

286. When we have eliminated the inconsistent combinations we can read them, "No combinations are," naming them. Or we can substitute for the expression, "No combinations are," the shorter phrase, "Nothing is."

287. We can read the uneliminated combinations, "All the combinations are," or "Every combination is," naming them. For these expressions we can substitute the shorter phrase, everything is, understanding that "everything" stands for every combination.

288. In order to solve this problem we suppose AB to be the only combination uneliminated in the Reasoning Frame.

Now, if everything is AB, then the combinations Ab, aB, ab are inconsistent and we eliminate them by making a figure 1 in those sections.

We can now read in the Reasoning Frame,

(1) Everything is AB

(2) Nothing is Ab or aB or ab, which can be reduced to, Nothing is Ab or a.

Hence the opposite of AB is Ab | a.

Everything is AB is an impossible proposition because it causes the elimination of a and b.

289. What is the opposite of a or b?

Make an AB diagram.

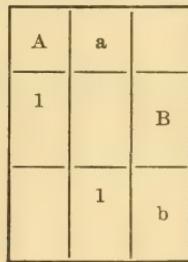


Fig. 65.

Now, if everything is aB or Ab, then the combinations AB, ab are inconsistent and we eliminate them by making a figure 1 in those sections.

We can now read in the Reasoning Frame,

(1) Everything is a or b

(2) Nothing is AB or ab. Hence the opposite of a or b is AB or ab.

290. What is the opposite of A or B? The expression of A or B means, A without B, or B without A, and it can be expressed Ab or aB.

Make an AB diagram.

A	a	
1		B
	1	b

Fig. 66.

Now, if everything is Ab or aB , then the combinations AB , ab are inconsistent and we eliminate them by making a figure 1 in those sections.

We can now read in the Reasoning Frame,

- (1) Everything is A or B
- (2) Nothing is AB or ab . Hence the opposite of A or B is AB or ab .

291. What is the opposite of ab ?

Make an AB diagram.

A	a	
1	1	B
1		b

Fig. 67.

Now, if everything is ab , then the combinations AB , Ab , aB are inconsistent and we eliminate them by making a figure 1 in those sections.

We can now read in the Reasoning Frame,

- (1) Everything is ab . This is an impossible proposition because A and B are eliminated.
- (2) Nothing is AB or Ab or aB . Hence the opposite of ab is AB or aB or Ab .

292. What is the opposite of A or BC ?

Make an ABC diagram.

AB	Ab	aB	ab	
1			1	C
		1	1	c

Fig. 68.

Now, if everything is A or BC, then the combinations ABC, aBc, abC, abc are inconsistent and we eliminate them by making a figure 1 in those sections.

We can now read in the Reasoning Frame,

- (1) Everything is A or BC.
- (2) Nothing is ABC or ABc or abC or abc

Hence the opposite of A or BC is ABC or aBc or abC or abc.

293. What is the opposite of ab or ac?

Make an ABC diagram.

AB	Ab	aB	ab	
1	1	1		C
1	1		1	c

Fig 69.

Now, if everything is ab or ac, then the combinations containing AB, Ab, aBC, abc are inconsistent and we eliminate them by making a figure 1 in those sections.

We can now read in the Reasoning Frame,

- (1) Everything is ab or ac
 - (2) Nothing is ABC or ABc or AbC or Abc or aBC or abc
- Hence the opposite of ab or ac is aBC or abc or ABC or ABc or AbC or Abc.

294. What is the opposite of ABC or ABD?

Make an ABCD diagram.

AB	Ab	aB	ab	
1	1	1	1	CD
	1	1	1	Cd
	1	1	1	cD
1	1	1	1	cd

Fig. 70.

Now, if everything is ABC or ABD, then the combinations containing ABCD, ABed, Ab, aB, ab are inconsistent and we eliminate them by making a figure 1 in those sections.

We can now read in the Reasoning Frame,

(1) Everything is ABC or ABD

(2) Nothing is ABCD or ABed or Ab or a. Hence the opposite of ABC or ABD, is ABCD or ABed or Ab or a.

295. What is the opposite of a or b or cd?

Make an ABCD diagram.

AB	Ab	aB	ab	
1			1	CD
1			1	Cd
1			1	cD
	1	1	1	cd

Fig. 71.

Now, if everything is a or b or cd, then the combinations containing ABC, ABcD, Abcd, aBed, ab are inconsistent and we eliminate them by making a figure 1 in those sections.

We can now read in the Reasoning Frame,

(1) Everything is a or b or cd

(2) Nothing is ABC or ABcD or Abcd or aBed or ab.

Hence the opposite of a or b or cd is ABC or ABcD or Abcd or aBed or ab.

296. Complex terms may be different without being opposites. They are different when one contains a positive term and the other contains the negative of that positive term, thus: ABC and AB_c are different without being opposites.

297. Terms which have the determinant in one replaced by the opposite determinant in the other, are called logical contraries, thus AbC, aBc are called contraries.

When we take a pair of contraries, e. g., AbC, aBc and get a definition of each letter stated in the remaining two letters, we have this interesting result, that the triplet of definitions furnished by one is equivalent to the triplet of definitions furnished by the other. Thus, from AbC we can get these definitions,

$$A = AbC$$

$$b = bAC$$

$$C = CAB$$

Make an ABC diagram.

AB	Ab	aB	ab	
1		3	2	C
3			3	
1	1		2	c
	2			

Fig. 72.

Now, if $A = AbC$, then the combinations AB_c, ABC, Abc are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $b = bAC$, then the combinations Abc, abc, abc are inconsistent and we eliminate them by making a figure 2 in those sections.

Again, if $C = CAB$, then the combinations ABC, aBC, abC are inconsistent and we eliminate them by making a figure 3 in those sections.

298. From aBc we can get these definitions:

$$a = aBc$$

$$B = Bac$$

$$c = cab$$

Make an ABC diagram.

AB	Ab	aB	ab	
2		2 1	1	C
2	3		1	c
3			3	

Fig. 73.

Now, if $a = aBc$, then the combinations abC , abc , aBC are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $B = Bac$, then the combinations ABC , ABc , aBC are inconsistent and we eliminate them by making a figure 2 in those sections.

Again, if $c = cab$, then the combinations ABc , Abc , abc are inconsistent and we eliminate them by making a figure 3 in those sections.

An examination of the two Frames now shows the equivalence of the given triplets of definitions.

299. Prof. Keynes in speaking of the obversion of complex propositions on p. 403, says, "The obverse, therefore, of all X is AB or ab , is no X is Ab or aB . This agrees with the conclusion which we have arrived at.

300. The question is, What is an equivalent expression for AB or AC ?

Make an ABC Diagram.

AB	Ab	aB	ab	
1		1	1	C
	1	1	1	c

Fig. 74.

Now, if everything = AB | AC, then the combinations containing ABC, Abc, a, are inconsistent and we eliminate them by making a figure 1 in those sections.

We can now read in the Reasoning Frame,

- (1) Everything = AB | AC
- (2) Everything = Ab | Ac
- (3) Nothing = ABC | Abc | a
- (4) C = Ab
- (5) c = AB

Hence (2) and (3) are equivalents for (1).

301. What is an equivalent expression for A or BC?

Make an ABC Diagram.

AB	Ab	aB	ab	
1			1	C
		1	1	c

Fig. 75.

Now, if everything = A | BC, then the combinations containing ABC, aBc, ab are inconsistent and we eliminate them by making a figure 1 in those sections.

We can now read in the Reasoning Frame.

- (1) Everything = A | BC
- (2) Everything = Ab | ABe, | aBC

- (3) Everything = A_c | AbC | aBC
 (4) Nothing = ABC | aBc | ab
 (5) Nothing = ac | ABC | abc

The result proves that (2), (3), (4) and (5) are equivalent expressions for (1).

302. What is an equivalent expression for A or aB?

Make an AB diagram.

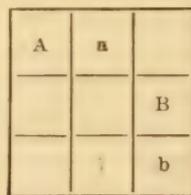


Fig. 76.

Now, if everything = A | aB, then the combination ab is inconsistent and we eliminate it by making a figure 1 in that section.

We can now read in the Reasoning Frame,

- (1) Everything = A | aB
 (2) Everything = B | Ab
 (3) Nothing = ab

The result proves that (2) and (3) are equivalent expressions for (1).

303. What is an equivalent expression for AC or AD or BD?

Make an ABCD diagram.

AB	Ab	aB	ab	
1	1	1	1	CD
1		1	1	Cd
1			1	cD
1	1	1	1	cd

Fig. 77.

Now, if everything = AC | AD | BD, then the combinations containing AB, ab, AbCD, Abed, aBC, aBcd are inconsistent and we eliminate them by making a figure 1 in those sections.

We can now read in the Reasoning Frame,

- (1) Everything = AC | AD | BD
- (2) Nothing = AB | ab | AbCD | Abed | aBC | aBed
- (3) Nothing = CD | cd | ABCd | ABcD | abCd | abcD

The result proves that (2) and (3) are equivalent to (1).

304. By the Law of Contradiction such an expression as Bb has no meaning and, therefore, such a phrase as A or Bb, simply means A. We can reject the Bb as surplusage.

305. By the Law of the Excluded Middle we can always affirm that A is AB or Ab, and consequently the phrase AB or Ab, means simply A. And again, A (B or b), means nothing but A. After working our problems we shall frequently, in reading our Reasoning Frames, find such combinations as AB or Ab. We can, in reading, always simplify them by omitting the B or b and calling the phrase simply A.

Whenever in a conclusion we have the equivalent for any letter, we can always substitute the letter, in reading, for the equivalent.

EXERCISES.

306. (1) What is the opposite of BA?
- (2) What is the opposite of Ab or aB?
 - (3) What is the opposite of ba?
 - (4) What is the opposite of A or Be?
 - (5) What is the opposite of A or B or cD?
 - (6) What is a complete equivalent for Ab or Ac?
 - (7) What is a complete equivalent for CABd or DAbe or DBea?

CHAPTER XI.

ELIMINATION.

307. Elimination may be considered as a process for getting rid of inconsistent propositions.

Given any proposition, we take letters to stand for the terms, and then we combine the letters by means of our Reasoning Frame, into all the possible combinations which can be made of them. Each one of these combinations represents a bundle of propositions.

308. When we eliminate a combination because it is inconsistent with our premises, we mean that the bundle of propositions contained in the eliminated combination is inconsistent or irrelevant for our present purposes. It does not follow that the combination is inconsistent with other premises,—the same combinations might in other cases be consistent which in the case before us are inconsistent.

309. When we have made all our possible combinations and before we have eliminated any of them, the various combinations have nothing to say as to whether they are true or false. Those that are true and those that are false depend upon the results of the eliminating process. After we have completed the eliminating process, the propositions remaining in the Frame are taken to be true because they are consistent with the premises and the premises are supposed to be true, and the propositions which are eliminated because they are inconsistent with the premises are supposed to be false because the premises are taken to be true.

310. But, after all, a supposition always lies behind a premise. When we say A is B, we really mean, If A is B, and hence we can say If A is B, such and such will be our conclusions. Absolute knowledge is not given to any mortal being.

311. Every premise will cause us to eliminate certain combinations (that is, propositions), because they are inconsistent, and we can say with certainty this much, that if our premise, for instance, AB, is true, surely the combination Ab is false in this case, because by the Law of Contradiction a thing cannot both be and not be at the same time, and if this is not certainty, then there is no such thing as logic. If A is B we can feel sure that there is no thing which has both names A and b. And, similarly, if A is B we can feel sure that no thing has the names a and B.

CHAPTER XII.

EXAMPLES CONTAINING THREE TERMS.

312. In order to illustrate the practical application of our system to logical problems containing three terms, it will be appropriate for us to work out a number of examples taken from other writers.

Granite is not a sedimentary rock;

Basalt is not a sedimentary rock.

Let. $A = \text{granite}$,

$b = \text{not-a-sedimentary rock}$,

$C = \text{Basalt}$.

The premises can be stated:

$$(1) A = Ab$$

$$(2) C = Cb$$

We repeat the subject in the predicate in order to have our symbolical propositions true when read both ways, and we say C is Cb because the order of the letters makes no difference— Cb means exactly the same thing as bC .

Make an ABC diagram:

AB	Ab	aB	ab	
1		2		C
2				
1				c

Fig. 78.

Now, if $A = Ab$, then the combinations ABC , ABc , are inconsistent, because they imply that $A = AB$, and we therefore eliminate them by making a figure 1 in those sections.

If $C = Cb$, then the combinations ABC , aBC , are inconsistent because they imply that $C = CB$ and we therefore eliminate them by making a figure 2 in those sections.

From the combinations which remain we can obtain the following definitions:

- (1) $A = AbC \mid Abc$, which interpreted is, Granite is not a sedimentary rock and is either Basalt or not Basalt.
- (2) $C = CAB \mid Cab$, which translated is, Basalt is not a sedimentary rock and is granite or not granite.
- (3) $B = Bac$, which translated is,

A sedimentary rock is not Granite and is not Basalt.

- (4) $a = aBc \mid abC \mid abc$, which translated is,
Whatever is not Granite is either a sedimentary rock and not-Basalt, or not a sedimentary rock and is Basalt, or is neither a sedimentary rock nor Basalt.

We might go on and give the definitions of all the other single terms and of all the compound terms which are to be found remaining in the diagram, but it is not necessary. This shows us how easy it is with one operation, to obtain all the information which is latent in the premises.

313. This example and several others which follow, are taken from Prof. Jevons.

All planets are subject to gravity;

Fixed stars are not planets.

Let $A = \text{planets}$,

$B = \text{fixed stars}$,

$C = \text{gravity}$.

The propositions can be stated,

$$A = AC$$

$$B = aB$$

Make an ABC diagram,

AB	Ab	aB	ab	
2				C
2	1			c
1				

Fig. 79.

Now, if $A = AC$, then the combinations ABC , AbC are inconsistent because they imply that $A = Ac$, and we therefore eliminate them by making a figure 1 in those sections.

Again, if $B = aB$, then the combinations ABC and ABc are inconsistent because they imply that $B = AB$, and we therefore eliminate them by making a figure 2 in those sections.

From the propositions which remain in the Reasoning Frame we can get the following definitions:

(1) $A = AbC$, which translated is,

Planets are not fixed stars and are subject to gravity.

(2) $B = aBC \mid aBc$, which interpreted is,

Fixed stars are not planets and they are or are not subject to gravity.

(3) $C = AbC \mid aBC \mid abC$, which translated is,

Whatever is subject to gravity is either a planet, and in that case it is not a fixed star or it is a fixed star, and then it is not a planet, or it is something which is neither a planet nor a fixed star.

We can simplify the last by saying, whatever is subject to gravity is either a planet, or a fixed star, or neither. Sometimes it requires considerable ingenuity to translate our combinations into popular English.

314. He that is of God, heareth my words: Ye therefore hear them not because ye are not of God.

Let $A =$ he that is of God,

$B =$ heareth my words,

$C =$ ye

$a =$ are not of God,

$b =$ hear them not.

The premises can be stated:

(1) $A = B$

(2) $B = A$

(3) $C = Ca$

Make an ABC diagram:

AB	Ab	aB	ab	
3	3 1	2		C
	1	2		c

Fig. 80.

Now, if $A = B$, then the combinations AbC , Abc are inconsistent, because they imply that $A = b$, and we therefore eliminate them by making a figure 1 in those sections.

Again, if $B = A$, then the combinations aBC , aBc are inconsistent because they imply that $B = a$, and we therefore eliminate them by making a figure 2 in those sections.

Again, if $C = Ca$, then the combinations ABC and AbC are inconsistent because they imply that $C = A$, and we therefore eliminate them by making a figure 3 in those sections.

From the combinations which remain we can obtain the following definitions:

(1) $A = ABC$, which translated is:

He that is of God heareth my words and is not ye.

(2) $B = BAc$, which translated is:

He that heareth my words is of God and is not ye.

(3) $C = Cab$, which translated is:

Ye are not of God and ye do not hear my words.

315. John is a man,

John is not a triangle.

Let $A = \text{John}$,

$B = \text{man}$,

$C = \text{triangle}$.

We can state the propositions:

(1) $A = AB$

(2) $A = Ac$

Make an ABC diagram:

AB	Ab	aB	ab	
2	2 1			C
		1		c

Fig. 81.

Now, if $A = AB$, then the combinations AbC , Abc are inconsistent, because they imply that $A = b$, and we therefore eliminate them by making a figure 1 in those sections.

Again, if $A = Ac$, then the combinations ABC , AbC are inconsistent because they imply that $A = C$, and we therefore eliminate them by making a figure 2 in those sections.

From the combinations which remain in the Reasoning Frame we can obtain the following definitions:

(1) $A = ABC$, which translated is:

John is a man and not a triangle.

(2) $B = aBC \mid aBc \mid ABe$, which translated is:

A man is either not-John and is a triangle, or a man is not-John and is not a triangle, or a man is John and not a triangle.

We can simplify this by saying that it means:

A man is either John or a triangle or neither.

(3) $C = aBC \mid abC$, which translated is:

A triangle is not-John, and is either a man or not a man.

316. All avaricious men refuse to give money;

This man refuses to give money.

Let $A =$ all avaricious men,

$B =$ refuse to give money.

$C =$ this man.

The premises can be stated thus:

(1) $A = AB$

(2) $C = CB$.

Make an ABC diagram.

AB	Ab	aB	ab	
	2 1		2	C
	1			c

Fig. 82.

Now, if $A = AB$, then the combinations AbC , Abc are inconsistent, because they imply that $A = b$, and we therefore eliminate them by making a figure 1 in those sections.

Again, if $C = CB$, then the combinations AbC , abC are inconsistent because they imply that $C = b$, and we therefore eliminate them by making a figure 2 in those sections.

From the combinations which remain we can obtain the following definitions:

(1) $C = CAB | CaB$, which translated is:

This man refuses to give money and he is either avaricious or not avaricious.

(2) $AB = ABC | A B c$, which interpreted is:

An avaricious man who refuses to give money is either this man, or he is not this man.

(3) $B = BAC | BA c | BaC | Ba c$, which translated is:

Those who refuse to give money are either an avaricious man who is this man, or an avaricious man who is not this man, or this man who is not an avaricious man, or men who are neither avaricious nor this man.

(4) $a = aBC | a B c | abc$, which translated is:

Those who are not avaricious are either those who refuse to give money and are this man, or those who refuse to give money and are not this man, or those who are neither.

(5) $b = abc$, which translated is:

Those who do not refuse to give money are neither avaricious nor this man.

317. A science which furnishes the mind with a multitude of useful facts deserves cultivation;

Logic is not such a science.

Let $A =$ a science which furnishes the mind with a multitude of useful facts;

$B =$ deserves cultivation.

$C =$ logic.

We can state the propositions thus:

$$(1) A = AB$$

$$(2) C = Ca.$$

Make an ABC diagram:

AB	Ab	aB	ab	
2	21			C
	1			c

Fig. 83.

Now, if $A = AB$, then the combinations AbC , Abc are inconsistent because they imply that $A = b$, and we therefore eliminate them by making a figure 1 in those sections.

Again, if $C = Ca$ then the combinations ABC , AbC are inconsistent because they imply that $C = CA$, and we therefore eliminate them by making a figure 2 in those sections.

From the combinations which remain we can get the following definitions:

(1) $AB = ABC$, which translated is:

A science which furnishes the mind with a multitude of useful facts and which deserves cultivation is not logic.

(2) $B = ABC | aBC | aBc$, which translated is:

Whatever deserves cultivation is either a science which furnishes the mind with a multitude of useful facts and is logic, or something which is not a science which furnishes the mind with a multitude of useful facts and which is logic, or something which is not a science which furnishes the mind with a multitude of useful facts and which is not logic.

(3) $C = CaB | Cab$, which translated is:

Logic is not a science which furnishes the mind with a multitude of useful facts and it deserves or it does not deserve cultivation.

(4) $b = abC | abc$, which interpreted is:

Whatever does not deserve cultivation is not a science which furnishes the mind with a multitude of useful facts, and it is logic or it is not logic.

318. Mont Blanc is the highest mountain in Europe;
The highest mountain in Europe is deeply covered with
snow.

Let $A = \text{Mont Blanc}$,

$B = \text{the highest mountain in Europe}$,

$C = \text{deeply covered with snow}$.

We can state the proposition thus:

- (1) $A = B$
- (2) $B = A$
- (3) $B = BC$.

In this case it is clear that if Mont Blanc is the highest mountain in Europe, the highest mountain in Europe is Mont Blanc, and as it is necessary that we should always read every meaning which is true on the face of a proposition, we therefore read $B = A$. The reason why we do not read $BC = B$, is because $BC = B$ would cause us to eliminate exactly the same combinations which the proposition $B = BC$ causes us to eliminate. It would therefore be unnecessary to read $BC = B$.

Make an ABC diagram:

AB	Ab	aB	ab	
	1	2		C
3	1	2	3	c

Fig. 84.

Now, if $A = B$, then the combinations AbC , Abc , are inconsistent, because they imply that $A = b$, and we therefore eliminate them by making a figure 1 in those sections.

Again, if $B = A$, then the combinations aBC and aBc are inconsistent, because they imply that $B = a$, and we therefore eliminate them by making a figure 2 in those sections.

Again, if $B = BC$, then the combinations ABC and aBc are inconsistent because they imply that $B = c$, and we therefore eliminate them by making a figure 3 in those sections.

From the combinations which automatically remain, we can obtain the following definitions:

(1) $A = BC$, which translated is:

Mont Blanc is the highest mountain in Europe and is deeply covered with snow.

(2) $B = AC$, which interpreted is:

The highest mountain in Europe is Mont Blanc and it is deeply covered with snow.

(3) $C = AB \mid Cab$, which translated is:

Whatever is deeply covered with snow is either Mont Blanc, which is the highest mountain in Europe, or it is not Mont Blanc, and it is something which is not the highest mountain in Europe.

(4) $a = abC \mid abc$, which translated is:

Whatever is not Mont Blanc is not the highest mountain in Europe, and it is deeply covered with snow, or it is not deeply covered with snow.

(5) $c = abc$, which translated is:

What is not deeply covered with snow is not Mont Blanc, and is not the highest mountain in Europe.

319. Sodium is a metal;

Metals conduct electricity.

Let A = sodium,

B = metal,

C = conducts electricity.

The propositions may be stated thus:

$$(1) A = AB$$

$$(2) B = BC$$

Make an ABC diagram:

AB	Ab	aB	ab	
	1			C
2	1	2		c

Fig. 85.

Now, if $A = AB$, then the combinations AbC, Abc, are inconsistent, because they imply that $A = b$, and we therefore eliminate them by making a figure 1 in those sections.

Again, if $B = BC$, then the combinations ABe, aBe are inconsistent, because they imply that $B = c$, and we therefore eliminate them by making a figure 2 in those sections.

From the combinations which remain we can obtain the following definitions:

(1) $A = ABC$, which translated is:

Sodium is a metal which conducts electricity.

(2) $B = ABC \mid aBC$, which translated is:

A metal conducts electricity, and it is either sodium or not sodium.

(3) $C = ABC \mid aBC \mid abC$, which translated is:

Whatever conducts electricity is either sodium or a metal, or neither.

(4) $c = abc$, which translated is:

Whatever does not conduct electricity is not sodium and is not a metal.

320. Neptune is a planet;

What has retrograde motion is not a planet.

This last premise in popular English could be expressed,

No planet has retrograde motion, or,

A planet has not retrograde motion.

Let $A = \text{Neptune}$,

$B = \text{planet}$,

$C = \text{retrograde motion}$.

The propositions may be stated thus:

$$(1) A = AB$$

$$(2) C = Cb$$

Make an ABC diagram:

AB	Ab	aB	ab	
2	1	2		C
	1			c

Fig. 86.

Now, if $A = AB$, then the combinations AbC , Abc , are inconsistent, because they imply that $A = b$, and we therefore eliminate them by making a figure 1 in those sections.

Again, if $C = Cb$, then the combinations ABC , aBC are inconsistent, because they imply that $C = CB$, and we therefore eliminate them by making a figure 2 in those sections.

We can obtain the following definitions from the combinations which remain:

(1) $A = ABe$, which translated is:

Neptune is a planet which has not retrograde motion.

(2) $B = ABe \mid abe$, which translated is:

A planet has not retrograde motion, and it is either Neptune or not Neptune.

(3) $C = abC$, which translated is:

Whatever has retrograde motion is not Neptune and is not a planet.

321. Whales are not true fish;

True fish respire water.

Let $A = \text{whale}$,

$B = \text{true fish}$,

$C = \text{respire water}$.

The propositions may be stated thus:

$$(1) A = Ab$$

$$(2) B = BC$$

Make an ABC diagram:

AB	Ab	aB	ab	
1				C
12		2		c

Fig. 87.

Now, if $A = Ab$, then the combinations ABC , ABc , are inconsistent, because they imply that $A = AB$, and we therefore eliminate them by making a figure 1 in those sections.

Now, if $B = BC$, then the combinations ABc , aBc , are inconsistent, because they imply that $B = Bc$, and we therefore eliminate them by making a figure 2 in those sections.

From the combinations which automatically remain we can obtain the following definitions:

(1) $A = AbC \mid Abc$, which translated is:

A whale is not a true fish and does or does not respire water.

(2) $B = BaC$, which translated is:

A true fish respires water and is not a whale.

(3) $C = CAB \mid CaB \mid Cab$, which translated is:

Whatever respires water is or is not a whale, and it is or is not a true fish.

(4) $c = cAb \mid cab$, which translated is:

Whatever does not respire water is not a true fish, and it is or is not a whale.

In the old logic this is called a syllogism in the mood Camestres. The old logic could not deal with this mood directly, it had to deal with it indirectly by reducing it to some other form. Similarly with the mood Baroco, which gave the old logicians considerable trouble.

322. The following is an example taken from Prof. Jevons:
All heated solids give continuous spectra;
Some nebulæ do not give continuous spectra.

Let A = all heated solids,

B = give continuous spectra,

C = some nebulæ.

The propositions may be stated thus:

$$(1) A = AB$$

$$(2) Cb = Cb$$

It is clear that "some nebulæ" referred to in the subject, mean the nebulæ which do not give continuous spectra; we can express this definitely by using the phrase Cb .

Make an ABC diagram:

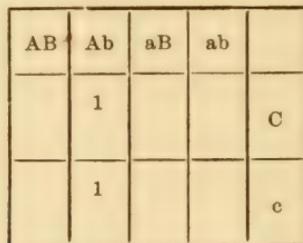


Fig. 88.

Now, if $A = AB$, then the combinations AbC , Abc , are inconsistent, because they imply that $A = Ab$, and we therefore eliminate them by making a figure 1 in those sections.

Again, if $Cb = Cb$, then no combinations are inconsistent with $Cb = Cb$.

From the combinations which remain we can obtain the following definitions:

(1) $A = ABC \mid ABc$, which translated is:

Heated solids give continuous spectra and are or are not nebulæ.

(2) $C = CAB \mid CaB \mid Cab$, which translated is:

Nebulæ are either heated solids giving continuous spectra, or not heated solids giving continuous spectra, or not heated solids not giving continuous spectra.

The conclusion which the old logic drew in regard to "nebulæ" was, "Some nebulæ are not heated solids." This is a popular definition of "nebulæ." The strict logical definition is the one given by us. The logical definition says that nebulæ are either heated solids giving continuous spectra, or not heated solids giving continuous spectra, or not heated solids not giving continuous spectra. But it does not tell us that some nebulæ are not heated solids,—that is a matter of fact which happens to be true in this case.

323. All fixed stars are self-luminous;

Some heavenly bodies are not self-luminous.

Let $A = \text{fixed-stars}$,

$B = \text{self-luminous}$,

$C = \text{heavenly-bodies}$.

The premises may be stated,

(1) $A = AB$

(2) $Cb = Cb$

$Cb = Cb$ may be translated, "Heavenly bodies which are not self-luminous are not self-luminous,"—a self-evident proposition.

Make an ABC diagram:

AB	Ab	aB	ab	
	1			C
	1			c

Fig. 89.

Now, if $A = AB$, then the combinations AbC , Abc are inconsistent, because they imply that $A = Ab$, and we therefore eliminate them by making a figure 1 in those sections.

From the combinations which remain we can obtain the following definitions:

- (1) $AB = ABC \mid ABc$, which translated is:

Fixed stars which are self-luminous are either heavenly-bodies or are not-heavenly-bodies.

- (2) $aB = aBC \mid aBc$, which translated is:

Things which are self-luminous and not-fixed-stars are either heavenly-bodies or not-heavenly-bodies.

- (3) $C = CAB \mid CaB \mid Cab$, which translated is:

Heavenly bodies are either fixed-stars and self-luminous, or they are self-luminous but not-fixed-stars, or they are neither fixed-stars nor self-luminous.

The conclusion which the old logic drew was, "Some heavenly bodies are not fixed stars."

Prof. Jevon's theory that logic is the substitution of similars, is a half truth. Logic is the art of finding equivalents and contraditories, consistent and inconsistent propositions. Of course, when we have found equivalents, then we can substitute one for the other.

324. Some metals are of less density than water;

All bodies of less density than water will float upon the surface of the water.

Let $A = \text{metals}$,

$B = \text{bodies of less density than water}$,

$C = \text{float upon the surface of the water}$.

The propositions may be stated:

$$(1) AB = AB$$

$$(2) B = C$$

$$(3) C = B$$

When we have the word "some" in the subject and predicate, we can state the proposition in the form of a self-evident proposition.

Make an ABC diagram:

AB	Ab	aB	ab	
	2		2	C
1		1		c

Fig. 90.

Now, if $AB = AB$, there are no combinations in the Frame which are inconsistent with it, and there is nothing to eliminate. The only object of stating a proposition which is in the form of $AB = AB$, is simply to posit the terms.

It means, simply, that there are, in this case, things which we call A and B in the Universe of Discourse. When we posit A, of course A affirms its own existence, but it does not necessarily deny the existence of anything else.

$AB = AB$ means that there are such things in the Universe of Discourse as AB, that is, things which have the names A and B.

Again, if $B = C$, then the combinations ABC and aBC are inconsistent because they imply that $B = c$, and we therefore eliminate them by making a figure 1 in those sections.

We stated a third proposition, viz.: $C = B$, because it is evident that whatever will float upon the surface of water is of less density than water, and in stating our propositions we should always remember to state every proposition which is, *prima facie*, true, and which can be gathered from the given propositions.

Now, if $C = B$, then the combinations AbC, abC, are inconsistent, because they imply that $C = b$, and we therefore eliminate them by making a figure 2 in those sections.

From the combinations which remain we can obtain the following definitions:

(1) $A = ABC | Abc$, which translated is:

Metals are either of less density than water and will float upon the surface of water, or they are not of less

density than water and will not float upon the surface of water.

- (2) $B = ABC \mid aBC$, which translated is:

Whatever is of less density than water is a metal or not a metal, and it will float upon the surface of water.

- (3) $C = ABC \mid aBC$, which translated is:

Whatever will float upon the surface of water is a metal or not a metal and it is of less density than water.

- (4) $a = aBC \mid abc$, which translated is:

Whatever is not a metal is either of less density than water and will float upon the surface of water, or it is of not less density than water and will not float upon the surface of water.

- (5) $b = Abc \mid abc$, which translated is:

Whatever is not of less density than water is either a metal and will not float upon the surface of water, or it is not a metal and will not float upon the surface of water.

- (6) $c = Abc \mid abc$, which translated is:

Whatever will not float upon the surface of the water is not of less density than water and is either a metal or is not a metal.

The conclusion that the old logic drew was, "Some metals will float upon the surface of water."

EXERCISES.

325. What conclusions can be drawn from the following pairs of premises?

$$(1) A = AB$$

$$C = Cb$$

$$(2) b = ba$$

$$c = ca$$

$$(3) a = aB$$

$$c = cA$$

$$(4) a = ab$$

$$b = bAc$$

$$(5) AB = AB$$

$$C = Ca$$

$$(6) \quad b = ba$$

$$c = cA$$

$$(7) \quad B = Ba$$

$$c = cd$$

$$(8) \quad aB = aB$$

$$b = ba$$

CHAPTER XIII.

DISJUNCTIVES.

326. In this chapter we shall consider Disjunctive propositions. Disjunctive terms are separated by the little conjunction "or," our sign for which is a short perpendicular mark, thus: |.

327. A Disjunctive proposition implies opposition, incompatibility, separation.

328. The Law of Identity holds good for Disjunctive propositions. The Law of Identity says that $A = A$. It is also true that $A \text{ or } B = A \text{ or } B$, and as the order of the terms makes no difference, it is also true that $B \text{ or } A = A \text{ or } B$.

329. Prof. Jevons in his Principles of Science says: "Few or no logicians except De Morgan, have adequately noticed the close relation between combined and disjunctive terms, viz: that every disjunctive term is the negative of a corresponding combined term and vice versa." Consider the term,

Malleable dense metal.

How shall we describe the class of things which are not malleable dense metals? Whatever is included under that term must have all the qualities of malleability, denseness and metallicity. Wherever one or more of the qualities is wanting, the combined term will not apply, hence the negative of the whole term is: "Not-malleable or not-dense or not-metallic."

Let $A =$ malleability,

$B =$ denseness,

$C =$ metallicity.

What is the negative of ABC?

Make an ABC diagram:

AB	Ab	aB	ab	
	1	1	1	C
1	1	1	1	c

Fig. 91.

Let us assume that ABC is the only combination in an ABC Reasoning Frame.

Now, if the only combination is ABC, then the combinations containing AB_c, Ab, a are inconsistent and we eliminate them by making a figure 1 in those sections.

We can now read in the Reasoning Frame:

- (1). The only combination is ABC.
- (2) No combination is AB_c | AbC | Abc | aBC | aBc | abC | abc.

Which can be reduced to:

Nothing = AB_c | Ab | a, hence, AB_c | Ab | a is the negative of ABC.

330. Prof. Jevons further says on the same page, 72,: "In the above, (i. e. not-malleable, not-dense or not-metallic) the conjunction "or" must necessarily be interpreted as unexclusive, for there may readily be objects which are both not-malleable and not-dense and perhaps not-metallic at the same time. If in fact, we were required to use "or" in a strictly exclusive manner, it would be requisite to specify seven distinct alternatives in order to describe the negative of a combination of three terms. The negative of four or five terms would consist of fifteen or thirty-one alternatives. This consideration alone, is sufficient to prove that the meaning of "or" cannot be always exclusive in common language,"

Now, in the example which he has given us it is requisite to specify seven distinct alternatives in order to describe the negative of a combination of three terms.

331. Prof. Venn in his *Symbolic Logic*, p. 280, in treating of this same subject, says: "The full contradictory of any given class expression may be defined as comprising 'all the rest' required to make up the Universe with which we are concerned. Thus, in the simplest case X and x are contradictories because X and $x = 1$. The complete theoretic process, therefore, for assigning the contradictory for any class expression involving two, three, four, etc., terms, would be to develop unity into its four, eight, sixteen, etc., elements, and then subtract from this the given expression. The remainder is the contradiction required."

332. An examination of any Reasoning Frame will convince the reader that it is impossible to eliminate all the combinations but one, without removing one or more letters, and, of course, as we have repeatedly shown, when we eliminate one letter entirely, we eliminate every combination in the Frame. Now, the logical meaning of eliminating every combination in the Reasoning Frame is that our premises are inconsistent. Everything = ABC is an impossible proposition.

333. The old logic took little notice of many forms of disjunctive propositions.

The disjunctive proposition may have a disjunctive subject or a disjunctive predicate, or both.

Prof. Jevons gives this proposition as an example of the doubly disjunctive form,

Solids or liquids or gases are electrics or conductors of electricity.

Let A = solids,

B = liquids,

C = gases,

D = electrics,

E = conductors of electricity.

Now, let us make an ABCDE diagram: To make an ABCDE diagram, we first make an ABCD diagram and then we divide the CD sections into those which are E and which are e, by drawing a horizontal line through the center of the CD sec-

tions, which gives us the CDE and CD_e sections; and by dividing the Cd sections into those which are E and those which are e, by drawing a horizontal line through the Cd sections, which gives us the CdE and the Cd_e sections, and by dividing the cd sections into those which are E and those which are e, by drawing a horizontal line through the cd sections, which gives us the cdE and the cd_e sections; and by dividing the cd sections into those which are E and e, by drawing a horizontal line through the cd sections, which gives us the cdE and the cd_e sections, thus:

AB	Ab	aB	ab	
				CDE
				CD _e
				CdE
				Cd _e
				cDE
				cD _e
				cdE
				cd _e
				cd _e

Fig. 92.

I assume that we can read the proposition backward, thus:
 Electrics or conductors of electricity are solids or liquids or gases.

The propositions can be stated thus:

- (1) A | B | C = D | E
- (2) D | E = A | B | C

Make an ABCDE diagram:

AB	Ab	aB	ab	
			1	CDE
2	2	2		CDe
2	2	2		CdE
			1	Cde
	1	1		cDE
2			2	cDe
2			2	cdE
	1	1		cde

Fig. 93.

The full logical expression of the proposition $A | B | C = D | E$ is:

$$Abc | aBc | abC = De | dE.$$

Now, if $A | B | C = D | E$, then the combinations containing $AbcDE$, $Abcde$, $aBcDE$, $aBcde$, $abCDE$, $abCde$, are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $D | E = A | B | C$, then the combinations containing $ABCDe$, $abcdE$, are inconsistent and we eliminate them by making a figure 2 in those sections.

From the combinations which remain we can get the following definitions:

- (1) $ABC = DE | de$
- (2) $ABc = DE | de$
- (3) $AbC = DE | de$
- (4) $Abc = De | dE$
- (5) $aBC = DE | de$
- (6) $aBc = De | dE$
- (7) $abC = De | dE$
- (8) $abc = DE | de$

They can be translated thus:

- (1) What is solid and liquid and gaseous is either both an electric and conductor of electricity or neither.
- (2) What is solid and liquid is either both an electric and conductor of electricity or neither.
- (3) What is both solid and gaseous is either both an electric and conductor of electricity or neither.
- (4) A solid is either an electric or a conductor of electricity.
- (5) What is both liquid and gaseous is either both an electric and conductor of electricity or neither.
- (6) A liquid is either an electric or a conductor of electricity.
- (7) A gas is either an electric or a conductor of electricity.
- (8) What is neither solid nor liquid nor gaseous is either both an electric and conductor of electricity or neither.

We can also get the following definitions:

- (1) $DE \mid de = AC \mid aBC \mid ABc \mid abc$,
- (2) $De \mid dE = Abc \mid aBc \mid abC$.

They can be translated thus:

- (1) What is both an electric and conductor of electricity or neither, is solid and gaseous or liquid and gaseous or solid and liquid or neither solid nor liquid nor gaseous.
- (2) What is an electric or a conductor of electricity is a solid or a liquid or a gas.

334. Given the disjunctive proposition, $A \mid B = C \mid D$, the question is, what propositions are equivalent to it?

Make an ABCD diagram:

AB	Ab	aB	ab	
	1	1		CD
				Cd
				cD
	1	1		cd

Fig. 94.

Now, if $A | B = C | D$, then the combinations containing $AbCD$, $Abed$, $aBCD$, $aBcd$, are inconsistent and we eliminate them by making a figure 1 in those sections.

We can now read in the Reasoning Frame the following equivalent propositions.

- (1) $A = AB | AbCd | AbcD,$
- (2) $a = ab | aBCd | abcD,$
- (3) $B = BA | BaCd | Baed,$
- (4) $b = ba | bAcD | bACd.$

Let us assume that the proposition,

$A | B = C | D$ can be read backward,

The two propositions can be stated thus:

- (1) $A | B = C | D$
- (2) $C | D = A | B$

Make an ABCD diagram:

AB	Ab	aB	ab	
	1	1		CD
2			2	Cd
2			2	cD
	1	1		cd

Fig. 95.

Now, if $A | B = C | D$, then the combinations containing $AbCD$, $Abed$, $aBCD$, $aBcd$, are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $C | D = A | B$, then the combinations containing $ABCd$, $ABCd$, $abCd$, $abcD$, are inconsistent and we eliminate them by making a figure 2 in those sections.

From the combinations which remain we can get the following definitions by translating the figures in the sections by the word "No."

- (1) No $A | B = CD | cd$
No $C | D = AB | ab$

- (2) No BD | bd = Ac | aC
 No AC | ac = B | D
 (3) No AD | ad = B | C
 No BC | bc = A | D

335. Given the proposition $ab = cd$, the question is, what propositions are equivalent?

Make an ABCD diagram.

AB	Ab	aB	ab	
			1	CD
			1	Cd
			1	cD
				cd

Fig. 96.

Now, if $ab = cd$, then the combinations containing abC, abcD, are inconsistent and we eliminate them by making a figure 1 in those sections.

We can now read in the Reasoning Frame, the following propositions, which taken together are equivalent to the given proposition.

- (1) CD = ACD | aBCD
 (2) Cd = CdA | CdaB
 (3) cD = cDA | cDaB

336. Let us assume that the proposition, $ab = cd$, can be read backward.

The two propositions can be stated thus:

- (1) $ab = cd$
 (2) $cd = ab$

Make an ABCD diagram:

AB	Ab	aB	ab	
			1	CD
			1	Cd
			1	cD
2	2	2		cd

Fig. 97.

Now, if $ab = cd$, then the combinations containing abC , $abcD$, are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $cd = ab$, then the combinations containing Acd , $aBcd$, are inconsistent and we eliminate them by making a figure 2 in those sections.

We can now read in the Reasoning Frame,

- (1) $ab = cd$
- (2) $cd = ab$
- (3) $A = AC \mid AcD$
- (4) $B = BC \mid BeD$

The appearance of these Reasoning Frames shows that the propositions,

$A \mid B = C \mid D$, and

$ab = cd$, do not correspond to each other.

337. In our system, after stating the propositions we eliminate in the Reasoning Frame the inconsistent propositions and read the results.

Equivalent propositions will have the same appearance in the Reasoning Frames.

Non-equivalent propositions will have a different appearance.

338. According to my view, a proposition resembles a box which contains many different articles. When we lift the cover of the box we can see what is in the box. So in our system, when we eliminate the inconsistent combinations we can see the contents of the given proposition.

339. When we wish to ascertain whether propositions are equivalent, all that is necessary is to eliminate the inconsistent combinations and if the same combinations have been eliminated, then the propositions are equivalent.

340. A proposition is the expression of a thought, and, by our system, we make thoughts visible. It then becomes an easy matter to read them and to tell exactly the extent to which they agree or disagree.

341. Given the disjunctive propositions,

$$A \mid B = C \mid D$$

$$C \mid D = A \mid B$$

the question now arises, What propositions are contradictory to the given propositions?

A complete contradictory will cause us to eliminate the combinations which the given propositions saved and to save the combinations which the given propositions eliminated.

342. By the use of the Reasoning Frame I have discovered an easy method of solving this problem. It is this:

From the eliminated combinations get a pair of propositions containing the definitions of a letter-term and its negative. Any letter-term will do. This pair of definitions will together make a full contradictory to the given propositions.

Make an ABCD diagram:

AB	Ab	aB	ab	
	1	1		CD
2			2	Cd
2			2	cD
	1	1		cd

Fig. 98.

Now, if $A \mid B = C \mid D$, then the combinations containing AbCD, Abed, aBCD, aBed, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $C \mid D = A \mid B$, then the combinations ABCd, ABcD, abCd, abcD are inconsistent, and we eliminate them by making a figure 2 in those sections.

From the eliminated combinations we can get the following pair of definitions:

- (1) $B = ACd \mid AcD \mid aCD \mid aed$
- (2) $b = ACD \mid Aed \mid aCd \mid acD$

Make an ABCD diagram:

AB	Ab	aB	ab	
1			2	CD
	2	1		Cd
	2	1		cD
1			2	cd

Fig. 99.

Now, if $B = ABCd \mid ABcD \mid aBCD \mid aBcd$, then the combinations containing ABCD | ABcd | aBCD | aBcd, are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $b = AbCD \mid Abcd \mid abCd \mid abcD$, then the combinations containing AbCd, AbcD, abCD, abcD are inconsistent, and we eliminate them by making a figure 2 in those sections.

The result proves that we have found a pair of propositions which are completely contradictory to the given propositions.

We can also read in the eliminated combinations the following pairs of propositions which are each equivalent to the given propositions.

- (1) $No BD \mid bd = A \mid C$, and
 $No AC \mid ac = B \mid D$.
- (2) $No CD \mid cd = A \mid B$, and
 $No AB \mid ab = C \mid D$.

- (3) $\text{No } B \mid D = AC \mid ac$, and
 $\text{No } A \mid C = BD \mid bd$.
(4) $\text{No } AD \mid ad = B \mid C$, and
 $\text{No } BC \mid bc = A \mid D$.

We will prove the last one.

Now, if $\text{No } AD \mid ad = B \mid C$, then the combinations containing ABCD, AbCD, aBcd, abCd, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Make an ABCD diagram:

AB	Ab	aB	ab	
	1	2		CD
2			1	Cd
1			2	cD
	2	1		cd

Fig. 100.

Again, if $\text{No } BC \mid bc = A \mid D$, then the combinations ABCd, aBCD, Abcd, abcD are inconsistent, and we eliminate them by making a figure 2 in those sections.

The result proves that the pair of propositions,

$$\text{No } AD \mid ad = B \mid C, \text{ and}$$

$$\text{No } BC \mid bc = A \mid D,$$

is equivalent to the given propositions,

$$A \mid B = C \mid D$$

$$C \mid D = A \mid B$$

343. The next question is, Given any alternative definition of a letter-term, can we reduce it to a set of categorical propositions which shall be equivalent to the given alternative proposition?

We can generally do so, and perhaps always.

From the last diagram we can get the following alternative definition of A:

$$A = ABCD \mid ABcd \mid AbCd \mid AbcD.$$

We can reduce this to the following equivalent categorical propositions:

- (1) $ABC = ABCD$
- (2) $ABc = ABed$
- (3) $AbC = AbCd$
- (4) $Abe = AbeD$

This quartet of categorical definitions is equivalent to the given alternative definition.

Make an ABCD diagram:

AB	Ab	aB	ab	
	3			CD
1				Cd
2				cD
	4			cd

Fig. 101.

Now, if $ABC = ABCD$, then the combination containing $ABCd$ is inconsistent, and we eliminate it by making a figure 1 in that section.

Again, if $ABc = ABed$, then the combination $ABcD$ is inconsistent, and we eliminate it by making a figure 2 in that section.

Again, if $AbC = AbCd$, then the combination $AbCD$ is inconsistent and we eliminate it by making a figure 3 in that section.

Again, if $Abe = AbeD$, then the combination $Abed$ is inconsistent, and we eliminate it by making a figure 4 in that section.

The result proves the equivalence of the quartet of categorical propositions with the given alternative proposition.

344. The reader will understand that a diagram is at first a blank diagram, and that we mark it as we proceed with our work, so that when he sees it he does not see the diagram as it

was first made, but as it is after we have finished our work. It will be proper for him to bear in mind, therefore, that before commencing work he is supposed to be looking at a blank diagram, and each section is supposed to be blank until he is told to put some mark in it.

345. Whenever we wish to obtain the alternative definition of a term which is common to several categorical propositions, all we have to do is to mark these sections in the Reasoning Frame which contain the several categorical propositions, and then read the definition of the common term by saying that it is one or the other of all these combinations which are marked with a sign.

346. And, again, if we wish to break up an alternative definition into categorical propositions, all that is necessary to do is to mark the sections which contain the several alternants in the alternative definition, with a sign, and then from those sections which are so marked we read the categorical definitions which they imply.

347. Let us take another example of a disjunctive proposition from Prof. Jevon's Principles of Science, p. 74: Senior's definition of wealth:

"Wealth is what is transferable, limited in supply and either productive of pleasure or preventive of pain."

Let A = wealth,

B = transferable,

C = limited in supply,

D = productive of pleasure,

E = preventive of pain.

The proposition can be stated,

$$A = ABC(D \mid E)$$

which when developed becomes,

$$A = ABCDe \mid ABCdE$$

Prof. Jevons gives another alternative, viz.: ABCDE, because he places a different meaning on the word "or" from that which I do. He says that ordinarily "or" means one or

the other or both, whilst I contend that the usual meaning of "or" is one or the other and not both.

We can also obtain from Senior's definition of wealth this proposition: That whatever is transferable and limited in supply and productive of pleasure or preventive of pain, is wealth.

It can be stated thus:

$$BC(D \text{ or } E) = ABC(D \text{ or } E)$$

which can be developed into,

$$BCDe | BCdE = ABCDe | ABCdE$$

Now let us make another ABCDE diagram:

AB	Ab	aB	ab	
1	5			CDE
	5	3		CDe
	5	4		CdE
2	5			Cde
6	5 6			cDE
6	5 6			cDe
6	5 6			cdE
6	5 6			cde

Fig. 102.

Now, if $A = ABCDe | ABCdE$, then the combination ABCDE is inconsistent, because it implies that $A =$ both D and E, and we therefore eliminate it by making a figure 1 in that section.

Again, if $A = ABCDe$ or $ABCdE$, then the combination ABCde is inconsistent because it implies that $A =$ neither D nor E, and we therefore eliminate it by making a figure 2 in that section.

Again, if $BCDe = A$, then the combination $aBCDe$ is inconsistent because it implies that $BCDe = a$ and we therefore eliminate it by making a figure 3 in that section.

Again, if $BCdE = A$, then the combination $aBCdE$ is inconsistent because it implies that $BCdE = a$ and we therefore eliminate it by making a figure 4 in that section.

It also appears on the face of the proposition that Wealth is transferable, which can be stated,

$$A = AB$$

We say $A = AB$ because it does not follow from "Wealth is transferable" that what is transferable is wealth, and therefore we cannot say that $B = A$.

It is not necessary that we should always state in our symbolical language, in the first instance, all the *prima facie* meanings of the given proposition. We may not perceive them when we first read the given proposition, but if we perceive them afterwards, we can still state them and eliminate the inconsistent propositions. One great advantage of this system is that as our knowledge increases we state our new knowledge in our symbolical language and deduce the consequences which follow. We are not limited to the propositions with which we started at first.

Now, if $A = AB$, then all the combinations which contain Ab are inconsistent because they imply that $A = b$, and we therefore eliminate them by making a figure 5 in those sections.

Again, it appears on the face of the given proposition that Wealth is limited in supply. We can state it thus:

$$A = AC$$

Now, if $A = AC$ then the combinations which contain Ac are inconsistent because they imply that $A = c$, and we therefore eliminate them by making a figure 6 in those sections.

From the combinations which remain we can get the following definitions:

- (1) $A = ABCDe \mid ABCdE$, which translated is:

Wealth is transferable, limited in supply, and is either productive of pleasure or preventive of pain.

(2) $CDe | CdE = AB | ab$, which translated is:

What is limited in supply and productive of pleasure but not preventive of pain, or what is limited in supply and preventive of pain but not productive of pleasure is either wealth and transferable, or it is neither.

We can also get definitions of the other terms in the given proposition, but as the definitions would contain so many alternatives they would be of little practical use.

348. The reader will have noticed that the rule for combining alternatives is simply to combine each alternant of one, with each alternant of the other.

349. Let us take another example of an alternative proposition from Prof. Jevons:

"Gems are either rare stones or beautiful stones."

Let $A = \text{gems}$,

$B = \text{rare stones}$,

$C = \text{beautiful stones}$.

The proposition can be stated thus:

$$A = ABC | AbC.$$

Make an ABC diagram:

AB	Ab	aB	ab	
1				C
	1			c

Fig. 103.

Now, if $A = ABC | AbC$, then the combinations ABC and AbC are inconsistent and we eliminate them by making a figure 1 in those sections. ABC is inconsistent because it implies that $A = \text{both } B \text{ and } C$, and AbC is inconsistent because it implies that $A = \text{neither } B \text{ nor } C$.

From the combinations which remain we can get the following definitions:

- (1) $A_c = BA_c$, which translated is:

A gem which is not a beautiful stone is rare.

- (2) $AC = bAC$, which translated is:

A gem which is a beautiful stone is not rare.

As a matter of fact, these definitions are not true. It therefore follows that our original proposition was not true.

The original proposition should have been:

"Gems are either rare or beautiful stones, or both."

Our proposition can be stated thus:

$$A = ABc \mid AbC \mid ABC$$

Make an ABC diagram:

AB	Ab	aB	ab	
				c
	1			c

Fig. 104.

Now, if $A = ABc \mid AbC \mid ABC$ then the combination Abc is inconsistent, because it means that $A =$ neither B nor C , and we therefore eliminate it by making a figure 1 in that section.

From the combinations which remain we can get the following definitions:

- (1) $A = ABc \mid AbC \mid ABC$, which translated is:

Gems are either rare but not beautiful stones or beautiful but not rare stones, or they are both beautiful and rare.

- (2) $B = AB \mid aBC \mid aBc$, which translated is:

A rare stone is either a gem or it is beautiful and not a gem, or it is neither.

The point I wish to make in giving this illustration is, that when a man means one or the other or both, he should say so,

or, if when he says one or the other he clearly means "one or the other or both," then we should state what he means and not what he says, otherwise our conclusions will be wrong.

350. Let us take another example from Prof. Jevons:

- (1) Red colored metal is either copper or gold.
- (2) Copper is dissolved by nitric acid.
- (3) This specimen is red colored metal.
- (4) This specimen is not dissolved by nitric acid.

Let A = this specimen,

B = red colored metal,

C = copper,

D = gold,

E = dissolved by nitric acid.

The propositions can be stated as follows:

- (1) B = BCd | BcD
- (2) C = CE
- (3) A = AB
- (4) A = Ae
- (5) C = Cd
- (6) D = Dc

Now make an ABCDE diagram:

AB	Ab	aB	ab	
4 5	5 4	5	5	CDE
1	3	1		
5	5	2		
2	2	1	2	CDe
1	3	5	5	
	4			
4	3			CdE
	3			
2	2	2	2	Cde
4	3			cDE
	4			
	3			cDe
4	4			cdE
1	3	1		
1	3	1		cde

Fig. 105.

Now, if $B = BCd \mid Bcd$ then the combinations which contain $BCD \mid Bcd$ are inconsistent because they imply that $B = CD \mid cd$, and we therefore eliminate them by making a figure 1 in those sections.

They are ABCDE, ABCDe, ABcdE, ABede, aABCDE, aBCDe, aBcdE, aBede.

Again, if $C = CE$, then all the combinations containing Ce are inconsistent because they imply that $C = e$, and we therefore eliminate them by making a figure 2 in those sections.

They are ABCDe, ABCde, AbCDe, AbCde, aBCDe, aBCde, abCDe, abcDe.

Now again, if $A = AB$, then all the combinations contain-

ing Ab are inconsistent because they imply that A = b and we therefore eliminate them by making a figure 3 in those sections.

They are AbCDE, AbCDe, AbCdE, AbCde, AbcDE, AbcDe, AbcdE, Abcde.

Now again, if A = Ae, then all the AE combinations are inconsistent because they imply that A = E, and we therefore eliminate them by making a figure 4 in those sections.

They are ABCDE, ABCdE, ABcDE, ABedE, AbCDE, AbCdE, AbcDE, AbedE.

Again, if C = Cd, then all the combinations which contain CD are inconsistent because they imply that C = D, and we therefore eliminate them by making a figure 5 in those sections.

They are ABCDE, AbCDE, aBCDE, abCDE, ABCDe, AbCDe, aBCDe, abCDe.

Again, if D = Dc, then all the combinations which contain CD are inconsistent, but it is unnecessary to eliminate them because they are already eliminated.

From the combinations which remain we can get the following definition:

A = ABcDe, which translated is:

This specimen is red colored metal, and not-copper, but
is gold and is not dissolved by nitric acid:

which can be contracted into,

This specimen is gold.

351. This process of reasoning is called *abcessio infiniti*, and it means the obtaining of a name by getting rid of the alternatives.

The reader will understand that the reason why so many sections in this diagram are not eliminated is, that they are not inconsistent with any information given to us in the original propositions. So far as data has been furnished us they are true propositions, but of course the definition of a or b or C or D or E or c or d or e, would contain so many alternants that the definitions would be of little practical use.

As we have stated elsewhere, whenever there is more than one combination containing a given letter, then the definition of that letter is always an alternative definition. The number of alternants in the alternative will be equal to the number of combinations which contain the letter which is to be defined.

352. Let us take the following example from Prof. Venn:

"The members of a Board were all of them either bondholders or shareholders, but not both, and the bondholders, as it happened, were all on the Board."

Let $A =$ member of the Board,

$B =$ bondholder

$C =$ shareholder.

The propositions can be stated thus:

- (1) $A = ABC \mid AbC$
- (2) $B = AB$

Make an ABC diagram:

AB	Ab	aB	ab	
1		2		C
	1	2		c

Fig. 106.

Now, if $A = ABC \mid AbC$, then all the other combinations of A are inconsistent.

There are four A combinations and we therefore eliminate the combinations ABC and AbC .

Again, if $B = AB$, then all the combinations which contain aB are inconsistent, because they imply that $B = a$ and we therefore eliminate them by making a figure 2 in those sections, which are aBC and aBc .

From the combinations which remain we can get the following definitions:

- (1) $C = CAB \mid CAB$, which translated is:

Shareholders are either members of the Board and not bondholders, or they are neither members nor bondholders,

which can be simplified,

"The shareholders are not bondholders."

We already know that by the Law of the Excluded Middle, that shareholders are either members of the Board or not members of the Board, and therefore we can drop this statement:

- (2) $B = ABe$, which translated is:

A bondholder is a member of the Board and not a shareholder.

- (3) $a = ab (C \mid c)$, which translated is:

A non-member of the Board is not a bondholder.

It is unnecessary to translate C or c ; we know by the Law of the Excluded Middle that anything is C or c .

- (4) $c = cAB \mid cab$, which translated is:

A non-shareholder is either a member of the Board and a bondholder, or he is neither.

Prof. Venn says that "this problem was proposed in examination and lecture rooms to some 150 students, as a problem in ordinary logic; it was answered by, at most, five or six of them. It was afterwards set as an example of Boole's Method, to a small class who had attended a few lectures on the nature of the Symbolic Methods; it was readily answered by half or more of their number."

The conclusion arrived at by Boole's method was, "No shareholders are bondholders." By our method we not only get that conclusion, but we get a great many more.

353. A Disjunctive proposition always implies that one of its alternants must be true, otherwise the whole proposition must be false.

354. I want to add a caution to the reader at this point, in regard to reasoning from Disjunctive propositions. The difficulty is, that in many cases we cannot be sure that the

alternants are complete, that is, that we have stated all of them. When we say A is B or C, the question arises whether it may not also be D or E. If we can say that A is only B or C, we shall have no trouble. But if it is a doubtful matter whether A is only B or C, then we cannot be sure of our conclusions.

355. In working our system we must get rid of every ambiguity, for if there is ambiguity in the premises, there will be ambiguity in the conclusions, and if there is no ambiguity in the premises, there cannot be any ambiguity in the conclusions, if our operations have been correctly performed.

356. Our system is perfect, but we are liable to make mistakes in working it. First, we may not understand the propositions which are given to us and hence we may interpret them wrongly.

Second, through inadvertence, we may omit to eliminate an inconsistent combination, or we may eliminate a combination which ought to stand. Third, in reading the combinations which remain, we may neglect to read one or more of them. We should take care not to make these mistakes.

357. When we are in doubt whether an alternative proposition contains all the alternants, we should so state the proposition that it will cover omitted cases, if any such there be. Thus, if the proposition given us is, A = B or C, and we think that possibly there are other things which "A" may be, then we should state it:

$$A = ABC \mid AbC \mid Abc,$$

which translated is:

$$A = B \mid C \mid \text{neither}.$$

The proposition as stated thus, simply denies that A = both B and C.

For instance, "Wealth must be either spent or hoarded; it is not hoarded, therefore it is spent." This sort of reasoning is fallacious, because wealth may be neither spent nor hoarded.

Again, "If it is spring, you are to blame for not sowing; if it is autumn, you are to blame for not reaping, but it is either

spring or autumn, therefore, you are to blame for either not sowing or not reaping." This also is fallacious, because the enumeration of the parts is not complete—it may be either summer or winter.

358. Where the enumeration is complete and the members are exclusive, i. e., only one of them can be true, then if any one of them is true, all the others will be inconsistent.

For instance, "this event occurred in spring, summer, autumn or winter; it occurred in spring, therefore it did not occur in any of the others."

And of course it follows that if one of the alternants is not true, then one of the others which remain must be true. Thus, "It did not occur in summer, therefore it must have occurred in one of the others."

359. Disjunctive propositions sometimes appear in this form:

Either B or C exists,

Let A = exists.

The proposition can be stated thus:

$$Bc \mid bC = A$$

Make an ABC diagram:

AB	Ab	aB	ab	
			1	C
				c
	1			

Fig. 107.

Now, if $Bc \mid bC = A$, then the combinations aBc , abC are inconsistent and we eliminate them by making a figure 1 in those sections.

We can now read in the Reasoning Frame:

$B \mid C = A$, which can be translated thus:

Either B | C = some existing thing.

360. Prof. Bain in "Deductive and Inductive Logic," p. 119, gives the following example of a disjunctive proposition:

"Either the witness is perjured or the prisoner is guilty."

Let A = witness,

B = perjured,

C = prisoner,

D = guilty.

The proposition can be stated thus:

$$A = B \mid C = D$$

Make an ABCD diagram:

AB	Ab	aB	ab	
1 2				CD
	2 1	2	2	Cd
	1			cD
	1			cd

Fig. 108.

I assume that the proposition can be read backward thus:

Either the prisoner is guilty or the witness is perjured.

It can be stated thus:

$$C = D \mid A = B$$

Now, if $A = B$, except where $C = D$, then the combinations containing ABCD, AbCd, Abc are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $C = D$, except where $A = B$, then the combinations containing ABCD, AbCd, aBCd, abCd, are inconsistent, and we eliminate them by making a figure 2 in those sections.

From the combinations which remain we can get the following definitions:

$$(1) A = B \mid C = D$$

$$(2) C = D \mid A = B$$

(3) $Ab = CD$, which can be translated,

If the witness is not perjured the prisoner is guilty.

(4) $Cd = AB$, which can be translated,

If the prisoner is not guilty the witness is perjured.

361. Dr. Keynes in his "Formal Logic," p. 314, gives an example of a disjunctive proposition and a categorical proposition, which is called in the old logic, the *modus ponendo tollens*. Thus:

$$A = B \mid C$$

$$A = B, \text{ therefore,}$$

$$A = \text{not} \cdot C$$

which can be stated thus:

$$A = ABC \mid AbC$$

$$A = BA$$

Make an ABC diagram:

AB	Ab	aB	ab	
1	2			C
	2			c
	1			

Fig. 109.

Now, if $A =$ either $ABC \mid AbC$, then the combinations ABC , AbC , are inconsistent because they imply that $A =$ either both B and C , or neither B and C , and we therefore eliminate them by making a figure 1 in those sections.

Again, if $A = AB$, then the combinations AbC , Abc , are inconsistent because they imply that $A = b$, and we therefore eliminate them by making a figure 2 in those sections.

From the combinations which remain we can get this definition,

$$A = ABC$$

which translated is,

$A = B$ and c , which can be contracted into,

$A = c$, because we are not obliged to give any more of a definition than suits our purpose.

362. In this case we have followed our usual interpretation of the word "or." If we had followed the interpretation that "or" means one or the other or both, the conclusion obtained in this case would not be true.

The definition of A would have been $A = C \mid c$.

The validity of the *modus ponendo tollens* depends upon the exclusive meaning of the word "or."

363. Dr. Keynes in his "Formal Logic," p. 312, gives this example of disjunctive reasoning, which in the old logic is called, *modus tollendo ponens*:

"A is either B or C

"A is not B

"Therefore A is C."

We can state the propositions thus:

$$(1) A = ABC \mid AbC$$

$$(2) A = Ab$$

Make an ABC diagram:

AB	Ab	aB	ab	
1				
2				C
2	1			c

Fig. 110.

Now, if $A = ABC \mid AbC$, then the combinations ABC, AbC are inconsistent because they imply that A = either both B and C or neither, therefore we eliminate them by making a figure 1 in those sections.

Again, if $A = b$, then the combinations ABC, AbC are inconsistent because they imply that A = B. We therefore eliminate them by making a figure 2 in those sections.

From the combinations which remain we can get the following definition:

$A = AbC$, which can be translated,

$$A = C$$

364. Let us take the following example:

- (1) Either $A = B$ | $C = D$, and conversely,
- (2) $A = b$.

Make an ABCD diagram:

AB	Ab	aB	ab	
1 2				CD
2	1	1	1	Cd
2	1			cD
2	1			cd

Fig. 111.

Now, if $A = B$, except where $C = D$, and conversely, then the combinations containing $ABCD$, $AbCd$, Abc , aCd , are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $A = b$, then the combinations containing AB are inconsistent, and we eliminate them by making a figure 2 in those sections.

We can now read in the Reasoning Frame,

$$C = D$$

365. A writer says, "Logicians have not, as a rule, given any distinctive recognition to arguments consisting of two disjunctive premises and a disjunctive conclusion, and Mr. _____ goes so far as to remark that 'both premises of a syllogism cannot be disjunctive, since from two assertions as indefinite as disjunctive propositions necessarily are, nothing can be inferred.' It is, however, clear that this is erroneous." And he gives this example (the lettering is mine):

"Either A is not true or B is true;

"Either C is not true or A is true;

"Therefore, either C is not true or B is true."

I assume that the premises can be stated thus:

$$(1) A = d \mid B = E$$

$$(2) C = f \mid A = D$$

Make an ABCDEF diagram:

ABC								
		3	3	6		6		DEF
4		4 3	3					DEf
2	2	3	3	6		6		DeF
2 4	2	4 3	3					Def
1 5	1	5		6		6		dEF
1	1							dEf
5		5		6		6		deF
								def

Fig. 112.

An ABCDEF diagram consists of sixty-four sections. It is made by dividing our square by seven equi-distant horizontal lines and by seven equi-distant perpendicular lines.

Now, if $A = d$, except where $B = E$, then the combinations containing ABdE are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $A = d$, except where $B = E$, then the combinations containing ABDe are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if $A = d$, except where $B = E$, then the combinations containing AbD are inconsistent, and we eliminate them by making a figure 3 in those sections.

Again, if $C = f$, except where $A = D$, then the combinations containing ACDf are inconsistent, and we eliminate them by making a figure 4 in those sections.

Again, if $C = f$, except where $A = D$, then the combinations containing $ACdF$ are inconsistent, and we eliminate them by making a figure 5 in those sections.

Again, if $C = f$, except where $A = D$, then the combinations containing aCF are inconsistent, and we eliminate them by making a figure 6 in those sections.

The result proves that the given premises are not inconsistent, and that there are a large number of inferences which can be drawn from the given propositions.

366. A writer says, "In a pure, alternative syllogism, both of the premises and the conclusions are alternatives, e. g.,

$C \text{ is } D, \text{ or } A \text{ is not } B,$

$E \text{ is } F, \text{ or } C \text{ is not } D,$

Therefore, $E \text{ is } F, \text{ or } A \text{ is not } B.$ "

The propositions can be stated thus:

(1) $C = CD \mid A = Ab$

(2) $E = EF \mid C = Cd$

The conclusion can be stated,

$E = EF \mid A = Ab$

Make an ABCDEF diagram:

ABC	ABc	AbC	Abc	aBC	aBc	abC	abc	
		1						DEF
2	2	$\frac{1}{2}$	2	2	2	2	2	DEf
		1						DeF
		1						Def
$\frac{1}{2}$				1		1		dEF
$\frac{1}{2}$	2			2		2		dEf
1	2	2	2	1	2	1	2	def
1				1		1		deF
1				1		1		def

Fig. 113.

Now, if $C = CD$, except where $A = Ab$, then the combinations containing $AbCD$, $ABCd$, aCd , are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $E = EF$, except where $C = Cd$, then the combinations containing $CdEF$, $CDEF$, cEf , are inconsistent, and we eliminate them by making a figure 2 in those sections.

We cannot read in the Reasoning Frame the conclusion given in the text.

Make an ABCDEF diagram:

ABC								
		1	1					DEF
1	1			1	1	1	1	DEF
								DeF
								Def
		1	1					dEF
1	1			1	1	1	1	dEf
								deF
								def

Fig. 114.

Now, if $E = EF$, except where $A = Ab$, then the combinations containing $AbEF$, $ABEf$, aEf , are inconsistent, and we eliminate them by making a figure 1 in those sections.

This Reasoning Frame now shows the logical expression of $E = EF | A = Ab$.

The difference in the appearance of the two Reasoning Frames is very striking, and it shows that the conclusion given in the text is not correct.

367. In working this system, remember that we look to see what a proposition denies, or, in other words, we seek for the

propositions which can be eliminated. We do not look for the propositions which agree with the given proposition, but we look for those which are inconsistent with it.

368. A writer gives this example of a disjunctive proposition: "Every blood vessel is either a vein or an artery."

Let $A = \text{blood vessel}$,

$B = \text{vein}$,

$C = \text{artery}$.

The proposition may be stated,

$$A = ABC \mid AbC$$

Make an ABC diagram:

AB	Ab	aB	ab	
1		7		
3		4	6	C
	2	5		
		8		c

Fig. 115.

Now, if $A = ABC \mid AbC$, then the combination ABC is inconsistent because it means that $A =$ both B and C, and we eliminate it by making a figure 1 in that section.

Again, if $A = ABC \mid AbC$, then the combination Abc is inconsistent because it implies that $A =$ neither B nor C, and we therefore eliminate it by making a figure 2 in that section.

From the combinations which remain we can get the following definitions:

(1) $Ab = C$, which translated is:

A blood vessel which is not a vein is an artery.

(2) $AB = ABC$, which translated is:

A blood vessel which is a vein is not an artery.

(3) $C = Ab \mid aB \mid ab$, which translated is:

An artery is a blood vessel and not a vein, or a vein and not a blood vessel, or it is neither.

Now, this is the logical definition of an artery derivable from the proposition which was given to us. It is not stated in the proposition which was given to us, that an artery is not a vein, but we can assume that an artery is not a vein and state it thus:

$$C = Cb.$$

Now, if $C = Cb$, then the combination ABC is inconsistent, because it implies that $C = B$, and we therefore eliminate it by making a figure 3 in that section.

Again, if $C = Cb$, then the combination aBC is inconsistent because it implies that $C = B$, and we therefore eliminate it by making a figure 4 in that section.

The definition of C now is:

$$C = CAB \mid Cab, \text{ which translated is:}$$

An artery is a blood vessel and not a vein, or it is neither.

This is now the logical definition from the propositions which have been given and assumed of an artery. It is not stated in the propositions given and assumed that an artery is a blood vessel, but we may assume that an artery is a blood vessel.

It can be stated thus:

$$C = CA$$

Now, if $C = CA$, then the combination aBC is inconsistent because it implies that $C = a$, and we therefore eliminate it by making a figure 5 in that section.

Again, if $C = CA$, then the combination abC is inconsistent because it implies that $C = a$, and we therefore eliminate it by making a figure 6 in that section.

Our definition of C now is,

$$C = CAB, \text{ which translated is:}$$

An artery is a blood vessel and not a vein, or, simplified,

An artery is a blood vessel.

It is not stated in the proposition given us or in the proposition assumed, that a vein is a blood vessel, but we may assume that a vein is a blood vessel, and it can be stated thus:

$$B = BA$$

Now, if $B = BA$, then the combination aBC is inconsistent because it implies that $B = a$, and we therefore eliminate it by making a figure 7 in that section.

Again, if $B = A$, then the combination aBc is inconsistent because it implies that $B = a$, and we therefore eliminate it by making a figure 8 in that section.

Our definition of B now is,

$B = BA_c$, which translated is:

A vein is a blood vessel and not an artery.

The combination abc remains automatically.

From it we can get the following definitions:

(1) $a = abc$, which translated is,

What is a not-blood vessel is neither a vein nor an artery.

(2) $bc = abc$, which translated is:

Whatever is neither a vein nor an artery is not a blood vessel.

This last definition is true if in the original proposition given us the alternates were all stated. We have worked out this example for the purpose of showing that it is not necessary to state, in the first instance, all the propositions which may be worked in the Reasoning Frame; we can pursue our course and as we perceive new propositions to be true, we can state them and work them out in the Frame.

We saw in working out the proposition "Every blood vessel is either a vein or an artery" that we obtained the proposition "A blood vessel which is not a vein is an artery." We can read this, "If any blood vessel is not a vein, then it is an artery."

369. From a disjunctive proposition we can usually get equivalent hypothetical propositions. But a writer says, "Disjunctive judgments cannot be reduced to hypotheticals." In this I think he is mistaken. The two propositions, "A blood vessel which is not a vein is an artery," "If a blood vessel is not a vein it is an artery," are equivalents.

I have no doubt that disjunctives can usually, and perhaps always, be reduced to hypotheticals and both to categoricals.

Let us make an ABC diagram, and eliminate the combinations ABC and Abc by making an X in those sections:

AB	Ab	aB	ab	
X				C
	X			c

Fig. 116.

There are now two A-combinations, that is, combinations containing A, in the Reasoning Frame. They are ABC, AbC. Now we can read them categorically:

$$A \text{ which is } B = c$$

$$A \text{ which is } b = C$$

Disjunctively we can read them,

$$A = \text{either } B \text{ and } c \mid C \text{ and } b$$

Hypothetically we can read them,

$$\text{If } A = B \text{ it } = c$$

$$\text{If } A = b \text{ it } = C$$

This tends to prove that the three terms categorical, disjunctive and hypothetical, refer to three different ways of reading the same propositions.

370. A writer says, "It has, however, already been pointed out that two negative propositions do not in any sense state an alternative, since the denial of an alternative is equivalent to the affirmation of a conjunction; hence, if a complex alternative proposition is defined as a proposition which states an alternative by means of an alternative predicate, then only affirmative propositions can fall into this category. Distinctions of quantity cannot be applied to true compound alternatives, nor can distinctions of quality."

If I understand the meaning of this statement, which I probably do not, the writer would deny the validity of propositions like these:

$$A \mid b = C \mid d, \text{ and conversely,}$$

$a \mid b = c \mid d$, and conversely.

If so, I think he is mistaken.

Let us make an ABCD diagram:

AB	Ab	aB	ab	
	2	2		CD
1			1	Cd
1			1	cD
	2	2		cd

Fig. 117.

$A \mid b = C \mid d$, means $AB \mid ab = CD \mid cd$, because $A \mid b$ means A without b, and that is the same as saying A with B, on the principle that two negatives make an affirmative. Without b is equivalent to B in this case. It is a general principle of logic that two negatives are equal to an affirmative; not not-A means A.

Now we would express A without B thus: Ab; and A or B means A without B or B without A, that is, Ab or aB.

Now, if A without B means Ab, then A without b must mean AB. AB means A without b.

Again, A or b means, b without A, that is, ab, so that the full expression of A or b is,

AB or ab

Again, the full expression of C or d is,

CD | cd

I assume that we can state the proposition,

$A \mid b = C \mid d$, conversely, thus:

$$(1) AB \mid ab = CD \mid cd$$

$$(2) CD \mid cd = AB \mid ab$$

Now, if $AB \mid ab = CD \mid cd$, then the combinations ABCd, ABcD, abCd, abcD are inconsistent because they say that $AB \mid ab = Cd \mid cD$, and we therefore eliminate them by making a figure 1 in those sections.

Again, if $CD \mid cd = AB \mid ab$, then the combinations $AbCD$, $aBCD$, $Abcd$, $aBcd$ are inconsistent, because they mean that $CD \mid cd = Ab \mid aB$, and we therefore eliminate them by making a figure 2 in those sections.

From the combinations which remain in the Reasoning Frame we can get the following definitions:

- (1) $Ab = Cd \mid cD$
- (2) $aB = Cd \mid cD$
- (3) $Cd = Ab \mid aB$
- (4) $cD = Ab \mid aB$
- (5) $AB = CD \mid cd$
- (6) $ab = CD \mid cd$
- (7) $CD = AB \mid ab$
- (8) $cd = AB \mid ab$

As all the letters remain in the Reasoning Frame our premises are consistent, and therefore the propositions are valid propositions.

371. Our second proposition was,

a or b is c or d , and conversely, which can be stated, thus:

- (1) $aB \mid Ab = cD \mid Cd$
- (2) $cD \mid Cd = aB \mid Ab$

Now, if $aB \mid Ab = cD \mid Cd$, then the combinations $AbCD$, $aBCD$, $Abcd$, $aBcd$, are inconsistent because they imply that $Ab \mid aB = CD \mid cd$, and we therefore eliminate them from an ABCD diagram, by making a figure 1 in those sections:

AB	Ab	aB	ab	
	1	1		CD
2			2	Cd
2			2	cD
	1	1		cd

Fig. 118.

Again, if $Cd \mid cD = Ab \mid aB$, then the combinations ABCd, ABcD, abCd, abcD, are inconsistent because they imply that $Cd \mid cD = AB \mid ab$, and we therefore eliminate them by making a figure 2 in those sections.

We have now eliminated the same combinations which the propositions,

$$\begin{aligned} A \mid b &= C \mid d, \text{ and} \\ C \mid d &= A \mid b \end{aligned}$$

caused us to eliminate; the definitions which can be obtained from the combinations which remain will be the same, and this proves that the proposition $a \mid b = c \mid d$, is equivalent to the proposition $A \mid b = C \mid d$.

372. Let us take this proposition:

a or b is c or D , and conversely,

We can state it thus:

- (1) $Ab \mid aB = cd \mid CD$
- (2) $cd \mid CD = Ab \mid aB$

Now, if $Ab \mid aB = cd \mid CD$, then the combinations AbCd, AbcD, aBCd, aBeD are inconsistent, because they imply that $Ab \mid aB = Cd \mid cD$, and we therefore eliminate them from an ABCD diagram by making a figure 1 in those sections.

AB	Ab	aB	ab	
2			2	CD
	1	1		Cd
	1	1		cD
2			2	cd

Fig. 119.

Again, if $cd \mid CD = Ab \mid aB$, then the combinations ABCD, ABcD, abCD, abed, are inconsistent because they imply that $cd \mid CD = AB \mid ab$, and we therefore eliminate them by making a figure 2 in those sections.

From the combinations which remain we can get the following definitions:

- (1) $Ab = AbCD \mid Abcd$
- (2) $aB = aBCD \mid aBcd$
- (3) $cd = cdAb \mid cdaB$
- (4) $CD = CDAb \mid CDaB$
- (5) $AB = ABCd \mid ABcD$
- (6) $ab = abCd \mid abcD$
- (7) $Cd = ABCd \mid abCd$
- (8) $cD = ABcD \mid abcD$

373. Let us take this proposition,

$a \mid B = C \mid D$, and conversely, which can be stated thus:

- (1) $ab \mid AB = Cd \mid cD$
- (2) $Cd \mid cD = ab \mid AB$

Make an ABCD diagram:

AB	Ab	aB	ab	
1			1	CD
	2	2		Cd
	2	2		cD
1			1	cd

Fig. 120.

Now, if $ab \mid AB = Cd \mid cD$, then the combinations $abCD$, $abcd$, $ABCD$, $ABcd$ are inconsistent, because they imply that $ab \mid AB = CD \mid cd$, and we therefore eliminate them by making a figure 1 in those sections.

Again, if $Cd \mid cD = ab \mid AB$, then the combinations $AbCd$, $AbeD$, $aBCd$, $aBcD$ are inconsistent, because they imply that $Cd \mid cD = Ab \mid aB$ and we therefore eliminate them by making a figure 2 in those sections.

We have now eliminated the same combinations which the proposition $a \mid b = c \mid D$, caused us to eliminate; the def-

inations are the same in both cases, and the propositions are therefore equivalent.

This way of elucidating these disjunctive propositions is, I think, entirely new.

374. We stated the definitions which we obtained in the last case, in the alternative form, but they can be reduced to categoricals. Whenever any letter appears in one combination only, it can be defined categorically by stating that it is the other letters in the combination. Thus, if we have the combination A_b_c , and that is the only combination in which c appears, the definition of c is, cA_b . Similarly, when we have any combination and that combination of letters appears only once, we can define it by the remaining letter or letters. Thus, if we have the combination $A_B_C D$ and the group $A_B C$ appears only in that combination, then we can say that $A_B C$ is $A_B C D$.

If we have the following combinations:

- (1) $A_B C D$
- (2) $A_B C e$

From these two combinations we can get the following categorical definitions:

- (1) $A_B C = A_B C D$
- (2) $A_B c = A_B C e$

or we can obtain the following disjunctive definition:

$$A_B = A_B C D \mid A_B C e$$

Categoricals containing common terms can always be combined into disjunctives, and disjunctives can generally be expanded into categoricals.

375. Let us take this example of a disjunctive proposition:

$$A \mid B \mid C = D, \text{ and conversely.}$$

Now, this means A without B and without C , or B without A and without C , or C without A and without B , is D .

It can be stated thus:

- (1) $A_b_c \mid aB_c \mid abC = D$
- (2) $D = A_b_c \mid aB_c \mid abC$

Make an $A_B C D$ diagram:

AB	Ab	aB	ab	
2	3	4		CD
			1	Cd
5			6	cD
	1	1		cd

Fig. 121.

Now, if $Abc \mid aBc \mid abC = D$, then the combinations $Abcd$, $aBcd$, $abCd$, are inconsistent, because they imply that the combinations Abc , aBc , $abC = d$, and we therefore eliminate them by making a figure 1 in those sections.

Again, if $D = Abc \mid aBc \mid abC$, then the combination $ABCD$ is inconsistent, because it implies that $D = ABC$, and we therefore eliminate it by making a figure 2 in that section.

Again, the combination $AbCD$ is inconsistent, because it implies that $D = AbC$, and we therefore eliminate it by making a figure 3 in that section.

Again, the combination $aBCD$ is inconsistent, because it implies that $D = aBC$, and we therefore eliminate it by making a figure 4 in that section.

Again, the combination $ABCd$ is inconsistent, because it implies that $D = ABC$, and we therefore eliminate it by making a figure 5 in that section.

Again, the combination $abcD$ is inconsistent, because it implies that $D = abc$, and we therefore eliminate it by making a figure 6 in that section.

We have now eliminated all the inconsistent combinations. From the combinations which remain we can get the following definitions:

- (1) $abC \mid Abc \mid aBc = D$
- (2) $ABC = ABCd$
- (3) $A B c = A B c d$
- (4) $AbC = AbCd$

- (5) $\text{Abc} = \text{AbcD}$
 (6) $\text{abC} = \text{abCD}$, etc.

376. Let us take this disjunctive proposition:

$a | b | C = D$ and conversely, which can be stated thus:

- (1) $\text{aBc} | \text{Abc} | \text{ABC} = \text{D}$
 (2) $\text{D} = \text{aBc} | \text{Abc} | \text{ABC}$

The reader will understand from previous explanations, that in this case a means without b and without C; and that without b means with B; and without C means with c, so that the full logical expression of a is aBc.

Again, b means without a and without C; without a means with A; without C means with c, so that the full logical expression of b is Abc.

Again, C means without a and without b, and that means, with A and with B, so that the full logical expression of C is ABC.

Make an ABCD diagram:

AB	Ab	aB	ab	
	2	2	2	CD
1				Cd
2			2	cD
	1	1		cd

Fig. 122.

Now, if $\text{aBc} | \text{Abc} | \text{ABC} = \text{D}$, then the combinations aBcd, Abcd, ABCd, are inconsistent, because they imply that $\text{Abc} | \text{aBc} | \text{ABC} = d$, and we therefore eliminate them by making a figure 1 in those sections.

Again, if $\text{D} = \text{aBc} | \text{Abc} | \text{ABC}$, then all the other combinations which contain D are inconsistent, and we eliminate them by making a figure 2 in those sections.

They are AbCD, aBCD, abCD, ABcD, abcD.

From the combinations which remain we can get the following definitions:

- (1) D = ABC | Abc | aBc
- (2) ABC = ABCD
- (3) ABc = ABcD
- (4) AbC = AbCd, etc.

377. Again, let us take this example:

a | b | c = D | E and conversely, which can be stated thus:

- (1) aBC | AbC | ABc = De | dE
- (2) De | dE = aBC | AbC | ABc

Make an ABCDE diagram:

AB	Ab	aB	ab	
1	1			CDE
2			2	CDe
2			2	CdE
	1	1		Cde
1				cDE
	2	2	2	cDe
	2	2	2	cdE
1				cde

Fig. 123.

Now, if aBC | AbC | ABc = De | dE, then the combinations aABCDE, aBCde, AbCde, ABCDE, ABcDE, ABcde, are inconsistent, because they imply that aBC | AbC | ABc = DE | de, and we therefore eliminate them by making a figure 1 in those sections.

Again, if $De \mid dE = aBC \mid AbC \mid ABc$, then all the other combinations which contain $De \mid dE$ are inconsistent. They are $ABCDe$, $abcdE$, and we therefore eliminate them by making a figure 2 in those sections.

From the combinations which remain we can get the following definitions:

- (1) $De = AbC \mid aBC \mid ABc$
- (2) $dE = AbC \mid aBC \mid ABc$
- (3) $aBC = De \mid dE$
- (4) $AbC = De \mid dE$
- (5) $Abc = DE \mid de$
- (6) $ABCD = ABCDE$
- (7) $ABCd = ABCde$
- (8) $ABcD = ABcDe$
- (9) $ABed = ABedE$, etc.

378. Let us now take still more complicated forms of Disjunctive propositions:

$$\begin{array}{c} A = B \mid C = D \\ C = D \mid A = B \end{array}$$

I interpret this to mean $A = B$, except where $C = D$, $A = B$ unless $C = D$, and $C = D$ except where $A = B$, or $C = D$ unless $A = B$.

There are four combinations of CD , viz: CD , Cd , cD , cd . Therefore, the proposition $A = B$, except where $C = D$, means $A = B$ where $C = d$, $\mid c = D$, $\mid c = d$. And the proposition $C = D$ except where $A = B$ means, $C = D$ where $A = b$, $\mid a = B$, $\mid a = b$.

Make an $ABCD$ diagram:

AB	Ab	aB	ab	
1 3				CD
	3	3	3	Cd
	2			cD
	2			cd

Fig. 124.

The proposition $A = B \mid C = D$, also means that the combination ABCD is inconsistent, because it means $A = B$ and $C = D$, and we therefore eliminate it by making a figure 1 in that section.

Now, if $A = B$, except where $C = D$, then the combinations $AbCd$, $AbeD$, $Abed$ are inconsistent, because they imply that $A = b$, except where $C = D$, and we therefore eliminate them by making a figure 2 in those sections.

Again, if $C = D$, except where $A = B$, then the combinations $ABCD$, $AbCd$, $aBCd$, $abCd$ are inconsistent, because they imply that $C = d$, except where $A = B$, and we therefore eliminate them by making a figure 3 in those sections.

From the combinations which remain we can get the following definitions:

- (1) $AB = ABCd \mid ABcD \mid ABed$
- (2) $CD = AbCD \mid aBCD \mid abCD$

which we can read:

$$A = B \mid C = D,$$

that is, $A = B$ in every case except where $C = D$, and $C = D$ in every case except where $A = B$.

This form of a disjunctive proposition gave the old logicians considerable trouble.

I think this method of solving it is quite new.

CHAPTER XIV.

OR.

379. The little word “or” has given as much trouble to logicians as any other word in the English language. Before proceeding to discuss its meaning, I thought best to give a number of practical examples to show our method of working Disjunctive propositions.

Many logicians have contended that the usual meaning of “or” was “one or the other or both.” Others have insisted that it meant “one or the other and not both.”

380. Prof. Hamilton said that all disjunctives are to be regarded as exclusive, that is, when we say, ‘All A is X or Y,’ we are not only justified in inferring that any A which is not X is Y, but also that any A which is X is not Y.’

381. Prof. Venn says, Symbolic Logic, p. 48, “Thus to say, ‘He is deceiver or deceived’ is by no means the same thing as to say, ‘He is deceiver and deceived.’”

Prof. Venn also says that “or” means “one or the other or both.”

382. When we take the position that “or” means one or the other and not both, except in those cases where it is expressly stated that “or” means “one or the other or both,” it is easy to be consistent in our interpretation of the word “or” wherever it occurs. But if we take the other interpretation, i. e., that it means one or the other or both, we shall find it difficult to maintain our consistency, and shall oscillate from one meaning to the other.

383. If a man should say, “Arches are circular or pointed or both,” we would naturally conclude that he did not know well what he was talking about, and I think that it is generally true, that when a man uses “or” in the sense of “one or the

other or both" he does so because he is more or less ignorant of his subject.

384. St. Aquinas held to the opinion that when "or" did not mean "one or the other and not both" that the proposition was false, and Kant held the same opinion. This meaning of "or" is called the "exclusive meaning; the other is called the "non-exclusive."

385. Prof. Boole also took the "exclusive" side, but Archbishop Whately, Prof. Mansel and J. S. Mill have taken the "non-exclusive" side.

Archbishop Whately gives this example:

"Virtue tends to procure us either the esteem of mankind or the favor of God."

386. Prof. Jevons says in "Principles of Science," p. 68, "I discuss this subject fully, because it is really the point which separates my logical system from that of Boole."

387. When a man thinks that "or" means "one or the other or both," when he comes to state an alternative proposition he ought to say of his subject that it is "one or the other or both." If he says, "Matter is solid or liquid" and means that it is "solid or liquid or both," then he ought to say expressly, that "matter is solid or liquid or both." If he says "one or the other or both" we can state that expression symbolically and solve our logical problems just as correctly and easily as when "or" is used in the exclusive sense.

388. There is another meaning to the word "exclusive" for if we say, "one or the other or both," then "or" means that only one of those alternatives can be true; whichever alternative is the true meaning, it excludes both the others.

389. Miss Jones, a very talented logician, says in "Elements of Logic," p. 117, that besides Kant and Hamilton, Thompson, Bain and Fowler take the exclusive view. She gives this example and says:

"Every ragged person either is poor or wishes to be thought poor.' This seems to me an extremely ingenious ex-

ample, and at first sight very telling on the side of unexclusive-
ness; but if its full meaning were expressed, would it not run
as follows: ‘Every ragged person is ragged either because he
is poor, or because he wishes to be thought poor,’ and in this
case the alternation is exclusive.”

She says, “Alternatives must always have some element of
exclusiveness, otherwise they have no logical value whatever.”

Prof. Keynes takes the unexclusive side.

390. In our system when we wish to state that $A = B \mid C$
but not both, we do it in this way:

$$A = Be \mid bC$$

And when we wish to state that $A = B \mid C$ or both, we can
do it in either of the following ways:

- (1) $A = Be \mid bC \mid BC$
- (2) $A = B \mid bC$

In the first case the combinations ABC and Abc are incon-
sistent; in (1) and (2) the combination Abc is the only incon-
sistent combination.

391. I think that “or” is indefinite and that sometimes it is
used exclusively and sometimes unexclusively. Unless the
unexclusive meaning is plainly indicated I always give it the
exclusive meaning.

EXERCISES.

392. (1) What is a complete equivalent for $a \mid b = c \mid d$?
 (2) What is a complete equivalent for $AB = CD$?
 (3) What is a complete equivalent for $a \mid B = c \mid D$?
 (4) What are the categorical equivalents for the following
alternative proposition:

$$a = aBCd \mid aBcd \mid abCD \mid abcD?$$

- (5) What are the categorical equivalents for the following
alternative proposition:

$$B = ABCD \mid ABcd \mid aBCd \mid aBcd?$$

- (6) What alternative proposition is an equivalent for the fol-
lowing categoricals:

$$ABCD, AbCD, aBCD?$$

(7) What alternative proposition is equivalent to the following categoricals:

$$\text{Acdb, dcba, bade?}$$

(8) What conclusions can be drawn from the following pair of propositions:

$$\text{C} = \text{B} \mid \text{A}$$

$$\text{B} = \text{BA}?$$

(9) What conclusions can be drawn from the following pair of propositions:

$$\text{B} = \text{either A} \mid \text{C}$$

$$\text{B} = \text{a}?$$

(10) What conclusions can be drawn from the following pair of propositions:

$$\text{A} \mid \text{D} = \text{B} \mid \text{E}$$

$$\text{C} \mid \text{F} = \text{A} \mid \text{D}?$$

(11) What conclusions can be drawn from the following pair of propositions:

$$\text{a} \mid \text{b} = \text{a} \mid \text{c}$$

$$\text{a} = \text{C}?$$

(12) What conclusions can be drawn from the following pair of propositions:

$$\text{A} \mid \text{B} \mid \text{C} = \text{d}$$

$$\text{d} = \text{A} \mid \text{B} \mid \text{C}?$$

(13) What conclusions can be drawn from the following pair of propositions:

$$\text{A} \mid \text{B} = \text{CD} \mid \text{cd}$$

$$\text{C} \mid \text{D} = \text{AB} \mid \text{ab}?$$

CHAPTER XV.

HYPOTHETICAL PROPOSITIONS.

393. Hypothetical propositions commonly indicate a certain amount of doubt and usually commence with the word "if," "when," "where," "whenever," "wherever," "given," "granted," "provided," "since," or some other similar conjunction.

394. "If A = B" is an indefinite statement. It denies nothing and has little if any more predicative force than an identical proposition such as AB = AB. It does not mean that A = B.

395. A hypothetical proposition can be converted into a categorical proposition by the use of the words "the case of."

If Cæsar was an usurper he deserved death, is a hypothetical proposition. It can be converted into the categorical proposition, the case of Cæsar being an usurper is a case of Cæsar deserving death.

396. A hypothetical proposition usually has a suppressed premise and when this is supplied we can convert the hypothetical proposition into two categorical propositions and then deduce the conclusion in the usual way. In the hypothetical proposition,

If Cæsar was an usurper he deserved death, the implied premise is,

All usurpers deserve death.

397. Let us take this example:

- (1) The case of Cæsar is the case of an usurper,
- (2) All usurpers deserve death, therefore Cæsar deserved death.

Let A = the case of Cæsar,

B = the case of an usurper,

C = deserved death.

The propositions can be stated thus:

- (1) $A = AB$
- (2) $B = BC$

Make an ABC diagram:

AB	Ab	aB	ab	
	1			C
2	1	2		c

Fig. 125.

Now, if $A = AB$, then the combinations containing Ab are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $B = BC$, then the combinations containing Bc are inconsistent and we eliminate them by making a figure 2 in those sections.

From the uneliminated combinations we can get these definitions:

- (1) $AB = ABC$, which can be translated

The case of Cæsar being the case of an usurper, is a case where Cæsar deserved death.

- (2) $c = cb$, which can be translated:

One not deserving of death is not the case of an usurper.

398. Let us take this example:

- (1) Cæsar was an usurper,

(2) All usurpers deserve death, therefore Cæsar deserved death.

Let $A = \text{usurper}$,

$B = \text{deserved death}$,

$C = \text{Cæsar}$.

The propositions can be stated thus:

- (1) $C = CA$

- (2) $A = AB$

Make an ABC diagram:

AB	Ab	aB	ab	
2		1	1	C
2				c

Fig. 126.

Now, if **C = CA**, then the combinations containing **Ca** are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if **A = AB**, then the combinations containing **Ab** are inconsistent and we eliminate them by making a figure 2 in those sections.

From the uneliminated combinations we can get this definition,

C = CB, which can be translated:

Therefore, Cæsar deserved death.

399. Let us take this example,

- (1) If Cæsar was an usurper he deserved death,
- (2) Cæsar was an usurper.

Let **A = Cæsar**,

B = usurper,

C = deserved death.

The propositions can be stated thus:

- (1) **AB = ABC**,
- (2) **A = AB**

Make an ABC diagram:

AB	Ab	aB	ab	
	2			C
1	2			e

Fig. 127.

Now, if $AB = ABC$, then the combination ABC is inconsistent and we eliminate it by making a figure 1 in that section.

Again, if $A = AB$, then the combinations containing Ab are inconsistent, and we eliminate them by making a figure 2 in those sections.

From the uneliminated combinations we can get this definition of A ,

$A = C$, which can be translated:

Therefore, Cæsar deserved death.

Whenever we have a symbolic proposition reading, If $A = B$, $A = C$, we can always state it in this categorical form,

$AB = ABC$

This means where we have AB we must have ABC .

400. Many logicians divide hypothetical propositions into hypotheticals and conditionals.

A conditional proposition relates to some prior circumstance, for example:

If a child is spoiled his parents suffer.

The hypothetical implies a relation which always exists, for example:

"If patience is a virtue, there are painful virtues," but in our system we treat them both alike.

401. Let us take this example:

- (1) If $A = b$ then $A = c$
- (2) $A = C$

The propositions can be stated thus:

- (1) $Ab = Abc$
- (2) $A = AC$

Make an ABC diagram:

AB	Ab	aB	ab	
	1			C
2	2			c

Fig. 128.

Now, if $Ab = Abc$, then the combination AbC is inconsistent and we eliminate it by making a figure 1 in that section.

Again, if $A = AC$, then the combinations containing Ac are inconsistent and we eliminate them by making a figure 2 in those sections.

From the uneliminated combinations we can get this definition of A,

$$A = AB$$

402. Let us take this example:

- (1) If $A = B$ then $A = C$
- (2) $A = b$

The propositions can be stated thus:

- (1) $AB = ABC$
- (2) $A = Ab$

In this case the minor premise denies the antecedent of the major premise. The rule is, that from the denial of the antecedent the truth of the consequent cannot be inferred.

Make an ABC diagram:

AB	Ab	aB	ab	
2				C
2				c
1				

Fig. 129.

Now, if $AB = ABC$, then the combination ABC is inconsistent and we eliminate it by making a figure 1 in that section.

Again, if $A = Ab$ then the combinations containing AB are inconsistent and we eliminate them by making a figure 2 in those sections.

From the uneliminated combinations we can get this definition of A :

$$A = C | c$$

403. Let us take this example:

- (1) If $A = B$ then $A = C$
- (2) $A = c$

Therefore, $A = b$

The propositions can be stated thus:

- (1) $AB = ABC$
- (2) $A = Ac$

In this case the minor premise denies the consequent of major premise. The rule is that from the denial of the consequent, the contradictory of the antecedent can be inferred.

Make an ABC diagram:

AB	Ab	aB	ab	
2	2			C
1				c

Fig. 130.

Now, if $AB = ABC$, then the combination ABC is inconsistent and we eliminate it by making a figure 1 in that section.

Again, if $A = Ac$, then the combinations containing AC are inconsistent and we eliminate them by making a figure 2 in those sections.

From the uneliminated combinations we can get this definition of A ,

$$A = Ab$$

404. Let us take the following example:

- (1) If $A = b$ then $A = c$
- (2) $A = b$, therefore,

$$A = c$$

The propositions can be stated thus:

- (1) $Ab = Abc$
- (2) $A = Ab$

Make an ABC diagram:

AB	Ab	aB	ab	
2	1			C
2	.			c

Fig. 131.

Now, if $Ab = Abc$, then the combination AbC is inconsistent and we eliminate it by making a figure 1 in that section.

Again, if $A = Ab$, then the combinations containing AB are inconsistent and we eliminate them by making a figure 2 in those sections.

From the uneliminated combinations we can get this definition of A:

$$A = Ac$$

The propositions in this case are equivalent to the propositions in the last preceding case, because they eliminate the same combinations.

405. Let us take this example:

- (1) If $A = B$ then $A = C$
- (2) $A = C$

The propositions can be stated thus:

$$(1) AB = ABC$$

$$(2) A = AC$$

In this case the minor premise affirms the consequent of the major premise. The rule is, that from the affirmation of the consequent, the truth of the antecedent cannot be inferred.

Make an ABC diagram:

AB	Ab	aB	ab	
				C
1	2			c
2				

Fig 132.

Now, if $AB = ABC$, then the combination ABC is inconsistent and we eliminate it by making a figure 1 in that section.

Again, if $A = AC$, then the combinations containing AC are inconsistent and we eliminate them by making a figure 2 in those sections.

From the uneliminated combinations we can get this definition of A ,

$$A = B | b.$$

406. Let us take this example from Archbishop Whately,

(1) If the first preachers of the gospel had displayed no miracles they could not have obtained a hearing.

(2) But they did obtain a hearing.

(1) Let $A = \text{first preachers of the gospel}$,

(2) $B = \text{displayed miracles}$,

(3) $C = \text{obtained a hearing}$.

The propositions may be stated thus,

$$(1) Ab = Abc$$

$$(2) A = AC$$

Make an ABC diagram:

AB	Ab	aB	ab	
	1			C
2	2			c

Fig. 133.

Now, if $Ab = Abc$ then the combination AbC is inconsistent and we eliminate it by making a figure 1 in that section.

Again, if $A = Ac$, then the combinations containing Ac are inconsistent and we eliminate them by making a figure 2 in those sections.

From the uneliminated combinations we can get this definition of A,

$A = AB$, which can be translated:

Therefore, the first preachers of the gospel displayed miracles.

407. Let us take this example from Prof. Bain:

- (1) If the education of certain children is neglected, then they will grow up ignorant,
 (2) The education of certain children has been neglected.
 (1) Let $A = \text{education of certain children}$,

- (2) $B = \text{neglected}$,
 (3) $C = \text{ignorant}$.

The propositions can be stated thus:

- (1) $AB = ABC$
 (2) $A = AB$

Make an ABC diagram:

AB	Ab	aB	ab	
	z			C
1	2			c

Fig. 134.

Now, if $AB = ABC$, then the combination ABC is inconsistent and we eliminate it by making a figure 1 in that section.

Again, if $A = AB$, then the combinations containing Ab are inconsistent and we eliminate them by making a figure 2 in those sections.

From the uneliminated combinations we can get this definition of A ,

$A = ABC$, which can be translated:

The education of certain children has been neglected and they will grow up ignorant.

408. Let us take this example from Prof. Bain:

- (1) If the weather continues fine we shall go to the country.
- (2) The weather continues fine.

Let $A = \text{weather}$,

$B = \text{continues fine}$,

$C = \text{we}$,

$D = \text{shall go to the country}$.

The proposition means, where we have AB we must have $ABCD$, and it may be stated thus:

- (1) $AB = ABCD$
- (2) $A = AB$

Make an $ABCD$ diagram:

AB	Ab	aB	ab	
	2			CD
1	2			Cd
1	2			cD
1	2			cd

Fig. 135.

Now, if $AB = ABCD$, then the combinations containing $ABCd$, ABC , are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $A = AB$, then the combinations containing Ab are inconsistent and we eliminate them by making a figure 2 in those sections.

From the uneliminated combinations we can get this definition:

$AB = ABCD$, which can be translated:

The weather continues fine, therefore, we are going into the country.

409. In hypothetical propositions the granting of the consequent does not prove the truth of the antecedent, for example:

"If he caught the infection he will die."

His death does not prove that he caught the infection, because there are many causes of death besides the one mentioned.

Let $A = \text{he}$,

$B = \text{caught the infection}$,

$C = \text{will die}$

We can state the premises thus:

$$(1) AB = ABC$$

$$(2) A = AC$$

Make an ABC diagram:

AB	Ab	aB	ab	
				C
2	2			c
1				

Fig. 136.

Now, if $AB = ABC$, then the combination ABC is inconsistent and we eliminate it by making a figure 1 in that section.

Again, if $A = AC$, then the combinations containing AC are inconsistent and we eliminate them by making a figure 2 in those sections.

From the uneliminated combinations we can get this definition of A ,

$$A = B | b$$

The result proves that the granting of the consequent has not established the truth of the antecedent, viz: He caught the infection.

410. Let us take this example:

(1) If force is expended an equivalent force will be generated.

(2) An equivalent force is generated.

(1) Let $A = \text{force}$,

$B = \text{expended}$.

$C = \text{equivalent force}$,

$D = \text{generated}$.

The premises can be stated thus:

(1) $AB = ABCD$

(2) $C = CD$

Make an ABCD diagram:

AB	Ab	aB	ab	
				CD
1	2	2	2	Cd
1				cD
1				cd

Fig. 137.

Now, if $AB = ABCD$, then the combinations containing $ABCd$, ABC , are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $C = CD$, then the combinations containing Cd are inconsistent and we eliminate them by making a figure 2 in those sections.

From the uneliminated combinations we can get this definition of A,

$A = B | b$, which can be translated:

Force is expended or not expended.

This furnishes another example of the rule that the affirmation of the consequent does not establish the truth of the antecedent.

In this case we know that as a matter of fact when force is generated, force was expended; that is a matter of fact which science teaches but it is not a logical conclusion from the given premises.

411. Let us take this example:

(1) If this river has tides, the sea into which it flows must have tides.

(2) This river has tides.

(1) Let $A = \text{this river}$,

(2) $B = \text{river tides}$,

(3) $C = \text{the sea into which it flows}$,

(4) $D = \text{sea tides}$.

The premises can be stated thus:

- (1) $AB = ABCD$
- (2) $A = AB$

Make an ABCD diagram:

AB	Ab	aB	ab	
	2			CD
1	2			Cd
1	2			cD
1	2			cd

Fig. 138.

Now, if $AB = ABCD$, then the combinations containing ABCd, ABC are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $A = AB$, then the combinations containing Ab are inconsistent and we eliminate them by making a figure 2 in those sections.

From the uneliminated combinations we can get this definition of A,

$A = BCD$, which can be translated,

The river has tides and the sea into which it flows has tides.

412. Let us take this example:

(1) If this river has tides, the sea into which it flows must have tides.

(2) The sea into which it flows has not tides.

I think the proposition if $A = B$ then $C = D$ means that the only AB there is, is the one which is CD.

The premises can be stated thus:

- (1) $AB = ABCD$
- (2) $C = Cd$

Make an ABCD diagram:

AB	Ab	aB	ab	
2	2	2	2	CD
1				Cd
1				cD
1				cd

Fig. 139.

Now, if $AB = ABCD$, then the combinations containing $ABCd$, ABC are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $C = Cd$, then the combinations containing CD are inconsistent and we eliminate them by making a figure 2 in those sections.

From the uneliminated combinations we can get this definition of A ,

$A = b$, which can be translated:

This river has not tides.

413. Hypothetical propositions are of no value except for the categorical information which they contain.

It has been said that in the hypothetical proposition,

If $A = B$, then it $= C$,

conveys the categorical information that

All $B = C$

but I think this is a mistake. The categorical information conveyed is, that where we have the case of AB , we have also a case of C .

414. Prof. Venn gives this good illustration:

"Suppose some one had said in 1852, 'If Louis Napoleon becomes emperor he will be crowned'."

While it is true as a matter of fact, that all emperors are crowned, this is not a logical inference from the proposition, because, as Prof. Venn well says, "If one were to say in 1852 'If Napoleon becomes emperor he will be perjured'," we could not logically say, all emperors are perjured.

415. Miss Jones says, in "Elements of Logic," p. 113, (I have changed the letters,) "What any conditional proposition,

If any A is B, that A is C,

seems to me to assert is, that the Cness of any A is an inference from its being AB, thus, any conditional would be universal and affirmative. I think also that it implies the existence of some A's are not B's."

416. Unless there is something to indicate to the contrary, I treat all hypothetical propositions, and disjunctive propositions, excepting disjunctive propositions having the form of $A = B | C = D$, as Universals. I do not, however, always repeat the subject on paper before the predicate, on account of the tediousness of the process.

By this time the reader will have learned that in the case of Universals, the subject is to be repeated before the predicate, at least in the mind, whether it is put down on paper or not.

417. I usually treat disjunctive propositions of the form, $A = B | C = D$, as being doubly universal or reciprocating propositions, i. e., they are to be read and worked both forward and backward, thus:

$$\begin{array}{l} A = B | C = D \\ C = D | A = B \end{array}$$

418. There is no doubt that the proposition,

If any A is B, that A is C, implies the existence of some A's which are not B's.

Make an ABC diagram.

AB	Ab	aB	ab	
				C
1				c

Fig. 140.

Now, if where $A = B$ that $A = C$, then the combination APc is inconsistent, because it implies that where $A = B$ $A = c$, and we eliminate it by making a figure 1 in that section.

The Reasoning Frame now shows that in two cases $A = b$.

419. In hypothetical propositions the "if" of the antecedent may generally be replaced by "when" or "where" or "whenever" or "wherever" or "in the case in which," at our option.

420. Dr. Keynes says that a hypothetical is composed of two propositions and that in the form,

If A is true, then C is true,

A and C stand for propositions and the words, "is true" are introduced to make this clear.

421. In the old logic the quality of the consequent determines the quality of the hypothetical proposition,

If $A = B$ $C = D$
is affirmative,

If $A = B$ $C = d$
is negative,

422. We are liable to make the mistake that the two propositions,

If $A = B$ $C = D$
If $A = B$ $C = d$

are contradictories. We have seen that the test of contradiction is the total elimination of a letter-term from the Reasoning Frame.

But these two propositions do not cause the total elimination of any letter-term. The one asserts that where $A = c$ $C = D$; the other asserts where $A = B$ $C = d$.

The inference which results is, that No $A = B$.

Make an ABCD diagram:

AB	Ab	aB	ab	
2				CD
1				Cd
2				cD
1				cd
2				

Fig. 141.

Now, if $A = B$ $C = D$, then the combinations containing ABCd, ABC, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $A = B$ $C = d$, then the combinations containing ABCD, ABC, are inconsistent, and we eliminate them by making a figure 2 in those sections.

The result proves that $A = b$, No $A = B$.

423. Hypothetical propositions can be converted.

Let us take this example:

If $A = B$ then $A = C$

It can be stated thus:

$$AB = ABC$$

Make an ABC diagram:

AB	Ab	aB	ab	
				C
				c
1				

Fig. 142.

Now, if $AB = ABC$, then the combination ABC is inconsistent, and we eliminate it by making a figure 1 in that section.

We can now read in the Reasoning Frame,

If $A = c$, then $A = b$

This is called the converse of the original proposition.

424. Let us take this example from Dr. Keynes' "Formal Logic," p. 219:

"If a straight line falling upon two other straight lines make the alternate angles equal to one another, these two straight lines shall be parallel."

Let A = a straight line falling upon,

B = two other straight lines

C = make the alternate angles equal,

D = parallel.

We have already seen that the denial of the consequent is the denial of the antecedent, so that we can get this proposition,

If these two straight lines are not parallel, a straight line falling upon them will not make the alternate angles equal to one another.

The premises can be stated thus:

- (1) ABC = ABCD
- (2) Bd = ABc

Make an ABCD diagram:

AB	Ab	aB	ab	
				CD
1		2		Cd
2				cd
			2	cD
				cd

Fig. 143.

Now, if $ABC = ABCD$, then the combination ABCd is inconsistent, and we eliminate it by making a figure 1 in that section.

If $Bd = ABc$, then the combinations containing aBd, ABCd, are inconsistent, and we eliminate them by making a figure 2 in those sections.

We can now get the following definitions in the Reasoning Frame:

- (1) $ABC = ABCD$
- (2) $Bd = ABe$
- (3) $ABe = BD | Bd$

which can be translated,

A straight line falling upon two other straight lines which does not make the alternate angles equal, then the two straight lines are parallel or not parallel.

425. I believe that all the problems in geometry can be solved by our system, and that its use by mathematicians will bring to light a great many new geometrical truths, but, not being a mathematician myself, if I discovered a new truth in geometry, I would not know it.

426. It has been said that either C is true or A is not true is a disjunctive equivalent for, if A is true, then C is true.

The propositions can be stated thus:

- (1) If $A = B$, $C = D$
- (2) $C = D | A = b$

Make an ABCD diagram:

AB	Ab	aB	ab	
	2			CD
21		2	2	Cd
21				cD
21				cd

Fig. 144.

Now, if where $A = B$ $C = D$, then the combinations containing ABCd, ABc, are inconsistent, and we eliminate them by making a figure 1 in those sections.

If $C = D | A = b$, and conversely, then the combinations containing ABCd, ABc, AbCD, aBCd, abCd, are inconsistent and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that the propositions are not equivalent because they do not eliminate the same combinations.

The proposition $C = D \mid A = b$, is a much more definite proposition than the other, because it eliminates more combinations. But (1) is an inference from (2) because all the combinations eliminated by (1) are eliminated by (2), but not conversely.

427. In "Formal Logic," p. 223, Dr. Keynes says: "Mr. McColl writes (the lettering is mine) the expression, If A then B may be read, A implies B, or if A is true B must be true. The statement If A then B, implies a or B. But it may be asked, are not the two statements really equal; ought we not therefore to write, If A then $B = a \mid B$? Now, if the two statements are really equivalent their denials will also be equivalent."

Let us see if this will be the case, taking as concrete examples:

'If he persists in his extravagance he will be ruined;' 'He will either discontinue his extravagance or he will be ruined.'

The denial of, If A then B is (the contradictory of If A then B), and this denial may be read, 'He may persist in his extravagance without necessarily being ruined.'

The denial of a or B is Ab, which may be read, 'He will persist in his extravagance and he will not be ruined.'

Now, it is quite evident that the second denial is a much stronger and more positive statement than the first. The first only asserts the possibility of the combination Ab; the second asserts the certainty of the same combination. The denials of the statement, If A then B and a or B, having thus been proved to be not equivalent, it follows that the statements If A then B and a or B are themselves not equivalent, and that though a or B is a necessary consequence of, If A then B, yet, If A then B is not a necessary consequence of a or B (see Mind, 1880, pp. 50-54; one or two slight verbal changes have been made in this quotation)."

Let us now take up this statement, "If A then B implies a or

B." The A in this case is understood to stand for a proposition. Let it stand for the proposition "A is B." Let the B in this case stand for the proposition "C is D."

We can now state, If A then B, thus:

$$(1) \text{ If } A = B \text{ then } C = D$$

and we can express the statement a or B, thus:

$$(2) A = b \mid C = D$$

The propositions can be stated thus:

$$(1) AB = ABCD$$

$$(2) A = b \mid C = D$$

Make an ABCD diagram:

AB	Ab	aB	ab	
	2			CD
21		2	2	Cd
21				cD
21				cd

Fig. 145.

Now, if $AB = CD$, then the combinations containing ABCd, ABe, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $A = b$, except where $C = D$, and conversely, then the combinations containing ABCd, ABe, AbCD, aCd, are inconsistent, and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that the two propositions are not equivalent because they do not eliminate the same combinations. But (1) is an inference from (2).

428. Dr. Keynes says in the note on p. 223, of "Formal Logic," "Mr. Welton accepts Miss Jones' view up to a certain point, but apparently does not recognize all that it involves and hence obtains inconsistent results.

He regards,

(1) A or B, i. e., A is B or C is D,

(2) If not-A then B, i. e., if $A = b$ $C = D$, as equivalents.

For the contradictory of (1) he gives,

(3) Neither A nor B, i. e., Neither $A = B$ nor $C = D$,

and he considers that (2) yields as its contradictory,

(4) If not-A then not-B, i. e., If $A = b$ $C = d$.

This again being equivalent in his view to

(5) A or not-B, i. e., $A = B$ or $C = d$."

The explanations following the propositions are mine.

Dr. Keynes further says, "But (3) and (5) are obviously not equivalents. We may take Mr. Welton's concrete example (Logic, p. 281).

The propositions,

(a) This pen is either cross-nibbed or corroded by the ink,
and,

(b) This pen is neither cross-nibbed nor corroded by the ink,
are given as contradictions, but (a) is regarded as equivalent
to,

(c) If this pen is not cross-nibbed, it is corroded by the ink.

And for the contradictory of (c) Mr. Welton would give,

(d) If this pen is not cross-nibbed, it is not corroded by the
ink.

But (b) and (d) are clearly not equivalent to one another."

Let us work out these examples,

(1) can be stated thus:

$$(1) A = B \mid C = D$$

$$(2) C = D \mid A = B$$

Make an ABCD diagram:

AB	Ab	aB	ab	
1 2				CD
	1 2	2	2	Cd
	1			cD
	1			cd

Fig. 146.

Mr. Welton's (1)

Now, if $A = B$, except where $C = D$, then the combinations containing ABCD, AbCd, Abc, are inconsistent, and we eliminate them by making a figure 1 in those sections.

If $C = D$, except where $A = B$, then the combinations containing ABCD, AbCd, aBCd, abCd, are inconsistent, and we eliminate them by making a figure 2 in those sections.

Mr. Welton's (2) can be stated thus:

$$(1) \text{ Ab} = \text{AbCD}$$

Make an ABCD diagram:

AB	Ab	aB	ab	
				CD
	1			Cd
	1			cD
	1			cd

Fig. 147.

Mr. Welton's (2).

Now, if $\text{Ab} = \text{AbCD}$, then the combinations containing AbCd, Abc, are inconsistent, and we eliminate them by making a figure 1 in those sections.

The Reasoning Frames now show that (1) and (2) are not equivalents. But (2) is an inference from (1).

Mr. Welton's (3) can be stated thus:

- (1) No $A = B \mid C = D$
- (2) No $C = D \mid A = B$

Make an ABCD diagram:

AB	Ab	aB	ab	
	2	2	2	CD
1				Cd
1				cD
1				cd

Fig. 148.

Mr. Welton's (3)

Now, if $No A = B$, except where $C = D$, then the combinations containing $ABCd$, ABC , are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $No C = D$, except where $A = B$, then the combinations containing $AbCD$, $aBCD$, $abCD$, are inconsistent, and we eliminate them by making a figure 2 in those sections.

Now, by combining (1) and (3) we can learn whether they are contradictions, for, if they are contradictions, they will cause the total elimination of some letter-term.

We will put the figure 1 in those sections which are eliminated by (1) and the figure 3 in those sections which are eliminated by (3).

Make an ABCD diagram:

AB	Ab	aB	ab	
1	3	3	3	CD
3	1	1	1	Cd
3	1			cD
3	1			cd

Fig. 149.

Mr. Welton's (1) and (3) combined.

The Reasoning Frame now shows that all the A's and C's are eliminated, and this proves that (1) and (3) are contradictions.

Mr. Welton's (4) can be stated thus:

$$(1) \text{Ab} = \text{AbCd}$$

Make an ABCD diagram:

AB	Ab	aB	ab	
	1			CD
				Cd
	1			cD
	1			cd

Fig. 150.

Mr. Welton's (4).

Now, if $\text{Ab} = \text{AbCd}$, then the combinations containing AbCD, Abc are inconsistent and we eliminate them by making a figure 1 in those sections.

Now, by combining (2) and (4) we can learn whether they are contradictions. We will put a figure 2 in those sections which Mr. Welton's (2) eliminated and a figure 4 in those sections which his (4) eliminated.

Make an ABCD diagram:

AB	Ab	aB	ab	
	4			CD
	2			Cd
	2			cD
	2			cd

Fig. 151.

Mr. Welton's (2) and (4) combined.

The Reasoning Frame now shows that (2) and (4) are not contradictions, because no letter-term has been eliminated. But they are inconsistent because neither one can be read in the Reasoning Frame. This will be explained later.

They say $A = B$, No $A = b$,

Mr. Welton's (5) can be stated thus:

- (1) $A = B$, | $C = d$
- (2) $C = d$, | $A = B$.

Make an ABCD diagram:

AB	Ab	aB	ab	
	21	2	2	CD
21				Cd
	1			cD
	1			cd

Fig. 152.

Mr. Welton's (5).

Now, if $A = B$, except where $C = d$, then the combinations containing ABCd, AbCD, Abc, are inconsistent, and we eliminate them by making a figure 1 in those sections.

If $C = d$, except where $A = B$, then the combinations containing ABCd, AbCD, aCD, are inconsistent, and we eliminate them by making a figure 2 in those sections.

The Reasoning Frames now show that (3) and (5) are not equivalents.

Let us now take Mr. Welton's (a).

This pen is either cross-nibbed or corroded by the ink.

Let $A =$ this pen,

$B =$ cross-nibbed,

$C =$ corroded by the ink.

The premise can be stated thus:

- (1) $A = ABc$ | AbC

Make an ABC diagram:

AB	Ab	aB	ab	
1				C
	1			c

Fig. 153.

Mr. Welton's (a).

Now, if $A = ABC \mid AbC$, then the combinations ABC, Abc, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Mr. Welton's (b).

This pen is neither cross-nibbed nor corroded by the ink, can be stated thus:

$$(1) A = Abc$$

Make an ABC diagram:

AB	Ab	aB	ab	
1	1			C
1				c

Fig. 154.

Mr. Welton's (b).

Now, if $A = Abc$, then the combinations containing AB, AbC, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Now, if (a) and (b) are contradictories, we can ascertain that fact by combining the two propositions. We will put a in the sections which (a) eliminates, and b in the sections which (b) eliminates.

Make an ABC diagram:

AB	Ab	aB	ab	
b	b			C
a				
b	a			c

Fig. 155.

Mr. Welton's (a) and (b) combined.

The result proves that (a) and (b) are contradictions because all the A's are eliminated.

Mr. Welton's (c).

If this pen is not cross-nibbed, it is corroded by the ink, can be stated thus:

$$(1) \text{ Ab} = \text{AbC}$$

Make an ABC diagram:

AB	Ab	aB	ab	
				C
	1			c

Fig. 156.

Mr. Welton's (c).

Now, if $\text{Ab} = \text{AbC}$, then the combination Abc is inconsistent, and we eliminate it by making a figure 1 in that section.

The Reasoning Frame now shows that (a) and (c) are not equivalent. But (c) is an inference from (a).

Mr. Welton's (d).

If this pen is not cross-nibbed, it is not corroded by the ink, can be stated thus:

$$(1) \text{ Ab} = \text{Abc}$$

Make an ABC diagram:

AB	Ab	aB	ab	
	1			C
				c

Fig. 157.

Mr. Welton's (d).

Now, if $Ab = Abc$, then the combination AbC is inconsistent, and we eliminate it by making a figure 1 in that section.

Now, by combining (c) and (d) we can ascertain whether they are contradictories.

Make c in the combination which (c) eliminates and d in the combination which (d) eliminates.

Make an ABC diagram:

AB	Ab	aB	ab	
	d			C
	c			c

Fig. 158.

Mr. Welton's (c) and (d).

The result proves that they are not contradictories because no letter-term is eliminated. But they are inconsistent because neither can be read in the Reasoning Frame. This will be explained later.

They say $A = B$

This pen is cross-nibbed.

429. The Reasoning Frames have led to this discovery. Given two hypotheticals with the same antecedent and inconsistent consequents, their combination yields a categorical contradictory to the antecedent, thus:

- (1) If $A = B, C = D$
- (2) If $A = B, C = d$ yields,
No $A = B$, thus:

Make an ABCD diagram:

AB	Ab	aB	ab	
2				CD
1				Cd
2 1				cD
2 1				cd

Fig. 159.

Now, if $A = B \ C = D$, then the combinations containing ABCd, ABC, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $A = B \ C = d$, then the combinations containing ABCD, ABC, are inconsistent, and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that,

No $A = B$.

CHAPTER XVI.

HYPOTHETICAL PROPOSITIONS CONTINUED.

430. I have also made this discovery:

Given two hypotheticals having the same consequents and inconsistent antecedents, their combination yields a categorical having the subject of the consequents and the opposite of the predicate of the consequents for its subject, and the opposite of the subject of the antecedents for its predicate, and they also yield a categorical having the opposite of the subject of the consequents for its subject and the opposite of the subject of the antecedents for its predicate, thus:

- (1) If $A = B$, $C = D$
- (2) If $A = b$, $C = D$, yields,

$$Cd = a$$

$$c = a$$

Make an ABCD diagram:

AB	Ab	aB	ab	
				CD
1	2			Cd
1	2			cD
1	2			cd

Fig. 160.

Now, if $A = B$, $C = D$, then the combinations containing ABCd, ABC, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $A = b$, $C = D$, then the combinations containing AbCd, Abc, are inconsistent, and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that,

$$Cd = a$$

$$c = a$$

431. Assume that disjunctive propositions of the form,

$$A = B \mid C = D$$

are reciprocating, that is, they are to be worked both ways, then the use of the Reasoning Frame shows that this kind of a disjunctive proposition is more definite than a hypothetical.

$$\text{Let (1) } A = B \mid C = D$$

$$(2) A = B \mid C = d$$

$$(3) A = b \mid C = D$$

(1) and (2) are contradictories.

(1) and (3) are contradictories.

(2) and (3) are not contradictories.

Make an ABCD diagram:

AB	Ab	aB	ab	
1	2	2	2	CD
2	1	1	1	Cd
	21			cD
	21			cd

Fig. 161.

Now, if $A = B \mid C = D$, then the combinations containing ABCD, AbCd, Abc, aCd are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $A = B \mid C = d$, then the combinations containing ABCd, AbCD, Abc, aCD, are inconsistent and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows the visible expression of the combination of (1) and (2) and it proves that they are contradictories because the letter C is totally eliminated.

Make an ABCD diagram:

AB	Ab	aB	ab	
1	3			CD
3	1	3 1	3 1	Cd
3	1			cD
3	1			cd

Fig. 162.

Now, if $A = B \mid C = D$, then the combinations containing ABCD, AbCd, Abc, aCd, are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $A = b \mid C = D$, then the combinations containing AbCD, ABCd, ABC, aCd, are inconsistent and we eliminate them by making a figure 3 in those sections.

The Reasoning Frame now shows the visible expression of the result of the combination of (1) and (3) and it proves that (1) and (3) are contradictions because the letter A is eliminated.

Make an ABCD diagram:

AB	Ab	aB	ab	
	3 2	2	2	CD
3 2		3	3	Cd
3	2			cD
3	2			cd

Fig. 163.

Now, if $A = B \mid C = d$, then the combinations containing ABCd, AbCD, Abc, aCD, are inconsistent and we eliminate them by making a figure 2 in those sections.

Again, if $A = b \mid C = D$, then the combinations containing $AbCD$, $ABCd$, ABC , aCd , are inconsistent and we eliminate them by making a figure 3 in those sections.

The result shows the visible expression of the combination of (2) and (3) and it proves that they are not contradictories because no letter term has been eliminated.

The combination yields these results:

- (1) No $A = c$
- (2) No $C = a$
- (3) $a = c$
- (4) $C = ABD \mid Abd$
- (5) $D = ABC \mid a$

432. If we consider disjunctive propositions of the form,

- (1) $A \mid B = C \mid D$, i. e., $Ab \mid aB = Cd \mid cD$, as Universals, then disjunctives having the form,
 - (2) $A \mid B = C \mid d$, i. e., $Ab \mid aB = CD \mid cd$
 - (3) $A \mid b = C \mid D$, i. e., $AB \mid ab = Cd \mid cD$, are not contradictories to,
- (1) $A \mid B = C \mid D$.

Make an ABCD diagram:

AB	Ab	aB	ab	
	2 1	$\frac{1}{2}$		CD
				Cd
	2	2		cD
	1	1		cd

Fig. 164.

Now, if $A \mid B = C \mid D$, then the combinations containing $AbCD$, $Abcd$, $aBCD$, $aBeD$, are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $A \mid B = C \mid d$, then the combinations $AbCD$, $AbcD$, $aBCD$, $aBeD$, are inconsistent and we eliminate them by making a figure 2 in those sections.

The result proves that (1) and (2) are not contradictories. They yield this result,

$$A \mid B = C \mid D.$$

Make an ABCD diagram:

AB	Ab	aB	ab	
3	1	1	3	CD
				Cd
				cD
3	1	1	3	cd

Fig. 165.

Now, if $A \mid B = C \mid D$, then the combinations AbCD, Abcd, aBCD, aBcd, are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $A \mid b = C \mid D$, then the combinations containing ABCD, ABcd, abCD, abcd are inconsistent and we eliminate them by making a figure 3 in those sections.

The result proves that (1) and (3) are not contradictories. they yield these definitions:

$$(1) C = d$$

$$(2) D = o$$

Make an ABCD diagram:

AB	Ab	aB	ab	
3	2	2	3	CD
				Cd
	2	2		cD
3			3	cd

Fig. 166.

Now, if $A | B = C | d$, then the combinations $AbCD$, $AbcD$, $aBCD$, $aBcD$, are inconsistent and we eliminate them by making a figure 2 in those sections.

Again, if $A | b = C | D$, then the combinations $ABCD$, $ABcd$, $abCD$, $abcd$, are inconsistent and we eliminate them by making a figure 3 in those sections.

The result proves that (2) and (3) are not contradictions.

They yield these results,

- (1) $A | b = C | D$
- (2) $A | B = d$

But, if we were to regard disjunctive propositions having the form,

$$A | B = C | D$$

i. e., $Ab | aB = Cd | cD$

as reciprocating propositions to be worked both ways, then propositions having the form,

$$A | B = C | d$$

i. e., $Ab | aB = CD | cd$

$$A | b = C | D$$

i. e., $AB | ab = Cd | cD$

would be contradictions of,

$$A | B = C | D$$

i. e., $Ab | aB = Cd | cD$

433. Dr. Keynes says in "Formal Logic," p. 225, "It will be observed that on neither interpretation are 'If A then C' and 'If A then c' true contradictions."

The propositions can be stated thus:

- (1) $AB = ABCD$.
- (2) $AB = ABCd$

Make an ABCD diagram:

AB	Ab	aB	ab	
2				CD
1				Cd
2				cD
1				cd

Fig. 167.

Now, if $AB = CD$, then the combinations containing ABCd, ABc, are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $AB = Cd$, then the combinations containing ABCD, ABc are inconsistent and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that the two propositions are not true contradictions because they do not eliminate any letter term. They are, however, inconsistent because neither one of them can now be read in the Reasoning Frame.

434. Dr. Keynes also says, "As a concrete example we may take the propositions,

- (1) If this pen is not cross-nibbed it is corroded by the ink.
- (2) If this pen is not cross-nibbed it is not corroded by the ink."

Let A = pen,

B = cross-nibbed,

C = corroded by the ink.

The propositions can be stated thus:

- (1) $Ab = AbC$
- (2) $Ab = Aba$

Make an ABC diagram:

AB	Ab	aB	ab	
	2			C
	1			c

Fig. 168.

Now, if $Ab = \neg AbC$, then the combination Abc is inconsistent and we eliminate it by making a figure 1 in that section.

Again, if $Ab = Abc$, then the combination AbC is inconsistent and we eliminate it by making a figure 2 in that section.

The Reasoning Frame now shows that the two propositions are not contradictories because no letter-term is eliminated. They are, however, inconsistent because neither one of them can now be read in the Reasoning Frame..

I quote from Dr. Keynes "Formal Logic" so often, because I consider it the best work there is on the old logic.

435. In the old logic a hypothetical having the form of,
 If A is B then C is D, is called an affirmative proposition.
 If A is B then C is not-D is called a negative hypothetical.

As I believe that all propositions are affirmative, I would suggest that propositions which in the old logic are called universal affirmatives, be called identifying propositions and propositions which in the old logic are called universal negatives, be called excluding propositions. But until this suggestion is adopted by logicians I shall use the term "negative proposition," although protesting against it. I think "differential proposition" would be a good name for a particular negative proposition.

When propositions are doubly universal, that is, of the form of Hamilton's U:

A is B and B is A,

I suggest that such propositions be called reciprocating propositions.

436. Unless there is something to indicate to the contrary, I consider identifying propositions to be universal.

Take the hypothetical,

If A is B, C is D, and I think it means

If All A is some B, then All C is some D.

437. Let us take this example,

(1) If patience is a virtue, there are painful virtues.

I assume that this means,

If patience is a virtue, then painful virtues exist

Let A = patience,

B = virtue,

C = painful,

D = exist.

The proposition can be stated thus:

$$AB = ABCD.$$

Make an ABCD diagram:

AB	Ab	aB	ab	
				CD
1				Cd
				cd
1				cD
				cd
1				cd

Fig. 169.

Now, if AB = ABCD, then the combinations containing ABCd, ABC, are inconsistent and we eliminate them by making a figure 1 in those sections.

The Reasoning Frame now shows the visible expression of the proposition,

If patience is a virtue, then painful virtues exist.

438. Let us take this example,

- (1) If a righteous God exists, then the wicked will not escape their just punishment.
- (2) If the wicked escape their just punishment, a righteous God does not exist.

Let A = a righteous God,

B = exists,

C = wicked,

D = escape their just punishment.

The propositions can be stated thus:

- (1) AB = ABCd
- (2) CD = CDAb

Make an ABCD diagram:

AB	Ab	aB	ab	
1				CD
2		2	2	
				Cd
				cD
1				cd
1				

Fig. 170.

Now, if AB = Cd, then the combinations containing ABCD, ABC, are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if CD = Ab, then the combinations containing ABCD, aCD, are inconsistent and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame shows that the two propositions are consistent, because both of them can be read in the Reasoning Frame.

439. Let us take this example,

- (1) If A is true, then C is not true,
- (2) If C is true, then A is not true,

The premises can be stated thus:

- (1) $AB = ABCd$
- (2) $CD = CDAb$

Make an ABCD diagram:

AB	Ab	aB	ab	
2 1		2	2	CD
				Cd
1				cD
1				cd

Fig. 171.

Now, if $AB = Cd$, then the combinations containing $ABCD$, ABC , are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $CD = Ab$, then the combinations containing $ABCD$, aCD , are inconsistent and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that these propositions are consistent.

440. Let us take the following example,

- (1) If $A = B$ then $C = D$
- (2) If $E = F$ then $A = B$, therefore,
If $E = F$ $C = D$

The propositions can be stated thus:

- (1) $AB = ABCD$
- (2) $EF = EFAB$

Make an ABCDEF diagram:

ABC								
	1	2	2	2	2	2	2	DEF
	1							DEf
	1							DeF
	1							Def
1	1	2	2	2	2	2	2	dEF
1	1							dEf
1	1							deF
1	1							def

Fig. 172.

Now, if $AB = CD$, then the combinations containing ABCd, ABC, are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $EF = AB$, then the combinations containing EFAb, EFa, are inconsistent and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that,

If $E = F$ then $C = D$.

441. Let us take the following example:

- (1) If water is salt it will not boil at 212 degrees,
 - (2) Sea water is salt, therefore,
- Sea water will not boil at 212 degrees.

I assume that sea water is water,

Let A = water,

B = salt,

C = boil at 212 degrees,

D = sea water.

The propositions can be stated thus:

$$(1) AB = ABC$$

$$(2) D = DB$$

$$(3) D = DA$$

Make an ABCD diagram:

AB	Ab	aB	ab	
1	2	3	$\frac{3}{2}$	CD
1				Cd
	2	3	$\frac{3}{2}$	cD
				cd

Fig. 173.

Now, if $AB = ABC$, then the combinations containing ABC are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $D = DB$, then the combinations containing Db are inconsistent and we eliminate them by making a figure 2 in those sections.

Again, if $D = DA$, then the combinations containing Da are inconsistent and we eliminate them by making a figure 3 in those sections.

From the uneliminated combinations we can get the following definitions:

(1) $D = Dc$, which can be translated,

Sea water will not boil at 212 degrees.

(2) $AB = ABC$, which can be translated,

Salt water will not boil at 212 degrees.

From the eliminated combinations we can get the following definitions:

(1) $No D = C$, which can be translated,

No sea water will boil at 212 degrees.

(2) $No AB = C$, which can be translated,

No salt water will boil at 212 degrees.

442. Let us take the following example:

- (1) Whenever C is D then E is F,
- (2) Whenever A is B then C is D, therefore,
Whenever A is B, E is F.

The propositions can be stated thus:

- (1) CD = CDEF
- (2) AB = ABCD

Make an ABCDEF diagram:

ABC	A _B c	A _b C	A _b c	aBC	aBc	abC	abc	
2								DEF
1	2	1		1		1		DEF
1	2	1		1		1		DeF
1	2	1		1		1		Def
2	2							dEF
2	2							dEf
2	2							deF
2	2							def

Fig. 174.

Now, if $CD = EF$, then the combinations containing CDEf, CDeF, CDef, are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $AB = CD$, then the combinations containing ABCd, A_Bc, are inconsistent and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that,

Whenever A = B, E = F.

This example is a syllogism in Barbara, taken from Dr. Keynes' "Formal Logic," p. 300.

443. Let us take the following example in Darapti:

- (1) "Whenever C is D, E is F,
- (2) "Whenever C is D, A is B, therefore,
Sometimes when A is B, E is F."

The propositions can be stated thus:

- (1) $CD = CDEF$
- (2) $CD = CDAB$

Make an ABCDEF diagram:

ABC	A _B c	A _b C	A _{bc}	a _B C	a _B c	a _b C	a _{bc}	
		2		2		2		DEF
1		21		21		21		DEf
1		21		21		21		DeF
1		21		21		21		Def
								dEF
								dEf
								deF
								def

Fig. 175.

Now, if $CD = EF$, then the combinations containing $CDEF$, $CDeF$, $CDef$, are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $CD = AB$, then the combinations containing $CDAb$, $CDaB$, $CDab$, are inconsistent and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that we can get this definition of AB ,

$AB = ABEF \mid ABEf \mid ABeF \mid ABef$, which the old logic would translate,

Sometimes when A is B, E is F,
but I consider this too positive a statement to make. The Reasoning Frame leaves the matter in doubt.

444. Let us take this example:

(1) Never when C is D is it the case that A is B.

(2) Whenever E is F, C is D, therefore,

Never when E is F, is it the case that A is B, therefore,

Never when A is B, is it the case that E is F.

The propositions can be stated thus:

(1) No CD = AB

(2) EF = EFCD.

Make an ABCDEF diagram:

AB ^c	ABc	A ^b C	Abc	aBC	aBc	abC	abc	
1	2		2		2		2	DEF
1								DEF
1								DeF
1								Def
2	2	2	2	2	2	2	2	dEF
								dEf
								deF
								def

Fig. 176.

Now, if No CD = AB, then the combinations containing ABCD are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if EF = CD, then the combinations containing EF^c, EF^d, are inconsistent and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that we can draw the following conclusions:

- (1) Never when E is F is it the case that A is B,
- (2) Never when A is B is it the case that E is F.

(Keynes' "Formal Logic," p. 303.)

445. Let us take the following example:

- (1) If $A = B$, $C = D$,
- (2) $C = d$, therefore, $A = b$.

The premises can be stated thus:

- (1) $AB = ABCD$
- (2) $C = Cd$.

Make an ABCD diagram:

AB	Ab	aB	ab	
2	2	2	2	CD
1				Cd
1				cD
1				cd

Fig. 177.

Now, if $AB = CD$, then the combinations containing $ABCd$, ABC are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $C = d$, then the combinations containing CD are inconsistent and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that,

$$A = Ab.$$

The above is an example in the *modus tollens*, (also called the destructive hypothetical syllogism.)

(Dr. Keynes' "Formal Logic," p. 304.)

446. Let us take the following example:

- (1) If $A = b$ then $C = D$,

(2) $C = d$, therefore,

$A = B$.

The premises can be stated thus:

(1) $Ab = AbCD$

(2) $C = Cd$.

Make an ABCD diagram:

AB	Ab	aB	ab	
2	2	2	2	CD
	1			Cd
	1			cD
	1			cd

Fig. 178.

Now, if $Ab = CD$, then the combinations containing $AbCd$, Abc , are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $C = d$, then the combinations containing CD are inconsistent and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that,

$A = AB$.

This example is in the *modus tollens* and corresponds to Camestres.

447. Let us take the following example:

(1) If $A = B$ then $C = d$

(2) $C = D$, therefore,

$A = b$

The premises can be stated thus:

(1) $AB = ABCd$

(2) $C = CD$

Make an ABCD diagram:

AB	Ab	aB	ab	
1				CD
2	2	2	2	Cd
1				cD
1				cd

Fig. 179.

Now, if $AB = Cd$, then the combinations containing $ABCD$, ABC , are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $C = D$, then the combinations containing Cd are inconsistent and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that,

$$A = Ab.$$

This example is in the *modus tollens* and corresponds to Cesare.

448. Let us take the following example:

- (1) If $A = b$, then $C = d$
- (2) $C = D$, therefore, $A = B$

The premises can be stated thus:

- (1) $Ab = AbCd$
- (2) $C = CD$

Make an ABCD diagram:

AB	Ab	aB	ab	
	1			CD
2	2	2	2	Cd
	1			cD
	1			cd

Fig. 180.

Now, if $Ab = Cd$, then the combinations containing $AbCD$, Abc , are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $C = D$, then the combinations containing Cd are inconsistent and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that $A = AB$.

449. Let us take the following example:

- (1) If $A = B$ then $C = D$
- (2) $C = D$, therefore,
 $A = B$.

The premises can be stated thus:

- (1) $AB = ABCD$
- (2) $C = CD$

Make an ABCD diagram:

AB	Ab	aB	ab	
				CD
21	2	2	2	Cd
1				cD
1				cd

Fig. 181.

Now, if $AB = CD$, then the combinations containing $ABCd$, ABC , are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $C = D$, then the combinations containing Cd are inconsistent and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that,

$$A = B \mid b$$

Hence, it is a fallacy to regard the affirmation of the consequent as justifying the affirmation of the antecedent.

450. Let us take the following pair of hypothetical syllogisms:

- (1) If $A = B$ then $C = D$
- (2) $C = d$, therefore, $A = b$
- (3) If $C = d$ then $A = b$
- (4) $C = d$, therefore, $A = b$.

The premises can be stated thus:

- (1) $AB = ABCD$
- (2) $C = Cd$
- (3) $Cd = CdAb$
- (4) $C = Cd$

Make an ABCD diagram:

AB	Ab	aB	ab	
4	4	4	4	
2	2	2	2	CD
3 1		3	3	Cd
1				cD
1				cd

Fig. 182.

Now, if $AB = CD$, then the combinations containing $ABCd$, ABC , are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $C = d$, then the combinations containing CD are inconsistent and we eliminate them by making a figure 2 in those sections.

Again, if $Cd = Ab$, then the combinations containing $ABCd$, aCd , are inconsistent and we eliminate them by making a figure 3 in those sections.

Again, if $C = d$, then the combinations containing CD are inconsistent and we eliminate them by making a figure 4 in those sections.

The Reasoning Frame now shows that the given syllogisms

are not contradictories because no letter-term is eliminated and it also shows that their combination yields the conclusions,

$$A = Ab$$

$$C = Cd$$

EXERCISES.

451. (1) What conclusions can be drawn from the following pair of hypothetical propositions,

$$\text{If } A = b \text{ then } A = c$$

$$B = c?$$

(2) What conclusions can be drawn from the following pair of propositions,

$$\text{If } a = b \text{ then } a = c$$

$$a = C?$$

(3) What conclusions can be drawn from the following pair of propositions,

$$\text{If } aB = aBc \text{ then } a = c$$

$$a = aB?$$

(4) What conclusions can be drawn from the following pair of propositions,

$$\text{If } a = b \text{ then } C = D$$

$$a = b?$$

(5) What conclusions can be drawn from the following pair of propositions,

$$\text{If } A = B \text{ then } c = d$$

$$c = D?$$

(6) What conclusions can be drawn from the following pair of propositions,

$$\text{If } A = AC \text{ then } A = AB$$

$$\text{If } A = AC \text{ then } A = b?$$

(7) What conclusions can be drawn from the following pair of propositions,

$$\text{If } A = Ab \text{ then } D = De$$

$$\text{If } A = Ab \text{ then } C = CE?$$

(8) What conclusions can be drawn from the following pair of propositions,

$$\text{If } A = AC \text{ then } B = Bd$$

If $A = AC$ then No $A = d$?

- (9) What conclusions can be drawn from the following pair of propositions,

If $A = b$ then $C = d$

If $E = f$ then $A = b$?

- (10) What conclusions can be drawn from the following pair of propositions,

If No $A = B$ then $C = D$

$C = d$?

- (11) What conclusions can be drawn from the following pair of propositions,

Never when $A = B$ is it the case that $C = D$

No $A = B$?

CHAPTER XVII.

DILEMMAS.

452. Dr. Keynes, a most accomplished logician, in "Formal Logic," p. 316, says: "The proper place of the dilemma amongst hypothetical and disjunctive arguments is difficult to determine, inasmuch as conflicting definitions are given by different logicians.

The following definition may be taken, perhaps, as on the whole the most satisfactory:

A dilemma is a formal argument containing a premise in which two or more hypotheticals are conjunctively affirmed, and a second premise in which the antecedents of these hypotheticals are alternately affirmed or their consequents alternately denied ."

453. Prof. Bain in his "Deductive and Inductive Logic," p. 121, defines a dilemma thus:

"The dilemma contains a conditional and a disjunctive proposition. If the antecedent of a conditional is made disjunctive, there emerges, what Whately calls a 'Simple Constructive Dilemma.' He gives this example:

"If either A or B is, C is,
Now, either A or B is,
Therefore, C is."

454. Let us take this example:

(1) If the barometer falls then there will be either wind or rain,

(2) The barometer is falling, therefore, there will be either wind or rain.

Let A = barometer,

B = falls,

C = wind,

D = rain.

The premises can be stated thus:

- (1) $AB = Cd \mid cD$
- (2) $A = AB$

Make an ABCD diagram:

AB	Ab	aB	ab	
1	2			CD
	2			Cd
	2			cD
1	2			cd

Fig. 183.

Now, if $AB = C \mid D$, then the combinations ABCD, ABcd, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $A = B$, then the combinations containing Ab are inconsistent, and we eliminate them by making a figure 2 in those sections.

From the uneliminated combinations we can get this definition:

$A = BCd \mid BcD$, which can be translated,

The barometer is falling and there will be wind or rain.

455. Let us take this example:

(1) If the barometer falls then there will be either wind or rain.

(2) It is not raining.

The premises can be stated thus:

- (1) $AB = C \mid D$
- (2) $AB = ABd$

Make an ABCD diagram:

AB	Ab	aB	ab	
21				CD
				Cd
2				cD
1				cd

Fig. 184.

Now, if $AB = C | D$, then the combinations containing ABCD, ABcd, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $AB = d$, then the combinations containing ABD are inconsistent, and we eliminate them by making a figure 2 in those sections.

From the uneliminated combinations in the Reasoning Frame we can get this definition,

$ABd = ABdC$, which can be translated,

If the barometer is falling and there is no rain, then there is wind.

456. Let us take this example:

(1) If the barometer falls then there will be either wind or rain.

(2) It is raining.

The premises can be stated thus:

(1) $AB = C | D$

(2) $AB = ABD$

Make an ABCD diagram:

AB	Ab	aB	ab	
1				CD
2				Cd
				cD
2 1				cd

Fig. 185.

Now, if $AB = C | D$, then the combinations ABCD, ABcd, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $AB = D$, then the combinations containing ABd are inconsistent, and we eliminate them by making a figure 2 in those sections.

From the uneliminated combinations we can get this definition:

$ABD = ABCD$, which can be translated,

If the barometer is falling and there is rain then there is no wind.

457. Let us take this example:

(1) If the barometer falls then there will be either wind or rain,

(2) There is wind.

The premises can be stated thus:

(1) $AB = C | D$

(2) $AB = ABC$

Make an ABCD diagram:

AB	Ab	aB	ab	
1				CD
				Cd
2				cD
2 1				cd

Fig. 186.

Now, if $AB = C | D$, then the combinations ABCD, ABcd, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $AB = C$, then the combinations containing ABC are inconsistent, and we eliminate them by making a figure 2 in those sections.

From the uneliminated combinations we can get this definition:

$ABC = ABCd$, which can be translated,

If the barometer is falling and there is wind, then there is no rain.

458. Let us take this example:

(1) If the barometer falls then there will be either wind or rain.

(2) There is no wind.

The premises can be stated thus:

(1) $AB = C | D$

(2) $AB = ABc$

Make an ABCD diagram:

AB	Ab	aB	ab	
2 1				CD
2				Cd
				cD
1				cd

Fig. 187.

Now, if $AB = C | D$, then the combinations ABCD, ABcd, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $AB = c$, then the combinations containing ABC are inconsistent, and we eliminate them by making a figure 2 in those sections.

From the uneliminated combinations we can get this definition:

$ABC = ABCD$, which can be translated,

If the barometer is falling and there is no wind, then there is rain,

459. Let us take this example:

(1) If the barometer falls then there will be either wind or rain.

- (2) There is no wind,
- (3) There is no rain,

The premises can be stated thus:

- (1) $AB = C | D$
- (2) $AB = ABC$
- (3) $AB = ABD$

Make an ABCD diagram:

AB	Ab	aB	ab	
2 1				CD
3				
				Cd
2				
				cD
3				
				cd
1				

Fig. 188.

Now, if $AB = C \mid D$, then the combinations ABCD, ABCd, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $AB = c$, then the combinations containing ABC are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if $AB = d$, then the combinations containing ABD are inconsistent, and we eliminate them by making a figure 3 in those sections.

From the uneliminated combinations we can get this definition:

A = Ab, which can be translated,
The barometer is not falling.

460. Let us take this example from Prof. Bain's Logic, p. 122.

(1) If the orbit of a comet is diminished, either the comet passes through a resisting medium or the law of gravitation is partially suspended.

(2) But the second alternative is inadmissible; hence, if the orbit of a comet is diminished, there is a resisting medium.

Let A = orbit of a comet,

B = diminished,

C = passes through a resisting medium,

D = law of gravitation,

E = partially suspended.

The premises can be stated thus:

- (1) $AB = ABC \mid ABDE$
- (2) $No D = E$

Make an ABCDE diagram:

AB	Ab	aB	ab	
$\frac{1}{2}$	2	2	2	CDE
				CDe
				CdE
				Cde
2	2	2	2	cDE
1				cDe
1				cdE
1				cde

Fig. 189.

Now, if $AB = C \mid DE$, then the combinations containing ABCDE, ABcDe, ABcd, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $No D = E$, then the combinations containing DE are inconsistent, and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that we can get this definition of AB:

AB = ABC, which can be translated,

If the orbit of a comet is diminished, there is a resisting medium.

461. Let us take this example:

(1) If a classical education is worth the cost, then either it must be pre-eminently fitted to develop the mental powers, or it must furnish exceedingly valuable information.

(2) But neither alternative can be maintained; hence, a classical education is not worth the cost.

Let A = classical education,

B = worth the cost,

C = pre-eminently fitted to develop the mental powers,

D = furnish exceedingly valuable information,

The premises can be stated thus:

$$(1) AB = ABC \mid ABD$$

$$(2) A = Ac$$

$$(3) A = Ad$$

Make an ABCD diagram:

AB	Ab	aB	ab	
3	3	—	—	CD
2 1	2	—	—	Cd
2	2	—	—	Cd
3	3	—	—	cD
1	—	—	—	cd

Fig. 190.

Now, if $AB = C \mid D$, then the combinations ABCD, ABcd, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $A = c$, then the combinations containing AC are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if $A = d$, then the combinations containing AD are inconsistent, and we eliminate them by making a figure 3 in those sections.

From the uneliminated combinations we can get this definition of A:

A = Ab, which can be translated,

A classical education is not worth the cost.

462. Let us take this example:

(1) If schoolmasters can claim exemption from Poor's rates, then it must be either by statute or by the common law.

(2) Now no statute exempts them.

(3) And the common law does not apply; hence they can claim no exemption from Poor's rates.

Let A = schoolmaster,

B = claim exemption from Poor's rates,

C = statutes,

D = the common law.

The premises can be stated thus:

$$(1) AB = ABC \mid ABD$$

$$(2) \text{No } C = AB$$

$$(3) \text{No } D = AB$$

Make an ABCD diagram:

AB	Ab	aB	ab	
2 1 3				CD
2				Cd
3				cD
1				cd

Fig. 191.

Now, if $AB = C \mid D$, then the combinations ABCD, ABcd, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $\text{No } C = AB$, then the combinations containing ABC are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if $\text{No } D = AB$, then the combinations containing ABD are inconsistent, and we eliminate them by making a figure 3 in those sections.

From the uneliminated combinations we can get this definition of A:

A = Ab, which can be translated,

Schoolmasters can claim no exemption from Poor's rates.

(Bain's "Deductive and Inductive Logic," p. 122.)

463. Let us take this example from Smart's "Manual of Logic," p. 169:

- (1) If Aeschines joined in the public rejoicings, then he is inconsistent.
- (2) If he did not, then he is unpatriotic.
- (3) But he either joined or not.

Therefore, either he is inconsistent or unpatriotic, or he is both inconsistent and unpatriotic.

Let $A = \text{Aeschines}$,

$B = \text{joined in the public rejoicings}$,

$C = \text{inconsistent}$,

$D = \text{unpatriotic}$.

The premises can be stated thus:

- (1) $AB = ABC$.
- (2) $Ab = Abd$
- (3) $A = B | b$

Make an ABCD diagram:

AB	Ab	aB	ab	
				CD
		2		cd
				cd
1				cd
1	2			cd

Fig. 192.

Now, if $AB = C$, then the combinations containing ABC are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $Ab = D$, then the combinations containing Abd are inconsistent, and we eliminate them by making a figure 2 in those sections.

The proposition $A = B | b$, has no logical force because it

denies nothing, and it does not cause us to eliminate any combination.

From the uneliminated combinations we can get this definition of A:

$A = Cd \mid cD \mid CD$, which can be translated,
Aeschines was inconsistent or unpatriotic, or both.

464. Let us take this example:

(1) If a science furnishes useful facts it is worthy of being cultivated.

(2) If the study of it exercises the reasoning powers, it is worthy of being cultivated.

(3) But either a science furnishes useful facts, or its study exercises the reasoning powers, therefore it is worthy of being cultivated.

Let $A =$ the study of a science,

$B =$ useful facts,

$C =$ worthy of being cultivated,

$D =$ exercises the reasoning powers.

The premises can be stated thus:

$$(1) AB = ABC$$

$$(2) AD = ADC$$

$$(3) A = B \mid D$$

Make an ABCD diagram:

AB	Ab	aB	ab	
3				CD
	3			Cd
3 21	2			cD
1	3			cd

Fig. 193.

Now, if $AB = C$, then the combinations containing ABC are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $AD = C$, then the combinations containing ADC are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if $A = B \mid D$, then the combinations containing ABD , AbD are inconsistent, and we eliminate them by making a figure 3 in those sections.

From the uneliminated combinations we can get this definition of A:

$A = AC$, which can be translated,

The study of science is worthy of being cultivated.

465. Let us take this example:

(1) If this man were wise, he would not speak irreverently of scripture in jest.

(2) If he were good, he would not do so in earnest.

(3) But he does it either in jest or earnest,

Therefore he is either not wise or not good.

Let $A =$ this man,

$B =$ wise,

$C =$ speak irreverently of scripture in jest,

$D =$ good,

$E =$ speak irreverently of scripture in earnest.

The premises can be stated thus:

$$(1) AB = ABC$$

$$(2) AD = ADE$$

$$(3) A = C \mid E$$

Make an ABCDE diagram:

AB	Ab	aB	ab	
2 1 3	3 2			CDE
1				CDe
3 1	3			CdE
1				Cde
2	2			cDE
3	3			cDe
				cdE
3	3			cde

Fig. 194.

Now, if $AB = c$, then the combinations containing ABC are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $AD = e$, then the combinations containing ADE are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if $A = C \mid E$, then the combinations containing ACE, Ace, are inconsistent, and we eliminate them by making a figure 3 in those sections.

From the uneliminated combinations we can get this definition of A:

$A = bCDe \mid bCde \mid BcdE \mid bcdE$, which can be translated,

This man is either good and not wise, or not wise and not good, or wise and not good.

466. On p. 317, Dr. Keynes gives the following example of a Simple Destructive Dilemma.

If A is B, C is D, and if A is B, E is F,

But either C is not D, or E is not F,

Therefore, A is not B.

The premises can be stated thus:

- (1) $AB = ABCD$
- (2) $AB = ABEF$
- (3) $C = d \mid E = f$

Make an ABCDEF diagram:

ABC	ABc	A b C	Abc	aBC	aBc	abC	abc	
3	1	3		3		3		DEF
2	1 2							DEF
3 2	1 2	3		3		3		DeF
3 2	2 1	3		3		3		Def
1	1 3		3		3		3	dEF
1 3 2	1 2	3		3		3		DEf
1 2	1 2							deF
1 2	1 2							def

Fig. 195.

Now, if $AB = CD$, then the combinations containing ABCd, ABC, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $AB = EF$, then the combinations ABEf, AB e , are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if $C = d \mid E = f$, then the combinations containing CdEf, CDEF, CDe, cEF, are inconsistent, and we eliminate them by making a figure 3 in those sections.

From the uneliminated combinations we can get this definition of A:

$$A = Ab$$

467. The method which I use to determine the combinations which the proposition $C = d \mid E = f$, will cause us to eliminate it as follows:

I make an ABCD diagram and letter it thus: I commence at the upper left-hand corner and over it write Cd; over the next file I write CD; over the next one cd, and over the last one cD. Against the top row I write Ef; against the second row EF; against the third row ef, and against the fourth row eF.

We can always letter the files and rows with any letters we please, provided we follow the method of changing one letter at a time, beginning with the last letter. But, of course, a positive letter and its negative cannot occupy the same section.

When disjunctive propositions are given to us to solve, in which the letters do not come in their regular order, or in which unusual letters, such as P, Q, R, are used, we can make the proper diagram and letter the files and rows as above directed, and then for the letters in the eliminated combinations substitute the letters A, B, C, etc., which we are accustomed to work with.

This course will tend to prevent our making mistakes in handling complex propositions.

Make a CdEf diagram as above directed:

Cd	CD	cd	cD	
1				Ef
2				
	1	2	2	EF
	1			ef
	1			eF

Fig. 196.

Now, if $C = d$, except where $E = f$, then the combinations containing CdEf, CDEF, CDe, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $E = f$, except where $C = d$, then the combinations containing $CdEf$, $CDEF$, cEF , are inconsistent, and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows us the combinations which the proposition,

$$C = d \mid E = f$$

will cause us to eliminate.

46S. The preceding dilemma should be distinguished from the following hypothetical syllogism:

- (1) If $A = B$, $C = D$ and $E = F$
- (2) But $C = d$ and $E = f$

Therefore, $A = b$

The premises can be stated thus:

- (1) $AB = ABCD$
- (2) $AB = ABEF$
- (3) $C = Cd$
- (4) $E = Ef$

Make an ABCDEF diagram:

ABC	ABc	AbC	A _b c	aBC	aBc	abC	abc	
4 2 3	2 1 4	4 3	4	4 3	4	4 3	4	DEF
3	1	3		3		3		DEF
2 3	2 1	3		3		3		DeF
3 2	2 1	3		3		3		Def
2 1 4	2 4 1	4	4	4	4	4	4	dEF
2 1	2 1							dEf
2 1	2 1							def
2 1	2 1							def

Fig. 197.

Now, if $AB = ABCD$, then the combinations containing $ABCd$, ABe , are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $AB = EF$, then the combinations containing ABe , ABe , are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if $C = d$, then the combinations containing CD are inconsistent, and we eliminate them by making a figure 3 in those sections.

Again, if $E = f$, then the combinations containing EF are inconsistent, and we eliminate them by making a figure 4 in those sections.

From the uneliminated combinations we can get this definition of A:

$$A = Ab.$$

469. Prof. Bain, in Logic p. 122, makes this very sensible remark,

"The dilemma, although occasionally a useful form, is perhaps oftener a snare."

Dilemmas are usually fallacies because of a defective enumeration of the different alternatives. Speakers are apt to give only those alternatives which they think will serve their purpose.

470. Let us take this example:

- (1) If $E = f$, then $A = b$
- (2) If $E = f$, then $C = d$
- (3) But either $A = B$ | $C = D$

Therefore $E = F$

The premises can be stated thus:

- (1) $Ef = EfAb$
- (2) $Ef = EfCd$
- (3) $A = B$ | $C = D$

Make an ABCDEF diagram:

ABC	ABc	AbC	Abc	aBC	aBc	abC	abc	
3			3					DEF
3 1 2	1 2	2	3 2	1 2	1 2	1 2	1 2	DEF
3			3					DeF
3			3					Def
			3	3	3		3	dEF
1	1 2	3	3 2	3 1	1 2	3 1	1 2	dEf
			3	3	3		3	deF
			3	3	3		3	def

Fig. 198.

Now, if Ef = Ab, then the combinations containing EfAB, Efa, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if Ef = Cd, then the combinations containing EfCD, EfC, are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if A = B | C = D, then the combinations containing ABCD, AbCd, Abc, aCd, are inconsistent, and we eliminate them by making a figure 3 in those sections.

From the uneliminated combinations we can get this definition of E:

$$E = EF$$

471. On p. 109, Archbishop Whately in "Easy Lessons in Reasoning," says:

"This kind of argument (dilemmas) was urged by the opponents of Don Carlos the Pretender to the Spanish throne, which he claimed as heir-male against his neice, the Queen, by virtue of the Salic law, excluding females, which was established (contrary to the ancient Spanish usage) by a former King of Spain,

and was repealed by King Ferdinand. They say, 'If a King of Spain has a right to alter the law of succession, Carlos has no claim; and if no King of Spain has that right, Carlos has no claim; but a King of Spain either has or has not such right; therefore (on either supposition) Carlos has no claim.'

Let A = King of Spain,

B = right to alter the law of succession,

C = Carlos,

D = claim.

The propositions can be stated as follows:

$$(1) AB = ABCd$$

$$(2) Ab = AbCd$$

$$(3) A = B | b$$

Make an ABCD diagram:

AB	Ab	aB	ab	
1	2			CD
				Cd
1	2			cD
1	2			cd

Fig. 199.

Now, if $AB = Cd$, then the combinations containing ABCD, ABC, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $Ab = AbCd$, then the combinations containing AbCD, Abc, are inconsistent, and we eliminate them by making a figure 2 in those sections.

The proposition $A = B | b$ has no predicative force, and does not cause the elimination of any combination.

From the uneliminated combinations we can get these definitions:

$$(1) AB = ABCd$$

$$(2) Ab = AbCd$$

These propositions can be translated thus:

- (1) If a King of Spain had a right to alter the law of succession, then Carlos has no claim.
- (2) If a King of Spain has not that right, then Carlos has no claim.

472. All the following disjunctive propositions are equivalents, because each causes us to eliminate the same combinations:

$$(1) A = B | C = D$$

$$(2) AB | ab = CD | cd$$

$$(3) B | C = A | B$$

$$(4) Ab | aB = Cd | cD$$

$$(5) Bc | bC = Ab | aB$$

$$(6) A | C = B | D$$

$$(7) Ac | aC = Bd | bD$$

$$(8) A | D = B | C$$

$$(9) Ad | aD = Bc | bC$$

$$(10) A | d = B | c$$

$$(11) AD | ad = BC | bc$$

$$(12) A | b = C | d$$

$$(13) A | c = B | d$$

$$(14) AC | ac = BD | bd$$

$$(15) a | B = c | D$$

$$(16) ab | AB = cd | CD$$

The eliminated combinations will be,

ABcD, ABCd, AbCD, Abcd, aBCD, aBcd, abCd, abcD

EXERCISES.

473. (1) What conclusions can be drawn from the following propositions?

If $A | B = C$, then $D = E$

$A | B = C$

(2) What conclusions can be drawn from the following pair of propositions?

$$\begin{aligned} \text{If } a &= d, \text{ then } B \mid C = E \\ C &= E \end{aligned}$$

(3) What conclusions can be drawn from the following propositions?

$$\begin{aligned} \text{If } A &= B, \text{ then } A = C \\ \text{If } C &= D, \text{ then } A = D \\ C &= D \end{aligned}$$

(4) What conclusions can be drawn from the following pair of propositions?

$$\begin{aligned} \text{If } B &= A, \text{ then } D = C \mid F = E \\ B &= A \end{aligned}$$

(5) What conclusions can be drawn from the following propositions?

$$\begin{aligned} \text{If } B &= A, \text{ then } D = C \\ \text{If } B &= A, \text{ then } F = E \\ D &= c \mid F = e \end{aligned}$$

(6) What conclusions can be drawn from the following propositions?

$$\begin{aligned} \text{If } E &= F, \text{ then } A = B \\ \text{If } E &= F, \text{ then } C = D \\ A &= b \mid C = d \end{aligned}$$

CHAPTER XVIII.

STATING PROPOSITIONS.

474. I think that the beginner in our system of logic is more likely to have trouble in stating propositions than in doing any other part of the work. This arises from the fact that the English language is very prolific in expressions which substantially assert the same idea in many different ways.

In order to assist the reader in stating propositions, I will give some examples of the manner in which terms and propositions should be stated.

In the first line we will state the term or proposition, and under it we will put its symbolic expression,

Not-A, not any A,

a.

A not-B, A except B, A omitting B,

A excluding B, A neglecting B, A without B, A setting aside B, A barring B,

Ab.

A and B, A also B, A along with B,

A likewise B, A moreover B,

A including B, A comprising B,

A embracing B, A containing B.

AB.

All A is all B,

$A = B$, $B = A$.

A alone is B alone, only A is only B,

Sole A is sole B, entire A is entire B,

$A = B$, $B = A$.

Some not-A is some not-B, Certain not A's are some not B's, one at least of the not-A's is not-B, Several not-A's are some not-B's, Most not-A's are some not-B's, Sometimes not-A is some not-B.

$ab = ab$.

All A is some B, Every A is some B,

Each A is some B, B only is A, B alone is A, B solely is A, Nothing but B is A, Nothing else than B is A:

$$A = AB.$$

No A is B, Not any A is B, A is never B, A is distinct from B, A always excludes B,

$$No\ A = B.$$

Provided not-A is not-B, Supposing not-A is not-B, Presuming not-A is not-B, Admitting not-A is not-B, Although not-A is not-B, Where not-A is not-B, When not-A is not-B, Wherever not-A is not-B:

$$If\ a = b.$$

A or B, not-A or not-B,

$$Ab \mid aB.$$

A or not-B:

$$AB \mid ab.$$

B or not-A:

$$AB \mid aB.$$

Unless A is B, A is C

$$A \text{ if } b, \text{ i. e., } Ab = AbC.$$

Some A is all B

$$BA = B.$$

Some B is all A

$$AB = A.$$

A is not BC

$$No\ A = BC.$$

A is not either B or C

$$No\ A = Bc \mid bC.$$

Except where, or, unless,

|.

A is in part B

$$BA = B.$$

A is partly not-B

$$bA = bA.$$

CHAPTER XIX.

READING.

475. In order to attain a proficiency in using the Reasoning Frame, it is desirable to learn to read it with ease. I will give some examples in reading for the purpose of assisting the reader to learn to read the various combinations in a Reasoning Frame.

The readings are so numerous that I cannot begin to give all of them, but I will give those which I think are the most useful.

These readings will also be useful in furnishing formulæ to the student when he comes to state and work out propositions in the Reasoning Frame.

476. Make an A diagram:

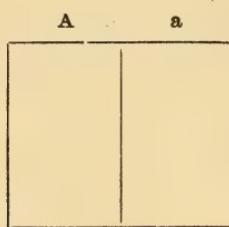


Fig. 200.

We can now read,

- (1) All A = All A
- (2) All a = All a
- (3) No a = A
- (4) Every A = A
- (5) No A = a

477. Make an AB diagram:

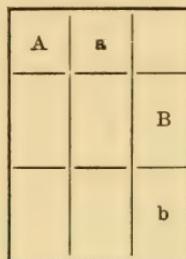


Fig. 201.

We can now read,

- (1) $A =$ either $B \mid b$
- (2) $b = A \mid a$ and conversely,
- (3) $AB = AB$
- (4) Not any $A = a$
- (5) Not any $A =$ either $aB \mid ab$

478. Make an AB diagram and eliminate the combination ab by making a figure 1 in it.

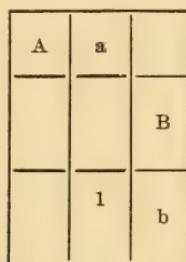


Fig. 202.

We can now read,

- (1) No $a = ab$ and conversely.
- (2) No $b = ab$ and conversely.
- (3) Not any $a = b$ and conversely.
- (4) $B = A \mid a$ and conversely.
- (5) Only $A = b$ and conversely.
- (6) B alone $= a$.
- (7) Every $a = B$ and conversely.
- (8) Each $b = A$ and conversely.

- (9) If $b = bA$, then $a = aB$ and conversely.
- (10) When $b = bA$, $a = aB$ and conversely.
- (11) When $a = aB$, $b = bA$ and conversely.
- (12) Provided $a = aB$, $b = bA$ and conversely.
- (13) $a = \text{No } b$.
- (14) $b = \text{No } a$.

479. Make an ABC diagram:

AB	Ab	aB	ab	
		1		C
			1	c

Fig. 203.

Eliminate the combinations aBC , abc , by making a figure 1 in those sections.

We can now read,

- (1) All $a =$ either bC | Bc and conversely.
- (2) Not any $a =$ either BC | bc and conversely.
- (3) BC | $bc = ABC$ | Abc and conversely.
- (4) $BC = BCA$ and conversely.
- (5) bC | $Bc =$ all a and conversely.
- (6) Not either BC | $bc = a$ and conversely.

480. Make an ABCD diagram:

AB	Ab	aB	ab	
				CD
1			1	Cd
1			1	cD
				cd

Fig. 204.

Eliminate the combinations ABCd, ABcD, abCd, abcD, by making a figure 1 in those sections.

We can now read,

- (1) AB = No Cd | cD.
- (2) Cd | cD = No AB.
- (3) All Cd | cD = Ab | aB.
- (4) No Cd | cD = AB | ab and conversely.
- (5) If A = B, C = D, c = d.

481. Make an ABCD diagram:

AB	Ab	aB	ab	
	1		1	CD
1		1		Cd
				cD
				cd

Fig. 205.

Eliminate the combinations ABCd, AbCD, aBCd, abCD by making a figure 1 in those sections.

We can now read,

- (1) All Bd | bD = Ac | aC.
- (2) No Bd | bD = AC | ac.

482. Make an ABCD diagram:

AB	Ab	aB	ab	
		1		CD
			1	Cd
1				cD
	1			cd

Fig. 206.

Eliminate the combinations ABCD, Abcd, aBCD, abCd.

We can now read,

- (1) All $A_c | aC = Bd | bD$.
- (2) No $A_c | aC = BD | bd$.
- (3) All $BD | bd = AC | ac$.
- (4) $ab = \text{No } Cd$.

483. Make an ABCD diagram:

AB	Ab	aB	ab	
1				CD
	1			Cd
		1		cD
			1	cd

Fig. 207.

Eliminate the combinations ABCD, AbCd, AbcD, Abcd.

We can now read,

- (1) No $AB = CD$, and conversely.
- (2) Never when $A = B, C = D$
- (3) $A = B$ or $CD = CD$
- (4) If $A = b$, then $C = D$

484. Make an ABCD diagram:

AB	Ab	aB	ab	
			1	CD
		1		Cd
			1	cD
1	1	1	1	cd

Fig. 208.

Eliminate the combinations abCD, aBCd, AbcD, Abcd, aBcd, abcd.

We can now read,

- (1) $a = D \mid b = C$ and conversely.
- (2) If $c = d$, then $A = B$

485. Make an ABCD diagram:

AB	Ab	aB	ab	
		1		CD
1	1		1	Cd
			1	cD
			1	cd

Fig. 209.

Eliminate the combinations $aBCD$, $ABCd$, $AbCd$, $abCd$, $abcD$, $abcd$.

We can now read,

- (1) $a = B \mid C = D$ and conversely.
- (2) If $a = b$, then $C = D$
- (3) If $C = d$, then $a = B$

486. Make an ABCD diagram:

AB	Ab	aB	ab	
		1	1	CD
				Cd
1		1		cD
1			1	cd

Fig. 210.

Eliminate the combinations $aBCD$, $abCD$, $ABCd$, $aBcd$, $ABcd$, $abcd$.

We can now read,

- (1) $a = d \mid c = b$ and conversely.
- (2) $ab = C \mid D$
- (3) $CD = A$
- (4) $cD = Ab \mid ab$
- (5) $aB = Cd \mid cd$
- (6) $Cd = Cd$
- (7) $Ab = Ab$
- (8) $Cd = A \mid a$
- (9) $Ab = C \mid c$

487. Make an ABCD diagram:

AB	Ab	aB	ab	
1			1	CD
	1	1		Cd
	1	1		cD
1			1	cd

Fig. 211.

Eliminate the combinations ABCD, ABcd, AbCd, AbcD, aBCd, aBcD, abCD, abcd.

We can now read,

- (1) $Ab \mid aB = CD \mid cd$ and conversely.
- (2) $Cd \mid cD = AB \mid ab$ and conversely.

Also the following pair:

- (3) $Ac \mid aC = BD \mid bd$ and conversely.
- (4) $Bd \mid bD = AC \mid ac$ and conversely

Also the following pair:

- (5) $Bc \mid bC = AD \mid ad$ and conversely.
- (6) $Ad \mid aD = BC \mid bc$ and conversely.

And the following pair:

- (7) $ad \mid AD = cB \mid Cb$ and conversely.
- (8) $cb \mid CB = aD \mid Ad$ and conversely.

488. Make an ABCD diagram:

AB	Ab	aB	ab	
1				CD
	1	1	1	Cd
	1			cD
	1			cd

Fig. 212.

Eliminate the combinations containing ABCD, AbCd, Abc, aCd.

We can now read,

- (1) $A = B \mid C = D$ and conversely.
- (2) $a = aCD \mid ac$
- (3) $c = cAB \mid ca$
- (4) If $A = b, C = D$

489. Make an ABCD diagram:

AB	Ab	aB	ab	
1			1	CD
	1	1		Cd
1				cD
	1	1	1	cd

Fig. 213.

Eliminate the combinations ABCD, ABcD, AbCd, Abcd, aBCd, aBcd, abCD, abcd.

We can now read the following pair of propositions:

- (1) $aD \mid Ad = cB \mid CB$ and conversely.
- (2) $ad \mid AD = cb \mid cb$ and conversely.

490. Make an ABCD diagram:

AB	Ab	aB	ab	
1		1		CD
	1		1	Cd
1		1		cD
	1		1	cd

Fig. 214.

Eliminate the combinations ABCD, ABcD, AbCd, Abcd, aBCD, aBcD, abCd, abcd.

We can now read the following pair of propositions:

- (1) $aD \mid AD = cb \mid Cb$
- (2) $cB \mid CB = Ad \mid ad$

491. Make an ABCD diagram:

AB	Ab	aB	ab	
				CD
				Cd
			1	cD
			1	cd

Fig. 215.

Eliminate the ac combinations.

We can now read,

- (1) Everything = $A \mid aC$
- (2) $c = cA$
- (3) $a = aC$
- (4) If $a = aC$, then $c = cA$

492. Make an ABCD diagram:

AB	Ab	aB	ab	
1				CD
	1	1	1	Cd
				cD
				cd

Fig. 216.

Eliminate the combinations containing ABCD, AbCd, aCd.
We can now read,

$$AB = AB \mid C = D$$

493. Make an ABCD diagram:

AB	Ab	aB	ab	
1				CD
	1			Cd
		1		cD
			1	cd

Fig. 217.

Eliminate the combinations containing ABCD, AbCd, Abc.
We can now read,

- (1) $A = B \mid CD = CD$
- (2) $Ab = CD$
- (3) $CD = Ab \mid a$

494. Make an ABCDEF diagram:

ABC								
1								DEF
	1	1	1	1	1	1	1	DEf
		1						DeF
			1					Def
				1				dEF
					1			dEf
						1		deF
							1	def

Fig. 218.

Eliminate the combinations containing ABCDEF, and all the combinations containing ABc, excepting the one containing ABcDEF, and all the combinations containing DEf, excepting the one containing ABCDEF.

The Reasoning Frame now shows the visible expression of the proposition,

$$AB = C \mid DE = F$$

495. Make an ABCDEF diagram:

ABC								
1								DEF
	1	1	1	1	1	1	1	DEF
	1	1	1	1	1	1	1	DeF
	1	1	1	1	1	1	1	Def
	1	1	1					dEF
	1	1	1					dEf
	1	1	1					deF
	1	1	1					def

Fig. 219.

Eliminate the combination containing ABCDEF, and also the combinations containing ABc, AbC, Abc, excepting those containing DEF; and also the combinations containing DEF, DeF, Def, excepting those containing ABC.

The Reasoning Frame now shows us the visible expression of the proposition,

$$A = BC \mid D = EF$$

496. Make an ABCDEF diagram:

ABC								
				1		1	1	DEF
				1		1	1	DEf
				1		1	1	DeF
				1		1	1	Def
1	1	1	1	1		1	1	dEF
					1			dEf
1	1	1	1	1		1	1	deF
1	1	1	1	1		1	1	def

Fig. 220.

Eliminate the combination containing aBcdEf, and also the combinations containing aBC, abC, abc, excepting those containing dEf; and also the combinations containing dEF, deF, def, excepting those containing ABc.

The Reasoning Frame now shows the visible expression of the proposition,

$$a = Bc \mid d = Ef$$

497. Make an ABCDEF diagram:

ABC								
		1	1			1	1	DEF
							1	DEF
		1	1		1	1	1	DeF
						1		Def
		1	1		1	1	1	dEF
						1		deF
		1	1		1	1	1	deF
						1		def
		1	1				1	def

Fig. 221.

Eliminate the combination containing abcDEF; and also the combinations containing AbC, Abc, abC, excepting those containing DEF; and also the combinations containing aBc, excepting those containing f; and also the combinations containing abcDeF, abedEF, abcedeF.

The Reasoning Frame now shows the visible expression of the proposition,

$$b = DE \mid ac = f$$

498. Make an ABCD diagram:

AB	Ab	aB	ab	
1	1			CD
1	1			Cd
	1	1		cD
	1	1		cd

Fig. 222.

Eliminate the combinations containing ABC, Ab, aBc.
We can now read,

$$A = B \mid B = C \text{ and conversely.}$$

499. Make an ABCD diagram:

AB	Ab	aB	ab	
	1	1	1	CD
1				Cd
	1			cD
	1			cd

Fig. 223.

Eliminate the combinations containing ABCd, AbCD, Abc, aCD.

We can now read,

- (1) If $A = b$, $C = d \mid A = B$
- (2) If $C = D$, $A = B \mid C = d$
- (3) $A = B \mid C = d$

500. Let us take this example:

$$a \mid D = B \mid C$$

The easiest method of finding the combinations which a proposition in this form will cause us to eliminate, is to first make an ABCD diagram, and letter it according to the circumstances of the case, remembering that a letter and its opposite cannot occupy the same section; eliminate the combinations which are inconsistent, then make another ABCD diagram. Letter it in the usual manner and transfer to it the eliminated combinations.

The given proposition can be stated thus:

$$(1) ad \mid AD = Bc \mid Cb$$

Make an ABCD diagram:

aD	ad	AD	Ad	
	1	1		BC
				Be
				bC
	1	1		bc

Fig. 224.

Now, if $a | D = B | C$, then the combinations containing aBCd, abcd, ABCD, AbcD, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Make another ABCD diagram:

AB	Ab	aB	ab	
1				CD
		1		Cd
	1			cD
			1	cd

Fig. 225.

Eliminate the combinations aBCd, abcd, ABCD, AbcD.

The Reasoning Frame now shows the visible expression of the proposition,

$$a | D = B | C$$

This method of lettering a Reasoning Frame, according to the exigencies of the occasion, in order to learn the combinations which are to be eliminated, is of frequent usefulness.

CHAPTER XX.

THE SYLLOGISM.

501. Thus far we have not discussed the Syllogism, because in our system, we have no use for it. But as a work on Logic without it would not be considered complete by the majority of educated people, we will give some space to this system of reasoning.

The Syllogism is the principal method of deductive inference, that is, an inference from the general to the particular, employed by the old logic. It should contain just three terms, a subject and predicate of the conclusion; another term called the middle term, which occurs in both premises.

502. It has three propositions, viz.: two premises and a conclusion. The premise containing the major term is called the major premise; the major term is the predicate of the conclusion. The premise containing the minor term is called the minor premise; the minor term is the subject of the conclusion.

503. Syllogisms are divided into four figures. The position of the middle term determines the figure. The middle term is the term which occurs in both premises and not in the conclusion.

504. In the first figure the middle term is subject in the major premise and predicate in the minor premise, thus:

$$B = C$$

$$A = B$$

Therefore, $A = C$

505. In the second figure the middle term is predicate in both premises, thus:

$$C = B$$

$$A = B$$

506. In the third figure the middle term is subject in both premises, thus:

$$\begin{aligned}B &= C \\B &= A\end{aligned}$$

507. In the fourth figure the middle term is predicate in the major and subject in the minor premise, thus:

$$\begin{aligned}C &= B \\B &= A\end{aligned}$$

508. The propositions in a syllogism are divided into four forms. I think that E is the only definite proposition in these four forms. A, I and O are indefinite and must be converted into definite propositions before we can use them. Until they are thus converted they should have no place in an exact logic.

509. The first is the Universal Affirmative:

“All men are animals,” $A = AB$,

and its symbol is A.

510. The second is the Universal Negative:

“No men are infallible,” $No A = B$,

and its symbol is E.

511. The third is the Particular Affirmative:

“Some men are wise,” $AB = AB$,

and its symbol is I.

512. The fourth is the Particular Negative:

“Some men are not religious,” $Ab = Ab$,

and its symbol is O.

513. The symbols A and I are taken from the latin word *affirmo*, “I affirm,” and E and O are taken from the latin word *nego*, “I deny.”

514. The first figure only, will yield conclusions in all the forms A, E, I, O. The second figure yields negative conclusions. The third figure yields particular conclusions. The fourth figure does not yield Universal Affirmative Conclusions.

515. The order of subject and predicate varies in the minds of persons according to the idea which they wish to convey.

“The best form of government is government by a plurality of persons,” and

“Government by a plurality of persons is the best form of government,”

would be stated in different figures, although both propositions have substantially the same meaning.

516. When the middle term embraces both the major and the minor terms, it naturally forms the predicate of both premises. This makes the second figure.

517. When the middle term is smaller than the major and minor terms it naturally forms the subject of both premises. This makes the third figure.

518. The syllogism is governed by six rules:

- (1) Every syllogism must have three and only three terms.
- (2) There must be three and only three propositions.
- (3) The middle term must be distributed (that is, taken altogether), at least once in the premises.
- (4) No term undistributed (i. e., taken partially) in the premises, must be distributed in the conclusion.
- (5) There can be no conclusion drawn from negative premises.
- (6) If one premise be negative, the conclusion must be negative.

519. The premises are so termed because they premise or go before the conclusion.

The conclusion is so named because it concludes, or shuts up in one the major and minor propositions.

520. In this syllogism,

“All horned animals ruminate” (major proposition).

“A sheep is a horned animal” (minor proposition).

Therefore, “A sheep ruminates” (conclusion).

“Sheep” is named the minor term because it is less extensive than “ruminating.”

“Ruminate” is the major term, because it includes “sheep.”

“Horned animals” is the middle or mean term.

521. The rule or maxim which is commonly called *dictum de omni et nullo*, by which Aristotle explains the validity of a syllogism in this form,

Every B is C

Every A is B

Therefore, every A is C,
is this: Whatever is predicated of a distributed term, whether affirmatively or negatively, may be predicated in like manner of everything embraced by it. This maxim, however, cannot be applied to all syllogisms. For instance, the dictum cannot be applied to this valid syllogism:

No savages have the use of metals,
The ancient Germans had the use of metals,
Therefore, they were not savages.

522. There are more rules which are used to test the validity of syllogisms:

- 1st. If two terms agree with one and the same third, they agree with each other.
- 2d. If one term agrees and another disagrees with one and the same third term, these two disagree with each other.

The first of these rules tests the validity of affirmative conclusions; the second, of negative conclusions.

523. Let us take this example and work it out by our system:

- (1) All horned animals ruminate,
- (2) A sheep is a horned animal,
- (3) Therefore a sheep ruminates.

Let A = sheep,

B = horned animal,

C = ruminates.

The propositions can be stated thus:

- (1) B = BC
- (2) A = AB
- (3) Therefore, A = AC

The term "ruminate" is a wider term than the term "horned animal," and in order that our proposition may be strictly accurate, we must reduce the term "ruminate" to an equality with the term "horned animal." We do this by adding the term "horned animals" to the term "ruminate," so that the term "horned animals ruminating," is the equivalent of the term "horned animal."

524. Where the predicate is larger than the subject, we reduce the predicate to the limits of the subject by adding the subject to the predicate. It is a kind of subtraction. By adding B, which stands for "horned animal," to C, which stands for "ruminating animals," we, in effect, subtract from "ruminating animals" all the not-horned animals, and this reduces the "ruminating animals" to the "horned animals." We thus have this paradox, that by adding a limited term to a more extensive term, we take away from the meaning of the extensive term. By adding a term we subtract from the meaning, except where the terms are equivalents.

To be formally accurate, we must always state our propositions so that they will be true when read either way. It can always be done by adding the subject to the predicate, where the predicate is larger than the subject.

Now, if $B = BC$, then the combinations containing Bc are inconsistent, because they imply that $B = c$, and we therefore eliminate them by making a figure 1 in those sections of an ABC Reasoning Frame:

AB	Ab	aB	ab	
	2			C
1	2	1		c

Fig. 226.

In our system, propositions in the form of $B = BC$, i. e., propositions where the subject is added to the predicate, are never worked backward.

Now, if $A = AB$, then the combinations containing Ab are inconsistent, and we eliminate them by making a figure 2 in those sections.

In examining the Reasoning Frame, we find that there is only one section containing the letter A, so that we can define

A by the other letters in that section; they are B and C. But we cannot say that A = BC, because that implies that BC is A; there is another BC which prevents us from saying that BC = A.

The definition of A is, A = ABC, but in translating our terms we need not repeat the subject, thus we can read,

A = ABC, which we can translate,

A sheep is a horned animal and is a ruminating animal.

Now the syllogism drops a part of this information and simply gives us,.

A sheep is a ruminating animal.

This is allowable because we are not obliged to give any more information than is necessary to serve the purpose in view.

When a letter is found in more sections than one, its definition will be in the form of a disjunctive proposition.

Now, the term B occurs in two combinations, viz.: ABC and aBC. We cannot say, when a term occurs in two combinations, that it is either one of them alone; our definition must be that it is one or the other; hence

B = BAC or BaC, which we can translate,

A horned animal is a sheep and a ruminating animal, or it is not a sheep but is a ruminating animal.

Now, where we have a term and its negative, like A and a, occurring in two alternants, we can omit translating them, and we can contract the definition just given into,

B = BC, which we can translate:

A horned animal is a ruminating animal.

Again, the letter C occurs in three combinations, viz.: ABC, aBC and abC, so that the definition of C is:

C = AB | aB | Cab

The reason why we do not repeat C before AB and aB is because AB = C and aB = C; but we repeat C before ab because there is another combination containing ab, so that we could not say that ab = C.

Now we can translate the definition of C thus:

A ruminating animal is either,

A sheep and a horned animal; or
 Not a sheep but a horned animal; or
 Not a sheep and not a horned animal.

The definition of a is,

$a = aBC \mid abC \mid abc$, which can be translated:
 An animal which is not a sheep is either a horned animal
 and a ruminating animal; or
 Not a horned animal but a ruminating animal; or
 Neither.

When the definition of b is,

$b = baC \mid bac$, then,

in this case we can omit the C and c and translate the definition of b, thus:

What is not a horned animal is not a sheep.

The definition of c is,

$c = cab$, which we can translate:

What is not a ruminating animal is neither a sheep nor
 a horned animal.

525. The reader can now see by this example, that the syllogism gives us a part of the information contained in two propositions having a common term. The reasoning process is a process of finding all the equivalents, inferences, consistents, inconsistents, and contradictions of given propositions. It is also a process of conversion; that is, by the reasoning process we can convert given propositions into many other equivalent forms. But the syllogism is a very imperfect instrument for effecting the possible conversions.

526. Let us take this example:

- (1) No savages have the use of metals,
- (2) The ancient Germans had the use of metals,
- (3) Therefore they were not savages.

Let A = savages,

B = the use of metals,

C = the ancient Germans.

Our propositions can be stated thus:

No A = B

C = CB

Therefore, $C = Ca$.

We state the first proposition in the form of

$$\text{No } A = B$$

It is evident that the proposition will read backward thus:

Those who have the use of metals are no savages.

Now, if $No A = B$, then the combinations containing AB are inconsistent, and we eliminate them by making a figure 1 in those sections of an ABC Reasoning Frame:

AB	Ab	aB	ab	
1	2		2	C
1				c

Fig. 227.

Again, if $C = CB$, then the combinations containing Cb are inconsistent, and we eliminate them by making a figure 2 in those sections.

An examination of the Reasoning Frame shows us that the definition of C is CaB , which we can translate:

The ancient Germans had the use of metals and were not savages.

The definition of A is,

$A = Abc$, which we can translate,

Savages do not have the use of metals, and are not ancient Germans.

The definition of B is,

$B = BaC \mid Bac$, which we can translate,

Those who had the use of metals were not savages.

The definition of a is,

$a = aBC \mid aBc$, which we can translate,

Those who are not savages, have the use of metals.

The definition of b is,

$b = bcA \mid bca$, which we can translate,
 Those who do not have the use of metals are not the
 ancient Germans.

The definition of c is,

$c = cAb \mid caB \mid cab$, which we can translate,
 Those who were not ancient Germans were either sav-
 ages without the use of metals; or
 not savages without the use of metals; or
 neither.

527. Let us take this example:

- (1) Some Europeans are Englishmen,
- (2) Some Englishmen are Londoners.

Let $A =$ Europeans,

$B =$ Englishmen,

$C =$ Londoners.

The premises can be stated thus:

- (1) $AB = B$
- (2) $BC = C$

Make an ABC diagram:

AB	Ab	aB	ab	
	2	1	2	C
			1	c

Fig. 228.

Now, if $B = AB$, then the combinations containing Ba are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $C = BC$, then the combinations containing bC are inconsistent, and we eliminate them by making a figure 2 in those sections.

From the uneliminated combinations we can get these definitions:

(1) $C = CAB$, which can be translated,
Londoners are Europeans and Englishmen.

(2) $Ab = Abc$, which can be translated,
Europeans who are not Englishmen are not Londoners.

(3) $a = abc$, which can be translated,
Those who are not Europeans are not Englishmen and
not Londoners.

(4) $b = bc$, which can be translated,
Those who are not Englishmen are not Londoners.

(5) $c = cAB \mid cAb \mid cab$, which can be translated,
Those who are not Londoners are either Europeans and
Englishmen, or Europeans who are not Englishmen,
or neither Englishmen nor Europeans.

(6) $A = AC \mid Ac$, which can be translated,
Europeans are either Londoners or not Londoners.

(7) $B = BC \mid Bc$, which can be translated,
Englishmen are either Londoners or not Londoners.

We can also read,

(8) No $C = b$, which can be translated,
No Londoners are not Englishmen.

(9) No $B = a$, which can be translated,
No Englishmen are not Europeans.

528. Let us take this example:

(1) Cornishmen are Englishmen,

(2) Some Englishmen are Londoners,

Let $A =$ Cornishmen,

$B =$ Englishmen,

$C =$ Londoners.

The premises can be stated thus:

$$(1) A = AB$$

$$(2) BC = C$$

Make an ABC diagram:

AB	Ab	aB	ab	
	1 2		2	C
	1			c

Fig. 229.

Now, if $A = AB$, then the combinations containing Ab are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $C = BC$, then the combinations containing bC are inconsistent, and we eliminate them by making a figure 2 in those sections.

We can now read,

(1) $b = bc$, which can be translated,

Those who are not Englishmen are not Londoners.

(2) $C = CB$, which can be translated,

Londoners are Englishmen.

(3) No $A = b$, which can be translated,

No Cornishmen are not Englishmen.

(4) No $C = b$, which can be translated,

No Londoners are not Englishmen.

529. Speaking of the syllogism, Prof. Bain, in his "Deductive and Inductive Logic," p. 207, says:

"1. It is the peculiarity of the syllogism that the conclusion does not advance beyond the premises. This circumstance has been viewed in two lights. On the one hand it is regarded as the characteristic excellence of the syllogism. On the other hand it is represented as constituting a *petitio principii*.

In the syllogism,

Men are mortal,

Kings are men,

Kings are mortal,

the conclusion seems already affirmed in the premises."

530. For myself, I regard the syllogism as useless, but I do not think that the syllogism can be criticised for "not advancing beyond the premises." It is true of all reasoning that it can "not advance beyond the premises." It can turn the premises into other forms; it can find many equivalents for the premises, but it cannot "advance beyond" them.

531. Prof. Bain again says:

2. "There remains a far more serious charge and one that takes us direct to the root of formal reasoning. Supposing there were any doubt as to the conclusion that 'Kings are mortal,' by what right do we proclaim, in the major, that 'All men are mortal,' Kings included?

It would be requisite seemingly, to establish the conclusion before we can establish the major.

In order to say, 'All men are mortal,' we must have found, in some other way, that all kings and all people are mortal. So that the conclusion first contributes its quota to the major premise, and then it takes it back again.

This is the dead-lock of the syllogism, the circumstance that has brought down upon it the charge of 'reasoning in a circle' (*petitio principii*). In point of fact we can hardly produce a more glaring case of that fallacy."

532. Neither do I consider this a serious charge against the syllogism. When the proposition, "All men are mortal" is given, logic does not question whether that is true or not. It is not a question of logic, it is a question of fact. Logic takes it for granted, assumes it to be true, and proceeds to find its various equivalents. There is no fallacy in the argument. A syllogism may be perfectly valid though the premises are false. We can draw the conclusions from false premises and the operation be logically valid. Logic has no power to pass on the truth or falsity of the premises; that is outside of its domain; its function is to draw the proper conclusions, that is, the conclusions which necessarily follow from the premises given.

533. Miss Jones in Elements of Logic, p. 161, makes a just criticism against the ordinarily accepted rules of the syllogism.

Speaking of terms and term-names she says:

"For if, in e. g., the syllogism,

All N's are Q's,

Some R's are N's,

Therefore, some R's are Q's.

We call (1) all N's, (2) some Q's, (3) some R's, (4) some N's, Terms, then the rule that in a valid syllogism we must have only three terms, excludes all syllogisms except those in Fig. 3, which have the middle term distributed twice. In the instance above taken we have four terms."

534. I think Miss Jones is correct in the position taken that all N's and some N's are two different terms. "All" and "some" are not equivalents; they refer to two different groups of things, and each group can be expressed logically by a different symbol, and this will give us four terms.

535. Lotze in his work on Logic, p. 114, says:

"Following Aristotle we give the name of Inference or Syllogism to any combination of two judgments for the production of a third and valid judgment which is not merely the sum of the two first. Such production would be impossible if the contents of the antecedent judgments, the two premises were entirely different; it is only possible if they both contain a common element M, the middle concept or *terminus medius*, which the one relates to S, the other to P (here M stands for middle term, S stands for subject, P stands for predicate)."

536. We can combine by our system two entirely different premises.

Let us take these propositions:

(1) Washington is the capital of the United States.

(2) Salt is chloride of sodium.

Let A = Washington

B = Capital of the United States.

C = Salt,

$D =$ chloride of sodium.

The propositions can be stated thus:

- (1) $A = B$
- (2) $B = A$
- (3) $C = D$
- (4) $D = C$

The question is: What inferences can we draw from these propositions?

Now, if $A = B$ then the combinations containing Ab are inconsistent, and we eliminate them by making a figure 1 in those sections of an ABCD Reasoning Frame:

AB	Ab	aB	ab	
	1	2		CD
3	13	23	3	Cd
4	41	24	4	cD
	1	2		cd

Fig. 230.

Again, if $B = A$, then the combinations Ba are inconsistent and we eliminate them by making a figure 2 in those sections.

Again, if $C = D$, then the combinations containing Cd are inconsistent and we eliminate them by making a figure 3 in those sections.

Again, if $D = C$, then the combinations containing Dc are inconsistent and we eliminate them by making a figure 4 in those sections.

From the combinations which remain we can get the following definitions:

- (1) $AB \mid ab = CD \mid cd$
- (2) $AD \mid ad = BC \mid bc$
- (3) $AC \mid ac = BD \mid bd,$

Which we can translate as follows:

- (1) Either Washington and the capital of the United States,

or neither, is salt and chloride of sodium or neither.

(2) Either Washington and chloride of sodium, or neither, is the capital of the United States and salt, or neither.

(3) Either Washington and salt, or neither, is the capital of the United States and chloride of sodium, or neither.

There are other definitions which can be obtained such as,
 $AB = ABCD \mid ABcd$, which can be translated,

Washington and the capital of the United States are
 salt and chloride of sodium or neither.

537. I give this example merely to show that it is possible to draw conclusions from entirely different premises. The usual way of obtaining the readings by our system, is to read the combinations which remain uneliminated in the Reasoning Frame, but as heretofore explained, we can also obtain readings by prefixing the word "No" to the eliminated combinations in which two or more letters are eliminated. Thus, we can take the example just given and get these conclusions by prefixing the word "No" to these eliminated combinations:

(1) No $A = Ab$,

which can be translated,

No Washington is not the capital of the United States.

(2) No $b = bA$,

which can be translated,

No not-capital of the United States is Washington.

(3) No $a = aB$,

which can be translated,

No not-Washington is the capital of the United States.

(4) No $B = Ba$,

which can be translated,

No capital of the United States is not-Washington.

(5) No $C = Cd$,

which can be translated,

No salt is not-chloride of sodium.

(6) No $d = Cd$,

which can be translated,

No not-chloride of sodium is salt.

(7) No $c = cD$,

which can be translated,

No not-salt is chloride of sodium.

(8) No $D = Dc$,

which can be translated,

No chloride of sodium is not-salt.

We could also get a good many more readings of this kind, such as:

No $AB = Cd$, etc., etc.

We could fill a chapter with the various combinations that can be read in this way from the eliminated combinations in this example, but it is hardly necessary to do so.

538. Prof. Venn in his *Symbolic Logic*, p. 402, makes the following very sensible remarks on the syllogism:

"We must frankly remark that from our point of view we do not greatly care for this venerable structure, highly useful though it be for purposes of elementary training in thought and expression, and almost perfect as it is technically, when regarded from its own standing point. But its ways of thinking are not ours, and it obeys rules to which we own no allegiance. To it the distinction between subject and predicate is essential; to us this is about as important as the difference between the two ends of a ruler which one may hold either way at will. To it the position of the middle term is consequently worth founding a distinction upon. To us this is as insignificant as is the order in which one adds up the figures in an addition sum. On the other hand, the distinction between Universal and Particular propositions, which to it is vital, is to us unimportant. There are reasons, nevertheless, for taking some account of the syllogism here; partly because the contrast of treatment will serve to emphasize this difference in the point of view, partly because the omission of any such references might possibly be taken as a confession of failure on the part of the *Symbolic logic*.

"Since the syllogism is a sound process, it must admit of some kind of treatment upon any scheme. There are two ways of

treating it. The method which would naturally be adopted by any one familiar with the use of symbols, but entirely ignorant of logical tradition, would probably be this. He would begin by rejecting all distinction of figure as utterly alien to his scheme; and as the common system admits that the other three figures can be reduced to the first, he would insist upon this simplification being made before he took the work in hand. That is, he would take account only of the first four moods. Then he would go on to reduce these by the consideration that, to his thinking, X and not-X being both classes of the same essential character, there was no occasion to formulate a distinction between moods which involved a negative and those which contained only affirmative premises. There would then remain only the distinction between a form which draws a universal and one which draws a particular conclusion."

539. Prof. Venn is a disciple of Boole and his work on Symbolic Logic is on the Boolean system, which is a sort of algebraical logic. The reader can see that in our system the distinction between subject and predicate is unessential; neither do we care anything about a middle term. The distinctions of figure and mood are useless to us, and we can treat negative and affirmative propositions with equal facility.

540. So far as the theory of logic is concerned, there is a strong analogy in many points between the Boolean system and ours.

541. Lotze in his work on Logic, vol. 2, p. 1, speaking of the syllogism, justly remarks:

"True conclusions, as Aristotle has observed, can be correctly drawn from false premises."

Every Laplander is a born poet,
Homer was a Laplander,
And therefore, by the first figure a poet.
All parasitic plants have red flowers,
No rose has red flowers,
Therefore, by the second figure roses are not para-
sitic plants.

Metals do not conduct electricity,
 All metals are non-fusible,
 And hence, according to the third figure, non-fusible substances exist which are non-conductors of electricity.

Alter Laplander into Greek, plants which have red flowers into plants which have exploding seed vessels, and write glass for metal, and in each example one premise will be true, while by inserting a new middle term in each case, you may make both premises true, but in every case the conclusion follows with neither more nor less validity.

Let T be a perfectly true proposition, S its subject and P its predicate; then a middle term, M, may be chosen at random, so long as the terms are arranged in both premises on the model of an Aristotelian figure. If this is done, the conclusion T will always follow according to the figure.

We shall see why this is universally true if we take as our middle term an abstract symbol, M, instead of a concrete term, thus:

All M are poets
 Homer was an M,
 All parasitic plants are M
 Roses are not M,
 All M are non-conductors
 All M are non-fusible.

What these symbolic premises tell us, is the relations in which S and P must stand to some middle term, if their conjunction, SP, is to be valid in the conclusion; and, conversely, these premises tell us that given any middle term, M, to which S and P are related as required, then the proposition SP must be valid. If the M is found, and so both the required premises established, then SP is valid, not merely in fact, but now also of necessity; on the other hand, if we could show that there exists no M to which S and P can stand in the requisite relation we shall know that SP was impossible, for no experience could give us SP as a fact; but if we have merely chosen a wrong M, then the case is different. The premises we have chosen will

not do, but there is no reason why there should not be some other M, the insertion of which will render the premises correct and so necessitate the conclusion SP.

If, again, we have correctly drawn a conclusion, SP, and that conclusion is unsound, there must be something false in the premises from which it follows. In a word, all cases where T is not given in direct perception, but deduced from premises, what really depends on the correctness of those premises is not the truth of T, but only our insight into that truth. Without correct premises, T cannot be true, but nevertheless it can be proved, and its truth is independent of any errors we may commit, when reflecting about it, and subsists even when conclusively deduced from premises materially false. This point deserves notice, for it is a common mistake in reasoning, to take the invalidity of the truth which is offered for T, as a proof of the falsehood of T itself, and to confuse the refutation of an argument with the disproof of a fact."

542. Let us take this example of a pretended syllogism:

Some men are kings

All cooking animals are men

Therefore all cooking animals are kings.

This example is said to exemplify the fallacy of undistributed middle. The middle term is "men," and it is not distributed, that is, taken altogether, in either the major or the minor premise.

The proposition

All cooking animals are men, means,

All cooking animals are some men,

but this group of "some men" is a different group from the group in "some men are kings."

Let A = men

B = kings

C = cooking animals.

The premises can be stated thus:

(1) BA = B

(2) C = CA

Make an ABC diagram:

AB	Ab	aB	ab	
		1 2	2	C
		1		c

Fig. 231.

Now, if $B = BA$, then the combinations containing aB are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $C = CA$, then the combinations containing aC are inconsistent, and we eliminate them by making a figure 2 in those sections.

From the uneliminated combinations we can get these definitions:

$$(1) C = CAB \mid CAB$$

which can be translated,

Cooking animals are men, and either kings or not kings.

$$(2) a = abc,$$

which can be translated,

What are not men are neither kings nor cooking animals.

From the eliminated combinations we can get these definitions:

$$(1) \text{No } B = a,$$

which can be translated,

No kings are not-men.

$$(2) \text{No } C = a,$$

which can be translated,

No cooking animals are not-men.

543. Prof. Bain remarks, p. 149: "We may have premises free from the last-named vice of undistributed middle, yet made to yield a false conclusion by overstepping the present rule (no term undistributed in the premises must be distributed

in the conclusion) or raising a term of particular quantity, in the premises, to the rank of universal quantity in the conclusion. To this error is given the name Illicit Process; and according as the unduly extended term occurs in the major or minor premise, the error is called Illicit Process of the Major, or Illicit Process of the Minor.

544. Let us take this example of the Illicit Process of the Minor:

All men are mortal

Some extended things are men

Therefore, all extended things are mortal

Let A = men

B = mortal

C = extended things.

The premises can be stated thus:

$$(1) A = AB$$

$$(2) CA = A$$

Make an ABC diagram:

AB	Ab	aB	ab	
	1			C
2	1	2		c

Fig. 232.

Now, if A = B, then the combinations containing Ab are inconsistent, and we eliminate them by making a figure 1 in those sections.

CA = A will cause us to eliminate the combinations Ac.

We eliminate them by making a figure 2 in those sections.

From the uneliminated combinations we can get this definition of C:

$$C = CAB \mid CaB \mid Cab,$$

which can be translated,

Extended things are either mortal men, or mortals and not-men, or neither men nor mortal.

545. Let us take the following example of Illicit Process of the Major:

All men are fallible

Some beings are not men

Therefore, no beings are fallible.

Let A = men

B = fallible

C = beings.

The premises can be stated thus:

$$(1) A = AB$$

$$(2) Ca = Ca$$

Make an ABC diagram:

AB	Ab	aB	ab	
	1			C
	1			c

Fig. 233.

Now, if A = AB, then the combinations containing Ab are inconsistent, and we eliminate them by making a figure 1 in those sections.

Ca = Ca will not cause us to eliminate any combinations.

From the uneliminated combinations we can get this definition of C:

$$C = CAB \mid CaB \mid Cab,$$

which can be translated,

Beings are either fallible men, or fallible not-men, or neither fallible nor men.

546. Prof. Bain, in "Deductive and Inductive Logic," p. 150, says:

"There can be no conclusion drawn from negative premises.

No men are gods

No trees are men

do not supply the materials for a deductive inference. The reason of this is already apparent from what has been said as to the applying proposition, which must always affirm. To know only that two things are each excluded from a third thing, is to know nothing concerning their mutual relation."

Let us work out the example given:

Let A = men

B = gods

C = trees.

The propositions can be stated:

(1) No A = B

(2) No C = A

Now, if No A = B, then the combinations containing AB are inconsistent, and we eliminate them by making a figure 1 in those sections of an ABC Reasoning Frame.

Make an ABC diagram:

AB	Ab	aB	ab	
1 2	2			C
1				c

Fig. 234.

Again, if No C = A, then the combinations containing AC are inconsistent, and we eliminate them by making a figure 2 in those sections. From the combinations which remain we can get the following definitions:

A = Abc

which can be translated,

Men are not Gods and not trees.

C = Ca

which can be translated,

Trees are not men.

$$B = aB$$

which can be translated,

Gods are not men.

The reader can see that it is as easy to draw conclusions from negative premises as it is to do so from affirmative premises.

547. We have already seen that by our system we can get all the conclusions which are contained in the premises.

Lotze justly criticises the barrenness of the syllogism. He says (p. 141):

“If we argue, ‘Heat expands all bodies; iron is a body, therefore, heat expands iron,’ or

All men are mortal

Caius is a man

Therefore, Caius is mortal,

everyone will feel the barrenness of this procedure, and will reply, ‘Undoubtedly heat expands all bodies, but each body in a different degree. Undoubtedly all men die, but the liability to die in one man is different from that in another. What we want to know for technical purposes or for administering a life insurance company is, how iron expands in distinction from lead, or how the mortality of Caius is to be estimated in distinction from other men.’ What good is it to say, ‘If a man is offended, he gets angry; Caius is a man, therefore, if he is offended he will get angry?’ ”

Now in our system we can state all the technical and scientific facts known in regard to the expansion of bodies by heat or the mortality of men, and in one operation draw all the conclusions which the facts would warrant.

548. Miss Jones, in “Elements of Logic,” p. 156, in criticism of the syllogism, says:

“It seems to me that Mill’s criticism of the *dictum de omni et nullo* is well founded, and that the canon merely amounts to saying that that which is predicated of every member or every portion of a class, may be predicated of any member or any portion of that class. For, when we say, ‘Whatever is predicated of a term distributed, may be predicated in like manner

of everything contained under it,' it seems clear that by that term is meant the term name, for in, e. g.,

Some men are mortals,

the subject "some men" is as much or as little (distributed) as in,

All men are mortals,

i. e. some class-name; and what is predicated of a class-name distributed is *ex vi termini*, predicated of each member of the class.

The dictum is not a canon of syllogism, if syllogism means formal mediate inference, nor even a statement of relations between different classes, but merely a formulation of the truth that if any object or objects belong to a class what can be said about the class distributively, can be said about it or them.

It seems to apply only in cases in which we are dealing with A, E, I, or O, propositions."

549. Miss Jones' position in the above quotation that "some men is as much distributed as all men" is correct. "Some men" constitute a group of beings just as much as "all men" do, and "some men" is to be taken altogether just as much as "all men."

One group we describe by "all," the other group we describe by "some." Logically the one group is as certain and definite as the other, and, as Miss Jones says, the one is as much or as little distributed as the other.

550. She further says: "But a canon of syllogism ought to apply, whatever terms and term-names we are dealing with, and whatever admissible arrangement of these we are considering. It ought to apply to syllogisms in the second, third and fourth figure, and to arguments in which all the terms are partial or single, as well as to syllogisms in figure 1, which have a class-name distributed for the subject name in the major premise.

When Prof. Jevons says (Principles of Science, p. 9, 3d Ed.), that 'The great rule of inference' is that 'so far as there

exists sameness, identity or likeness, what is true of one thing will be true of another,' I do not think he helps us much. For in any purely formal affirmative inference by categorical syllogism, it is not 'two things' that are named by the terms in each proposition, but one thing or group; and in a whole syllogism not two things, or three things, or six things, but one thing, or one thing and part of that same thing."

551. The reader will recall the statement made in the early part of the book, that in a proposition the subject and predicate were simply names for one thing. I am glad to know that so clear a thinker as Miss Jones agrees with me.

552. She further says: "The denomination and the application of the two terms in any affirmative proposition must be absolutely identical, and where there are more than three terms in an affirmative syllogism, the extra ones must be identical in denomination with part of the denomination of some of the three, e. g., in

All N's are Q's

Some R's are N's, Some R's are Q's

we have four terms, viz.:

- (1) All N's
- (2) Some Q's
- (3) Some N's
- (4) Some R's,

but we have not three or more things, but one group of things, viz. the Q's that are N's and a group that is all or a part of this, the R's that are N's."

553. And again, I am glad to call attention to the fact that Miss Jones' position agrees with mine when she says that "the two terms (i. e. the subject and predicate) in any affirmative proposition must be absolutely identical." The reader will remember that we have repeatedly called attention to the fact that in stating propositions we must so state them that they will read backward or forward.

554. But is Miss Jones consistent when she says, "And in

negative propositions and syllogisms we have only two things or two things and a part of one of them?"

In our system it makes no difference in this respect, whether a proposition is affirmative or negative, the subject and predicate must be identical and must both be names for the same thing, otherwise there could not be such a thing as a proposition. In making a proposition the mind holds in its grasp a thing or a group of things, and in order to describe it, or express a judgment in regard to it, the mind gives it one name and then another name and connects the two names together by the word "is" in a simple categorical proposition. It makes no difference whether the thing or the group of things, or the condition or state, is given an affirmative or negative name, the subject and the predicate, that is, the names of the thing or the condition or the state or whatever it is that the mind is holding in its grasp, must necessarily refer to the same thing, condition, or state, etc. The mind cannot be occupied with two things at once, any more than physically the same thing can occupy different places at the same time, or, to put it in other words, two bodies cannot occupy the same space at the same time.

555. Again, she says: "Jevons' rule has the further fault of being reducible to tautology, for, so far as there exists sameness, identity or likeness, what is true of one thing will be true also of another," can only mean "two things that are like, in as far as they are like" (two things cannot be identical or the same).

In this rule some of the important terms are in themselves ambiguous, and they are very loosely used. In employing likeness, identity, sameness, what is true, as he does in his canon, Jevons errs, it seems to me, in two ways:

(1) He confuses things that differ (qualitive likeness and quantitative identity—this confusion runs through his whole account of Inference);

(2) His phraseology implies a distinction where there is no difference, and thus a real tautology wears the guise of significant assertion.

When (*Op. Cit.*, p. 10,) he says, "In speaking of measuring extended objects, that we obviously employ the axiom that whatever is true of a thing as regards its length, is true of its equal in length, the absolute uselessness of the axiom seems clear—it amounts to no more than this, what is true of a given length in one case, is true of that length in another case, but this is only equivalent to saying that a given length is a given length, a form of words which has no predicative force. The matter is not mended by the further discussion (*Op. Cit.*, p. 17, and following pages) of logical inference, which appears to me to be spoiled all through by the ever-recurring confusion,

- (1) Between quantitative identity (what Jevons would perhaps call numerical sameness or identity) and qualitative likeness;
- (2) Between what I have called Independent and Dependent Propositions (to which latter class all mathematical equations belong).

There is perhaps also some confusion between terms and application (denotation) of terms. The implication that two things can be so similar point for point, as to be capable of being logically substituted one for another, seems due to (1)."

Miss Jones here refers to the confusion between quantitative identity and qualitative likeness. I entirely agree with her that Prof. Jevons' principle of reasoning, which he calls the Substitution of Similars, is useless in logic.

556. Again she says: "It seems to me that Jevons was constantly on the very verge of escaping from the first of the confusions indicated above, but that somehow he always just missed doing so."

CHAPTER XXI.

THE FIGURES.

557. The figures of the syllogism are merely different ways of stating it; just as the same act of reasoning may be stated categorically or hypothetically, so also can the syllogism be expressed in either of the four figures.

558. First Figure.

No true lover of pleasure is a true philosopher,
The Epicureans were lovers of pleasure,
Therefore they were not true philosophers.

Let A = true lover of pleasure,
B = true philosopher,
C = Epicureans.

The premises can be stated thus:

- (1) No A = B
- (2) C = CA.

Now, if No A = B, then the combinations containing AB are inconsistent and we eliminate them by making a figure 1 in those sections of an ABC Reasoning Frame:

AB	Ab	aB	ab	
1		2	2	C
1				c

Fig. 235.

Again, if C = CA, then the combinations containing Ca are inconsistent and we eliminate them by making a figure 2 in those sections.

From the combinations which remain we can get

$C = Cb$, which can be translated
The Epicureans were not true philosophers.

559. Second Figure.

No true philosopher is a lover of pleasure,
The Epicureans were lovers of pleasure,
Therefore, they were not true philosophers.

Let $A = \text{true philosopher}$,
 $B = \text{lovers of pleasure}$,
 $C = \text{Epicureans}$.

The premises can be stated thus:

- (1) $\text{No } A = B$
- (2) $C = CB$

Now, if $\text{No } A = B$, then the combinations containing AB are inconsistent and we eliminate them by making a figure 1 in those sections of an ABC Reasoning Frame:

AB	Ab	aB	ab	
1	2		2	C
1				c

Fig. 236.

Again, if $C = CB$, then the combinations containing Cb are inconsistent and we eliminate them by making a figure 2 in those sections.

From the combinations which remain we can get,
 $C = Ca$ which can be translated,
The Epicureans were not true philosophers.

560. Third Figure.

No lover of pleasure is a true philosopher,
Lovers of pleasure were Epicureans,
Therefore the Epicureans were not true philosophers.
Let $A = \text{lovers of pleasure}$,
 $B = \text{true philosophers}$,

$C = \text{Epicureans.}$

I assume that the proposition, Lovers of pleasure were Epicureans, means,

Some lovers of pleasure were Epicureans.

The premises can be stated thus:

- (1) $\text{No } A = B$
- (2) $CA = C.$

Make an ABC diagram:

AB	Ab	aB	ab	
1		2	2	C
1				c

Fig. 237.

Now, if $\text{No } A = B$, then the combinations containing AB are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $C = CA$, then the combinations containing Ca are inconsistent and we eliminate them by making a figure 2 in those sections.

From the uneliminated combinations we can get this definition of C,

$$C = CAB,$$

which can be translated,

The Epicureans were lovers of pleasure and not true philosophers.

The Epicureans were not true philosophers.

561. Fourth Figure.

No true philosopher is a lover of pleasure,

Lovers of pleasure were Epicureans,

Therefore the Epicureans were not true philosophers.

Let A = true philosophers,

B = lovers of pleasure,
 C = Epicureans,

The premises can be stated thus:

- (1) No A = B
- (2) CB = C

Make an ABC diagram:

AB	Ab	aB	ab	
1	2		2	C
1				c

Fig. 238.

Now, if No A = B, then the combinations containing AB are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if C = B, then the combinations containing Cb are inconsistent and we eliminate them by making a figure 2 in those sections.

From the uneliminated combinations we can get this definition of C,

$$C = CaB,$$

which can be translated,

The Epicureans were lovers of pleasure and not true philosophers.

562. Let P stand for the major term; M for the middle term; S for the minor term, then we can state the four figures thus:

Fig. 1.

$$M = P$$

$$S = M$$

Fig. 2.

$$P = M$$

$$S = M$$

Fig. 3.

$$M = P$$

$$M = S$$

Fig. 4.

$$P = M$$

$$M = S$$

563. Speaking of the special rules of the figures and the determination of the legitimate moods in each figure, Dr. Keynes, in "Formal Logic," p. 264, says:

"It may first of all be shown that certain combinations of the premises are incapable of yielding a valid conclusion in any figure."

564. From two particular propositions no conclusions can be drawn. Particular propositions which must be stated in the form of "AB = AB" have no inconsistents in the Reasoning Frame, and when nothing can be eliminated from the Reasoning Frame no conclusions can be drawn. It seems to me that the word "some" should be banished from the vocabulary of those who wish to reason exactly.

565. Let A = universal affirmative

I = particular affirmative

E = universal negative

O = particular negative

"Then there can be sixteen combinations of premises, the major premises being stated first, thus:

AA	IA	EA	OA
AI	II	EI	OI
AE	IE	EE	OE
AO	IO	EO	OO

But, according to the rules of the syllogism, EE, OE, OO, which contain two negative premises, yield no conclusions, and II, IO, OI, which contain two particular premises, yield no conclusions."

566. According to the special rules of the syllogism, in Fig. 1, the minor premise must be affirmative, (2) The major premise must be universal.

These rules leave four combinations in Fig. 1, viz: AA, AI, EA, EI. AA will yield the conclusion A or I; EA either E or O; AI only I; EI only O. This leaves six moods, which do not offend against any of the rules of the syllogism, viz: AAA, AAI, AII, EAE, EAO, EIO. The last letter in each combination stands for the conclusion.

567. Dr. Keynes further says, p. 270: "In this figure (Fig. 1) it is possible to prove conclusions of all the forms, A, E, I, O,

and it is the only figure in which a universal conclusion can be proved.

In Fig. 2 only negatives can be proved, and, therefore, it is chiefly used for purposes of disproof, for example:

Every real natural poem is naive.

Those poems of Ossian which Macpherson pretended to discover are not naive, but sentimental. Hence they are not real natural poems."

Let A = real natural poems

B = naive

C = poems of Ossian, etc.

The premises can be stated thus:

$$(1) \quad A = AB$$

$$(2) \quad C = Cb$$

Now if $A = AB$, then the combinations containing Ab are inconsistent, and we eliminate them by making a figure 1 in those sections of an ABC Reasoning Frame:

AB	Ab	aB	ab	
2	1	2		C
	1			c

Fig. 239.

Again, if $C = Cb$, then the combinations containing CB are inconsistent, and we eliminate them by making a figure 2 in those sections. From the combinations which remain we can get this definition:

C = Ca

which can be translated,

Those poems of Ossian, etc., are not real natural poems.

568. Figure 2 is also called the exclusive figure because it is used to successively exclude various suppositions. This process is called *abscissio infiniti*.

569. Let us take this example of the third figure:

Socrates is wise

Socrates is a philosopher

Therefore, some philosophers are wise.

Let A = Socrates

B = wise

C = philosopher.

The proposition can be stated thus:

$$(1) A = AB$$

$$(2) A = AC.$$

Now, if $A = AB$, then the combinations containing Ab are inconsistent, and we eliminate them by making a figure 1 in those sections of an ABC Reasoning Frame:

AB	Ab	aB	ab	
	1			C
2	1 2			c

Fig. 240.

Again, if $A = AC$, then the combinations containing Ac are inconsistent, and we eliminate them by making a figure 2 in those sections. From the combinations which remain we can get the following definition of C:

$$C = CAB \mid CaB \mid Cab$$

which can be translated,

Philosophers are either Socrates and wise, or not-Socrates and wise, or not-Socrates and not-wise.

I suppose the old logic would translate it in this way:

Some philosophers are wise and some philosophers are not wise;

Some philosophers are Socrates and some philosophers are not Socrates.

These conclusions are extra-logical.

CHAPTER XXII.

THE MOODS.

570. There are three propositions in a syllogism and they differ in quantity and quality.

These differences are indicated by the symbols A, E, I, O, which stand respectively for,

Universal Affirmative, Universal Negative,
Particular Affirmative, Particular Negative.

These differences are said to determine the mood of the syllogism. Now, as there are four kinds of propositions, and three propositions in each syllogism, there can be sixty-four different combinations.

571. If any one of the four of the above-named kinds of propositions A, E, I, O, be the major premise, each one of these majors may have four different minor premises, and these sixteen pairs of premises may each have four different conclusions, and four time four times four equals sixty-four. But the rules of the syllogism reject the moods which have negative premises, and particular premises, and some moods for other faults, so that out of the sixty-four possible moods the syllogism allows only eleven, viz.:

AAA, AAI, AEE, AEO, AII, AOO, EAI, EAO, EIO, IAI, OAO.

572. The moods of the syllogism which are allowed as mentioned in the preceding paragraph, are not allowable in every figure, because a mood might violate some of the rules of the syllogism in one figure and not in another. By applying the moods to each figure, the old logicians have found that each figure will admit six moods only, but several of these are useless because they draw a particular conclusion when a universal might have been drawn.

573. Let us take this example:

All human creatures are entitled to liberty,

All slaves are human creatures,

Therefore some slaves are entitled to liberty.

Let A = all human creatures,

B = entitled to liberty,

C = all slaves.

The premises can be stated thus:

$$(1) A = AB$$

$$(2) C = CA$$

Now, if $A = AB$, then the combinations containing Ab are inconsistent, and we eliminate them by making a figure 1 in those sections of an ABC Reasoning Frame:

AB	Ab	aB	ab	
	1	2	2	C
	1			c

Fig. 241.

Again, if $C = CA$, then the combinations containing Ca are inconsistent, and we eliminate them by making a figure 2 in those sections.

From the combinations which remain we can get the following definition of C:

C = CAB, which can be translated,

All slaves are human creatures and entitled to liberty.

574. The conclusion drawn by the old logic is not always the strict logical conclusion which necessarily follows from the premises. Five moods out of the twenty-four are neglected because they have particular conclusions when universals might have been drawn by the old logic.

575. For the remaining nineteen moods names have been devised to distinguish the mood and its figure. In these names the three vowels represent the quality and quantity of the propositions and the consonants represent the figures.

576. The first figure has four moods. The first mood is AAA, and its name is Barbara; in a certain way this is an arbitrary designation; it has no meaning except to point out a certain mood. The three A's in it mean that the mood is composed of three universal affirmative propositions.

An example is:

All men are fallible,

All kings are men.

Therefore all kings are fallible.

Let $A = \text{All men}$,

$B = \text{fallible}$,

$C = \text{kings}$.

The premises can be stated thus:

(1) $A = AB$

(2) $C = CA$

Now, if $A = AB$, then the combinations containing Ab are inconsistent, and we eliminate them by making a figure 1 in those sections of an ABC Reasoning Frame:

AB	Ab	aB	ab	
	1	2	2	C
	1			c

Fig. 242.

Again, if $C = CA$, then the combinations containing Ca are inconsistent, and we eliminate them by making a figure 2 in those sections.

From the combinations which remain we can get the following definition of C:

$C = CAB$, which can be translated.

All kings are fallible.

577. The second mood of the first figure is EAE, and its name is Celarent. An example is:

No men are gods,
All kings are men,
Therefore, no kings are gods.

Let A = men,

B = gods,

C = kings.

The premises can be stated thus:

$$(1) \text{No } A = B$$

$$(2) C = CA$$

Now, if No A = B, then the combinations containing AB are inconsistent, and we eliminate them by making a figure 1 in those sections of an ABC Reasoning Frame:

AB	Ab	aB	ab	
1		2	2	C
1				c

Fig. 243.

Again, if C = CA, then the combinations containing Ca are inconsistent, and we eliminate them by making a figure 2 in those sections.

From the combinations which remain we can get the following definition of C:

C = CAB, which we can translate,

Kings are not gods.

By reading the eliminated combinations, ABC, aBC, we can get,

"No C = B"

which can be translated,

No kings are gods.

578. The third mood is AII, and its name is Darii. An example is:

All men are fallible,

Some beings are men,

Therefore some beings are fallible.

Let A = men,

B = fallible,

C = beings.

The premises can be stated thus:

$$(1) A = AB$$

$$(2) CA = A$$

Now, if $A = AB$, then the combinations containing Ab are inconsistent, and we eliminate them by making a figure 1 in those sections of an ABC Reasoning Frame:

AB	Ab	aB	ab	
	1			C
2	21			c

Fig. 244.

Again, if $CA = A$, then the combinations containing Ac are inconsistent, and we eliminate them by making a figure 2 in those sections.

From the combinations which remain we can get the following definition of C:

$$C = CAB \mid CaB \mid Cab$$

which we can translate,

Beings are either fallible men, or fallible not-men, or not fallible not-men.

By reading the eliminated combinations we can get:

$$\text{No } A = b,$$

which can be translated ,

No men are infallible; no infallibles are men.

579. The fourth mood of the first figure is EIO, and its name is Ferio. An example is:

No men are gods,

Some beings are men,

Therefore some beings are not gods.

Let A = men,

B = gods,

C = beings.

The premises can be stated thus:

$$(1) \text{No } A = B$$

$$(2) CA = A$$

Now, if No A = B, then the combinations containing AB are inconsistent, and we eliminate them by making a figure 1 in those sections of an ABC Reasoning Frame:

AB	Ab	aB	ab	
1				C
12	2			c

Fig. 245.

Again, if A = C, then the combinations containing Ac are inconsistent, and we eliminate them by making a figure 2 in those sections.

From the combinations which remain we can get the following definition of C:

$$C = CAB \mid CaB \mid Cab$$

which can be translated,

Beings are either men and not gods; or gods and not men; or neither.

580. The second figure has four moods. The first mood of the second figure is EAE, and its name is Cesare. An example is:

No gods are men,

All kings are men,

Therefore no kings are gods.

Let A = gods,

B = men,

$C = \text{kings}$.

The premises can be stated thus:

- (1) $\text{No } A = B$
- (2) $C = CB$

Now, if $\text{No } A = B$, then the combinations containing AB are inconsistent, and we eliminate them by making a figure 1 in those sections of an ABC Reasoning Frame:

AB	Ab	aB	ab	
1	2		2	C
1				c

Fig. 246.

Again, if $C = CB$, then the combinations containing Cb are inconsistent, and we eliminate them by making a figure 2 in those sections.

From the combinations which remain we can get the following definition of C:

$$C = CaB$$

which we can translate,

Kings are not gods.

By reading the eliminated combinations we can get,

$$\text{No } B = A$$

which we can translate,

No men are gods.

We can also get,

$$\text{No } C = b,$$

which we can translate,

No kings are not-men.

581. The second mood of the second figure is AEE, and its name is Camestres.

An example is:

All kings are men,

No gods are men,
Therefore no gods are kings.

Let A = kings,
B = men,
C = gods.

The premises can be stated thus:

- (1) A = AB
- (2) No C = B

Now, if A = AB, then the combinations containing Ab are inconsistent, and we eliminate them by making a figure 1 in those sections of an ABC Reasoning Frame:

AB	Ab	aB	ab	
2	1	2		C
	1			c

Fig. 247.

Again, if No C = B, then the combinations containing CB are inconsistent, and we eliminate them by making a figure 2 in those sections.

From the combinations which remain we can get the following definitions:

$$C = Cab$$

which we can translate,

Gods are not kings,

$$A = ABC$$

which we can translate,

Kings are not gods.

From the eliminated combinations we can get:

$$\text{No } A = b$$

which we can translate,

No kings are not-men; not-men are no kings.

$$\text{No } C = A$$

which we can translate,

No gods are kings; no kings are gods.

No C = B

which we can translate,

No gods are men; no men are gods.

Camestres varies but slightly from Celarent, EAE.

582. An example of Celarent is:

No men are gods,

All kings are men,

Therefore no kings are gods.

which we have worked out in paragraph 577.

583. The third mood of the second figure is EIO and its name is Festino. An example is:

No gods are men,

Some beings are men,

Therefore some beings are not gods.

Let A = gods,

B = men,

C = beings.

The premises can be stated thus:

(1) No A = B

(2) CB = B

Now, if No A = B, then all the combinations containing AB are inconsistent, and we eliminate them by making a figure 1 in those sections of an ABC Reasoning Frame:

AB	Ab	aB	ab	
1				C
1 2		2		c

Fig. 248.

Again, if B = CB, then the combinations containing Bc are

inconsistent, and we eliminate them by making a figure 2 in those sections.

From the combinations which remain we can get the following definition of C:

$$C = CAB \mid CaB \mid Cab,$$

which we can translate:

Beings are either gods and not men, or men and not gods, or neither.

584. The fourth mood of the second figure is AOO, and its name is Baroco.

An example is:

All gods are men,

Some beings are not men,

Therefore some beings are not gods.

Let A = gods

B = men

C = beings.

The premises can be stated thus:

$$(1) A = AB$$

$$(2) Cb = Cb$$

Now, if A = AB, then all the combinations containing Ab are inconsistent and we eliminate them by making a figure 1 in those sections of an ABC Reasoning Frame:

AB	Ab	aB	ab	
	1			C
				c
	1			

Fig. 249.

From the combinations which remain we can get the following definition of C:

$$C = CAB \mid CaB \mid Cab,$$

which we can translate:

Beings are either gods and men; or men and not gods; or neither.

The eliminated combination Ab , can be read:

$$\text{No } A = b,$$

which we can translate:

No gods are not men; no not-men are gods.

This mood gave the old logicians a good deal of trouble. They tried to show its validity by a process called the *reductio ad impossible*. They showed that the conclusion cannot be supposed to be false without contradicting one of the premises, and the premises are supposed to be true.

585. The third figure has six moods. The first mood of the third figure is AAI, and its name is Darapti.

An example is:

All men are fallible,

All men are living beings,

Therefore, some living beings are fallible.

Let $A = \text{men}$,

$B = \text{fallible}$,

$C = \text{living beings}$.

The premises can be stated thus:

$$A = AB$$

$$A = AC$$

Now, if $A = AB$, then the combinations containing Ab , are inconsistent and we eliminate them by making a figure 1 in those sections of an ABC Reasoning Frame:

AB	Ab	aB	ab	
	1			C
2	1 2			c

Fig. 250.

Again, if $A = AC$, then the combinations containing Aa ,

are inconsistent and we eliminate them by making a figure 2 in those sections.

From the combinations which remain we can get the following definition of C:

$$C = CAB \mid CaB \mid Cab,$$

which can be translated,

Living beings are either men and fallible or not men and fallible; or neither.

From the eliminated combinations we can get,

$$\text{No } A = b$$

which can be translated,

No men are infallible; no infallibles are men.

$$\text{No } A = c,$$

which can be translated,

No men are not-living beings; no not-living beings are men.

586. The second mood of the third figure is IAI, and its name is Disamis.

An example is:

Some men are kings,

All men are fallible beings,

Therefore some fallible beings are kings.

Let A = men

B = kings

C = fallible beings

The premises can be stated thus:

$$(1) AB = B$$

$$(2) A = AC.$$

Now, if B = AB, then the combinations containing aB are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if A = AC, then the combinations containing Ac are inconsistent and we eliminate them by making a figure 2 in those sections of an ABC Reasoning Frame:

AB	Ab	aB	ab	
		1		C
2	2	1		c

Fig. 251.

From the combinations which remain we can get the following definition of C:

$$C = CAB \mid CAB \mid Cab,$$

which can be translated,

Fallible beings are men or kings or neither.

From the eliminated combinations we can get,

$$\text{No } A = c,$$

which can be translated,

No men are infallible beings; no infallible beings are men. I assume that not-fallible means infallible.

$$\text{No } B = c,$$

which can be translated,

No kings are infallible.

587. The third mood of the third figure is AII, and its name is Datusi. An example is:

All men are fallible

Some men are kings,

Therefore some kings are fallible beings.

Let A = men

B = fallible

C = kings.

The premises can be stated thus:

$$(1) A = AB$$

$$(2) AC = C.$$

Now, if A = AB, then all the combinations containing Ab are inconsistent and we eliminate them by making a figure 1 in those sections of an ABC Reasoning Frame:

AB	Ab	aB	ab	
	1	2	2	C
	1			c

Fig. 252.

Now, if $C = AC$, then the combinations containing aC are inconsistent and we eliminate them by making a figure 2 in those sections.

From the combinations which remain we can get this definition of C:

$$C = CAB,$$

which we can translate,

Kings are fallible men.

588. The fourth mood of the third figure is EAO, and its name is Felapton.

An example is:

No men are gods,

All men are living beings

Therefore some living beings are not gods.

Let A = men,

B = gods,

C = living beings.

The premises can be stated thus:

$$(1) \text{ No } A = B$$

$$(2) \quad A = AC$$

Now, if $\text{No } A = B$, then all the combinations containing AB are inconsistent and we eliminate them by making a figure 1 in those sections of an ABC Reasoning Frame:

AB	Ab	aB	ab	
1				C
1 2	2			c

Fig. 253.

Again, if $A = AC$, then the combinations containing Ac are inconsistent and we eliminate them by making a figure 2 in those sections.

From the combinations which remain we can get the following definition of C:

$$C = CAB \mid CaB \mid Cab,$$

which we can translate:

Living beings are either men and not gods, or gods and not men, or neither.

The eliminated combinations can be read:

$$\neg No\ A = B,$$

which we can translate,

No gods are men,

$$\neg No\ A = c$$

which we can translate,

No men are not-living beings; no not-living beings are men.

589. The fifth mood of the third figure is OAO, and its name is Bocardo. An example is:

Some men are not kings,

All men are fallible,

Therefore some fallible beings are not kings.

Let $A = \text{men}$,

$B = \text{kings}$,

$C = \text{fallible}$.

The premises can be stated thus:

$$(1) Ab = Ab$$

$$(2) A = AC$$

Ab = Ab has no contradictions.

If **A = AC**, then all the combinations containing **Ac** are inconsistent and we eliminate them by making a figure 2 in those sections of an ABC Reasoning Frame.

AB	Ab	aB	ab	
				C
1	1			c

Fig. 254.

From the combinations which remain we can get the following definition of C:

$$C = CAB \mid CAb \mid CaB \mid Cab,$$

which we can translate:

Fallible beings are fallible beings.

The full translation of the definition of C, i. e.,

Fallible beings are men and kings, or men and not kings,

or not-men and kings or neither,

does not tell us anything more than,

Fallible beings are fallible beings.

The eliminated combinations can be read,

No A = c, which we can translate:

No men are not-fallible beings, i. e.,

No men are infallible, and, no infallible beings are men.

590. The sixth mood of the third figure is EIO, and its name is Ferison.

An example is:

No men are gods,

Some men are living beings,

Therefore some living beings are not gods.

Let A = men,

B = gods,

C = living beings.

The premises can be stated thus:

- (1) No A = B
- (2) AC = AC

Now, if No A = B, then the combinations containing AB are inconsistent and we eliminate them by making a figure 1 in those sections of an ABC Reasoning Frame.

AB	Ab	aB	ab	
1				C
1				c

Fig. 255.

AC = AC has no contradictions in the Reasoning Frame.

From the combinations which remain we can get the following definition of C:

$$C = CAB \mid CaB \mid Cab,$$

which we can translate:

Living beings are either men and not gods, or gods and not men, or neither.

591. The fourth figure has five moods. The first mood of the fourth figure is AAI, and its name is Bramantip. An example is:

All kings are men,

All men are fallible,

Therefore some fallible beings are kings

Let A = kings,

B = men,

C = fallible.

The premises can be stated thus:

- (1) A = AB
- (2) B = BC

Now, if A = AB, then the combinations containing Ab are inconsistent and we eliminate them by making a figure 1 in those sections of an ABC Reasoning Frame:

AB	Ab	aB	ab	
	1			C
2	1	2		c

Fig. 256.

Again, if $B = BC$, then the combinations containing Bc are inconsistent and we eliminate them by making a figure 2 in those sections.

From the combinations which remain we can get the following definition of C:

$C = CAB | CaB | Cab$, which we can translate:

Fallible beings are either kings and men, or men and not-kings, or neither.

From the eliminated combinations we can get:

No $A = b$,

which we can translate:

No kings are not-men; not-men are no kings.

No $A = c$

which we can translate,

No kings are infallible; no infallible beings are kings.

592. The second mood of the fourth figure is AEE, and its name is Camenes. An example is:

All kings are men

No men are gods

Therefore, no gods are kings.

Let $A = \text{Kings}$,

$B = \text{men}$

$C = \text{gods}$

The premises can be stated thus:

(1) $A = AB$

(2) No $B = C$

Now, if $A = AB$, then all the combinations containing Ab are inconsistent, and we eliminate them by making a figure 1 in those sections of an ABC Reasoning Frame:

AB	Ab	aB	ab	
2	1	2		C
	1			c

Fig. 257.

Again, if $No\ B = C$, then all the combinations containing BC are inconsistent, and we eliminate them by making a figure 2 in those sections. From the combinations which remain we can get the following definitions of C and c :

$$C = Cab$$

which we can translate,

Gods are not kings

$$c = cAB \mid caB \mid cab$$

which we can translate,

Whatever things are not-gods are kings and men, or men and not-kings, or neither.

From the eliminated combinations we can get:

$$No\ A = b$$

which we can translate,

No kings are not-men; no not-men are kings;

$$No\ C = A$$

which we can translate,

No kings are gods; no gods are kings.

593. The third mood of the fourth figure is IAI, and its name is Dimaris. An example is:

Some living beings are men

All men are fallible

Therefore, some fallible objects are living beings.

Let A = living beings

B = men

C = fallible

The premises can be stated thus:

$$(1) AB = B$$

$$(2) B = BC$$

Now, if $B = AB$, then the combinations containing aB are inconsistent, and we eliminate them by making a figure 1 in those sections.

If $B = BC$, then all the combinations containing Bc are inconsistent, and we eliminate them by making a figure 2 in those sections of an ABC Reasoning Frame:

AB	Ab	aB	ab	
		1		C
2		12		c

Fig. 258.

From the combinations which remain we can get the following definition of C:

$$C = CAB \mid CAB \mid Cab$$

which we can translate,

Fallible beings are human living beings, or living beings not-men, or neither.

The eliminated combinations can be read:

$$No\ c = B$$

which we can translate,

No infallible beings are men; no men are infallible.

594. The fourth mood of the fourth figure is EAO, and its name is Fesapo. An example is:

No gods are men

All men are living beings

Therefore, some living beings are not gods.

Let $A = \text{gods}$

$B = \text{men}$

$C = \text{living beings}$

The premises can be stated thus:

(1) No $A = B$

(2) $B = BC$

Now, if No $A = B$, then all the combinations containing AB are inconsistent, and we eliminate them by making a figure 1 in those sections of an ABC Reasoning Frame:

AB	Ab	aB	ab	
1				C
1 2		2		c

Fig. 259.

Again, if $B = BC$, then all the combinations containing Bc are inconsistent and we eliminate them by making a figure 2 in those sections.

From the combinations which remain we can get the following definition of C :

$$C = CAB \mid CaB \mid Cab,$$

which we can translate:

Living beings are either gods and not men, or men and not gods, or neither.

From the eliminated combinations we can get,

$$\text{No } A = B,$$

which we can translate:

No gods are men; no men are gods.

$$\text{No } B = c,$$

which we can translate:

No men are not-living beings; no not-living beings are men.

595. The fifth mood of the fourth figure is EIO, and its name is Fresison.

An example is:

No gods are men,

Some men are living beings,

Therefore some living beings are not gods.

Let A = gods,

B = men,

C = living beings.

The premises can be stated thus:

(1) No A = B

(2) BC = BC

Now, if No A = B, then all the combinations containing AB are inconsistent and we eliminate them by making a figure 1 in those sections of an ABC Reasoning Frame:

BC = BC has no contradictions.

AB	Ab	aB	ab	
1				C
1				c

Fig. 260.

From the combinations which remain we can get the following definition of C:

C = CAB | CaB | Cab,

which we can translate:

Living beings are either gods and not men, or men and not gods, or neither.

The eliminated combinations can be read,

No A = B, which we can translate:

No gods are men; no men are gods.

I think the reader will be ready to agree with me that many of these moods are trifling variations of the moods of the first

figure. The examples given are taken from Prof. Bain's "Deductive and Inductive Logic."

596. The special rules for the legitimate moods of the second figure, are:

- (1) One premise must be negative,
- (2) The major premise must be universal.

The special rules for the legitimate moods of the third figure are

- (1) The minor premise must be affirmative,
- (2) The conclusion must be particular.

The special rules for the legitimate moods of the fourth figure are:

(1) If the major premise is affirmative, the minor premise must be universal.

(2) If either premise is negative, the major premise must be universal.

(3) If the minor premise is affirmative, the conclusion must be particular.

597. According to the old logic, wherever a universal conclusion could be drawn, a particular conclusion might also be inferred.

The old logic holds that "some" may be logically inferred from "all." I do not think this is logical. Logically the mind infers from positive to negative, or from negative to positive. Now "some" is neither synonymous with "all," nor do "all" and "some" stand in the relation of positive and negative to each other.

The opposite of "all" is "not all;" the opposite of "some" is "not some."

598. The moods of the syllogism which allow of a particular conclusion where a universal conclusion could have been drawn, are called Subaltern moods, and the conclusions are called Weakened conclusions.

The Subaltern moods are of no practical importance and are generally omitted.

599. A syllogism which has two universal premises from which a particular conclusion is drawn, is called a Strength-

ened syllogism, with the exception of AEO in the fourth figure. The premises are stronger than they need to be to draw the particular conclusion.

Thus if we had,

All A = B

All A = C

Therefore some C = B

the conclusion could have been obtained from the premises,

All A = B

Some A = C

or from the premises,

Some A = B

All A = C.

600. There are some forms which are not formal syllogisms in the strictest sense, though their correctness is immediately evident to most persons. Miss Jones gives this example:

A is greater than B

B is greater than C

Therefore A is greater than C.

This reasoning depends upon the assumption that,

If B is greater than C, what is greater than B is greater than C.

Let A ≡ A

B = B

C = greater than B

D = greater than C

The premises can be stated thus:

(1) A = AC

(2) B = BD

(3) C = CD

Now, if A = AC, then the combinations containing Ac are inconsistent and we eliminate them by making a figure 1 in those sections of an ABCD Reasoning Frame:

AB	Ab	aB	ab	
				CD
23	3	23	3	Cd
1	1			cD
$\frac{1}{2}$	1	2		cd

Fig. 261.

Again, if $B = BD$, then the combinations containing Bd are inconsistent and we eliminate them by making a figure 2 in those sections.

Again, if $C = CD$, then the combinations containing Cd are inconsistent and we eliminate them by making a figure 3 in those sections.

From the combinations which remain we can get the following definition of A:

A = AD, which we can translate,

A is greater than C.

601. In order to aid the student to remember the nineteen valid moods of the syllogism certain curious lines have been invented.

They are,

Barbāră, Cēlarent, Dārlī, Ferīōque, prioris; Cēsārē, Cāmēstres, Festīnō, Bārōkō, secundæ; Tertia, Dārapti, Dīsāmis, Dātisi, Fēlapton, Bōcardo, Fērisōn, habet; Quarta insuper addit Brāmantip, Cāmēnes, Dīmāris, Fēsāpo, Frēsison.

602. The first line indicates the four moods of the first figure. The second line indicates the four moods of the second figure. Then come the six moods of the third figure, and lastly the five moods of the fourth figure.

603. The letter s indicates that the proposition indicated by the vowel preceding the letter s is to be converted simply.

604. The letter p indicates that the proposition is to be converted *per accidens* or by limitation.

605. The letter m which is derived from the Latin word *mutare*, which means to change, indicates that the premises of the syllogism are to be transposed.

606. The letter k indicates that the mood must be proved by the process called the *reductio ad impossible*.

607. The capital letters B, C, D, F, which are the first consonants in the names of the moods of the first figure, indicate that the other moods beginning with the same initial letters are reducible to the mood of the first figure with the same initial letter. Thus, Cesare, Camestres, Camenes, are reducible to Celarent, Darapti, Disamis, Datisi and Dimaris to Darii. Felapton, etc, to Ferio, and so forth. The foregoing rules are taken from Prof. Jevons.

608. Aristotle did not recognize the fourth figure. It is supposed to have been invented by Galen, and hence it is frequently called the Galenian figure. Its use has been frequently condemned by the old logicians.

Father Clark in his Logic, p. 337, says:

"Ought we to retain it? If we do, it should be as a sort of syllogistic Hylot, to show how low the syllogism can fall when it neglects the laws on which all true reasoning is founded, and to exhibit it in the most degraded form which it can assume without being positively vicious. Is it capable of reformation? Not of reformation but of extinction. Where the same premises in the first figure would prove a universal affirmative, this feeble caricature of it, is content with a particular; where the first figure draws its conclusion naturally and in accordance with the forms into which human thought instinctively shapes itself, this perverted abortion forces the mind to an awkward and clumsy process which rightly deserves to be called 'inordinate and violent'."

609. Dr. Keynes says, "Thomson's ground of rejection is, that 'in the fourth figure the order of thought is wholly inverted, the subject of the conclusion had only been a predicate, whilst the predicate had been the leading subject in the premise. Against this the mind rebels and we can ascertain

that the conclusion is only the converse of the real one, by proposing to ourselves similar sets of premises to which we shall always find ourselves in the first figure, with the second premise first." (Laws of Thought, p. 176).

610. When one of the premises of a syllogism is omitted and it is understood, without being expressed, the syllogism is called Enthymeme; so also when the conclusion is left to be understood. If the major premise is omitted it is called an Enthymeme of the first order.

611. If the minor premise is omitted it is called an Enthymeme of the second order.

612. If the conclusion is omitted it is called an Enthymeme of the third order.

613. Dr. Keynes gives the following example of the three orders:

First, Balbus is avaricious and therefore he is unhappy.
Second, All avaricious persons are unhappy and therefore Balbus is unhappy.

Third, All avaricious persons are unhappy and Balbus is avaricious.

614. A chain of syllogisms is called a Polysyllogism.

615. In any syllogism, the conclusion of which is the premise of a preceding syllogism, the preceding syllogism is called a Prosylllogism, and the one which contains the conclusion is called an Episylllogism. The same syllogism may be both an Episylllogism and a Prosylllogism. When the same syllogism proceeds from Prosylllogism to Episylllogism it is called progressive. When the process is reversed, that is from Episylllogism to Prosylllogism, then the reasoning is called regressive.

616. An Epicheirema is a Polysyllogism in which one or more Prosylllogisms are merely indicated, thus:

All B = D because it = C

All A = B

Therefore all A = D

This would be called a single Epicheirema.

The following is a double Epicheirema:

All A = B because it = C

All D = A because all E = A

Therefore all D = B

The above argument can be stated thus:

$$(1) A = AB$$

$$(2) A = AC$$

$$(3) D = DA$$

$$(4) E = EA$$

Now, if A = AB, then the combinations containing Ab are inconsistent and we eliminate them by making a figure 1 in those sections of an ABCDE Reasoning Frame.

AB	Ab	aB	ab	
	—	—	—	—
	1	3	3 4	CDE
		4		
	—	—	—	—
	1	3	3	CDe
	1			
	.	4	4	CdE
	—	—	—	—
	1			Cde
	1			
2	2	4 3	3 4	cDE
	—	—	—	—
	1			
2	2	3	3	cDe
	—	—	—	—
	1			
2	2	4	4	cdE
	—	—	—	—
	2	1		
	2			cde

Fig. 262.

Again, if $A = AC$, then the combinations containing Ac are inconsistent and we eliminate them by making a figure 2 in those sections.

Again, if $D = DA$, then the combinations containing Da are inconsistent and we eliminate them by making a figure 3 in those sections.

Again, if $E = EA$, then the combinations containing Ea are inconsistent and we eliminate them by making a figure 4 in those sections.

From the combinations which remain we can get the following definition of D:

$$D = DB.$$

617. Where in a Polysyllogism the conclusions, except the final one, are omitted, and each two successive propositions contain a common term, the argument is called a Sorites.

618. There are two kinds of Sorites, the Aristotelian and the Gloclenian. In the Aristotelian the first premise contains the subject of the conclusion. In the Gloclenian the first premise contains the predicate of the conclusion.

619. The following is an example of the Aristotelian:

$$\text{All } A = B$$

$$\text{All } B = C$$

$$\text{All } C = D$$

$$\text{All } D = E$$

Therefore all $A = E$.

620. The following is an example of the Gloclenian:

$$\text{All } D = E$$

$$\text{All } C = D$$

$$\text{All } B = C$$

$$\text{All } A = B$$

Therefore all $A = E$.

621. Dr. Keynes gives the following rules of the Sorites:

- “(1) Only one premise can be negative; and if one is negative it must be the last.”
- “(2) Only one premise can be particular; and if one is particular it must be the first.”

- "(1) There cannot be more than one negative premise, for if there were.....since a negative premise in any syllogism, necessitates a negative conclusion.....we should in analyzing the Sorites, somewhere come upon a syllogism containing two negative premises. Again if one premise is negative, the final conclusion must be negative, hence P must be distributed in this conclusion, therefore, it must be distributed in this premise, i. e., the last premise, which must accordingly be negative. If any premise, then, is negative, this is the one.
- "(2) Since it has been shown that all premises, except the last, must be affirmative, it is clear that if any, except the first, were particular, we should somewhere commit the fallacy of undistributed middle."

622. In the examples given, the syllogisms in the Sorites are in Figure 1. A question has been raised as to whether there could be Sorites in Figures 2 or 3. Dr. Keynes says that there can be, and in this he is correct. He gives the following examples, (the lettering is mine):

"Some A is not B

C is B

D is C

E is D

Therefore some A is not E."

"Some D is not E

D is C

C is B

B is A

Therefore some A is not E."

623. Dr. Keynes further says that the first Sorites given are in Figure 2 and in the mood Baroco, and the syllogism in the last Sorites given are in Figure 3 and in the mood Bocardo.

624. The special rules given for the Sorites apply only to Figure 1.

625. Let us take the following example of a Sorites:

$$DE = DE$$

$$D = DC$$

$$C = CB$$

$$B = BA$$

Therefore $AE = AE$.

$DE = DE$ has no contradictions.

If $D = DC$, then the combinations containing Dc are inconsistent and we eliminate them by making a figure 1 in those sections of an ABCDE Reasoning Frame.

AB	Ab	aB	ab	
	2	3	2	CI E
	2	3	2	CDe
	2	3	2	CdE
	2	3	2	Cde
1	1	13	1	cDE
1	1	13	1	cDe
		3		cdE
		3		cde

Fig. 263.

Again, if $C = CB$, then the combinations containing Cb are inconsistent and we eliminate them by making a figure 2 in those sections.

Again, if $B = BA$, then the combinations containing Ba , are inconsistent and we eliminate them by making a figure 3 in those sections.

From the combinations which remain we can get the following definition of AE :

$$AE = AE$$

626. Dr. Keynes gives several examples like the preceding one, in which a particular conclusion is drawn. But as a particular amounts to a tautologous proposition, such as $A = A$, the Sorites yielding only particular conclusions are of no consequence.

627. We gave an example of an argument which read,

A is greater than B

B is greater than C

Therefore A is greater than C.

There are an indefinite number of other arguments which have a somewhat similar form and can be worked out on parallel lines, thus:

A equals B

B equals C

Therefore A equals C

It is assumed here that if B equals C, whatever is equal to B, will be equal to the equal of B. Similar arguments are:

A is a contemporary of B

A is the brother of B

A is to the right of B

A is in tune with B

and so on.

628. These kinds of arguments are not syllogisms, and yet they are valid arguments. Yet Archbishop Whately says:

"Syllogism is the form to which all correct reasoning may be ultimately reduced."

629. Prof. Ray makes the following claim:

"The syllogism is the type of all valid reasoning; for no reasoning will be valid unless it can be thrown into the form of a syllogism."

630. Spalding says,

"The syllogism is the norm of all inferences whose antecedent is more complex, and all such inferences may by those who think it worth while, be resolved into a series of syllogisms." (Logic, p. 158.)

631. J. S. Mill says:

"All reasoning by which from general propositions previously admitted other propositions equally or less general, are inferred, may be exhibited in some of the above forms," i. e., the syllogistic moods. (Logic, p. 191.)

CHAPTER XXIII.

PROPOSITIONS.

632. We have already said more or less on the subject of propositions, but in this chapter we purpose to treat the subject a little more fully.

A proposition is an act of the judgment giving two names to the one object in thought. The fundamental principle of the old logic that a proposition is the expression of the relation between the whole and its parts, is incorrect.

633. Every proposition has two terms, the subject and the predicate, which are connected by the copula "is." "Is" joins the two names which constitute the subject and the predicate together; it affirms the existence of the names or of the thoughts which are represented by the names.

634. The theory of the old logic that "is not" is a copula and denies the predicate of the subject, is incorrect; the "not" is a part of the predicate name.

635. Propositions are said to be opposed to each other when they have the same subject and the same predicate and at the same time the subject and predicate differ in quality or quantity or both.

636. A universal affirmative and a universal negative having the same subject and predicate are termed by the old logic, Contraries.

637. A universal affirmative and a particular affirmative, a universal negative and a particular negative are called Subalterns.

638. A particular affirmative and a particular negative are called Sub-contraries. Contraries and Sub-contraries differ in quality.

639. Subalterns differ in quantity.

640. Contradictories differ both in quality and quantity. Contradictories cannot both be true, for it cannot be true that "All men are mortal," and that "No man is mortal."

Sub-contraries cannot both be false; "Some horses are black" and "Some horses are not black" are sub-contraries; both propositions cannot be false.

641. Subalterns both may be true or false, thus:

All men are liable to mistakes, and,

Some men are liable to mistakes,

are both true.

No men are liable to mistakes, and,

Some men are not liable to mistakes,

are both false.

642. In Contradictories, if one is true, the other is false. If "All men are mortal" is true, then "No men are mortal" is false.

643. Propositions are said to be converted when their terms are transposed. As,

"Some cowards are boasters,"

"Some boasters are cowards."

644. A universal negative can be converted simply:

"No vegetables are stones,

"No stones are vegetables."

645. Particular affirmatives can be converted simply:

"Some men are tall,

"Some tall things are men."

646. A particular negative must be treated as a particular affirmative:

"Some members of the university are not learned,"

"Some not-learned are members of the university."

This example illustrates very clearly the statement already made that the word "not" belongs to the predicate and not to the copula.

647. An universal affirmative must have the quantity of the predicate in conversion explicitly stated, thus:

"All birds are animals,"

"Some animals are birds."

In the sentence "All birds are animals" it is implied, but not expressed, that all birds are some animals.

648. A proposition is not true when any one of the statements which it contains is false, as:

"Cæsar was put to death in the 610th year of Rome, by those whose lives he spared when conquered."

Cæsar was put to death in the 710th year of Rome.

The foregoing definitions are taken from the old logic, and in general are true.

649. According to the old logic, all universal propositions distribute the subject. No particular propositions distribute the subject. All negatives distribute the predicate. No affirmatives distribute the predicate.

I think this is a mistake. Usually a negative predicate is indefinite and undistributed. "Some men are not tall" means some men are some not tall things. "No men are immortal" means, all men are some not immortals, i. e., some mortals.

Take this example:

"All men are all rational animals."

The old logic said that it was merely accidental that "all rational animals" was a distributed term, and that it was not implied in the form of the expression.

650. Prof. Bain says, "Every proposition must be either true or false, and so on the other hand, nothing else can be, strictly speaking, either true or false. In colloquial language, however, true and false are often more loosely applied, as when men speak of the true cause of anything, meaning the real cause; the true heir, that is, the rightful heir; a false prophet, that is, a pretended prophet. A true or false argument, meaning a valid or apparent argument. A man true or false to his friend, meaning faithful or unfaithful."

651. Hobbes says, in his account of Categorical Propositions, that "the predicate is the name of the same thing of which the subject is a name," and Prof. Venn says, "What the statement (Plovers are lap-wings, clematis vitalba is travellers' joy) really means is that a certain object has two certain names belonging to it."

I am glad to be able to quote Hobbes and Prof. Venn on my side of this question.

652. The copula “is” has no relation to time; it expresses merely the fact that a certain thing has two names. If any other tense of the verb “to be” is used in a proposition, it is either understood as being equivalent in meaning to the present tense and the difference of tense is regarded merely as a matter of grammatical propriety, or else if the idea of time modifies the sense of the whole proposition, then this fact is one of the terms, and we can express it by “at that time” or similar words, as “This man was honest,” i. e., was honest at that time.

653. Some logicians have thought that the logical effect of “is” was equational, that is, it had the meaning of “is equal to.” But as in a proposition we are only speaking of one thing the equational theory is not correct. If we were talking of two different things, there would be some basis for the idea of equality.

654. The old logic places considerable stress on the order of the premises in the syllogism, and there has been a great deal of discussion on the subject. Aristotle held that the major premise containing the predicate of the conclusion should stand first; but logically it makes not the slightest difference which comes first. Of course we must state one after the other, but the order of statement can make no difference in logical results. It seems to me that Aristotle’s plan is not the best one. I like the order,

A is B

B is C

Therefore A is C

much better than I do the inverted order,

All B’s are C’s

All A’s are B’s

Therefore all A’s are C’s

655. Prof. Bain in "Logic," p. 159, says that a syllogism with two singular premises is not a genuine syllogism. Thus:

Socrates fought at Delium,

Socrates was the master of Plato,

Therefore the master of Plato fought at Delium.

He says that the proposition, "Socrates was the master of Plato and fought at Delium" compounded out of the two premises, is nothing more than a grammatical abbreviation."

In no way, therefore, can a syllogism with two singular premises be viewed as a genuine syllogistic or deductive inference.' If this is true, then I think no syllogisms are genuine, because we can always combine into one statement, the information contained in the two premises of a syllogism.

656. Speaking of negation Prof. Bain says, in "Logic," p. 57, "The negative of a real property or thing is also real. If negation be simply the remainder when one thing is subtracted from a universe containing more than one, such negation is no less a positive reality than the so-called positive, in fact, positive and negative must always be ready to change places.

657. Prof. Jevons, in "Principles of Science," p. 63, speaking of negative propositions, says, "It would be a mistake, however, to suppose that the real occurrence of negative terms in both premises of a syllogism, renders them incapable of yielding a conclusion. The old rule informed us that from two negative premises no conclusion could be drawn. But it is a fact that the rule in this bare form does not hold universally true, and I am not aware that any precise explanation has been given of the condition under which it is or is not imperative."

658. Referring to the above I would say that the old rule holds good in the case of particular propositions where both terms are indefinite. I do not know of any other cases where it holds good, but in the case stated above the rule makes no difference, because you cannot draw any definite conclusion from two wholly indefinite premises, no matter whether they are affirmative or negative.

659. Prof. Jevons further says, "Consider the following example:

Whatever is not metallic is not capable of powerful magnetic influence.

Carbon is not metallic.

Therefore carbon is not capable of powerful magnetic influence.

Here we have two distinctly negative premises, and yet they yield a perfectly valid conclusion. The syllogistic rule is actually falsified in its bare and general statement."

660. The word "not" is the word commonly used for a negative term, but the words "except," "omitting," "excluding," "but not," "only if not," have the same logical effect.

Take the sentence "Lawyers, not chancery solicitors," and we could substitute for the word "not" "except," or "omitting" or "excluding," or "but not," or "only if not," and the logical effect would be the same.

661. I suggest that two propositions which when worked out in the Reasoning Frame can both be read be called Consistents. That when one destroys a combination necessary to the other they be called Inconsistents. That when they eliminate a letter-term they be called Contradictories. That when one eliminates every combination which the other saved and saves every combination which the other eliminated, they be called Perfect Contradictories. That when each eliminates and saves the same combinations that the other did, they be called Equivalents.

662. The terms A and a are opposites but not contradictories. These terms can stand for propositions and then one is a necessary inference of the other. Propositions may be contradictories, but terms cannot be.

663. Given the proposition,

$$\text{All } A = \text{all } B$$

its negative equivalent is

$$\text{All } b = \text{all } a.$$

To take a concrete example,

If all salt is all chloride of sodium, its negative equivalent is,

Whatever is not chloride of sodium is not salt.

Let us take the proposition,

All A = some B,

which can be stated thus:

(1) A = AB

Make an AB diagram:

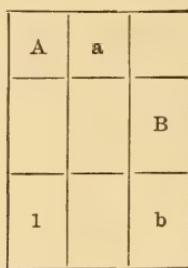


Fig. 264.

Now, if A = AB, then the combination Ab is inconsistent, and we eliminate it by making a figure 1 in that section.

We can now read in the Reasoning Frame:

$$b = ba$$

and this is the negative equivalent of the given proposition.

We can also read the following consistent propositions:

$$(1) B = A \mid a$$

$$(2) a = B \mid b$$

664. Not every consistent proposition is an inference, at least I make a distinction between consistent propositions and inferences. I call a consistent proposition one which can be read anywhere in the Reasoning Frame.

665. An inference is a consistent proposition which can be read in the uneliminated combinations, and which also eliminates some of the combinations, and no others, eliminated by the principal proposition or inferend, as Miss Jones calls it. I think this is an important discovery.

Let us take the proposition:

$$A = ABC$$

Make an ABC diagram:

AB	Ab	aB	ab	
	1			C
1	1			c

Fig. 265.

Now, if $A = ABC$, then the combinations containing Ab , Ac , are inconsistent, and we eliminate them by making a figure 1 in those sections.

The following propositions are inferences:

$$c = ca$$

$$b = ba$$

$$B = a \mid AC$$

$$C = a \mid AB$$

In this case the negative equivalent of the given proposition is a compound proposition, viz:

$$b = ba \text{ and } c = ca$$

$AB = ABC$ is an inference.

A consistent combination is:

$$bc = bea$$

this is also an inference, because it would eliminate one of the eliminated combinations and no one of the uneliminated combinations.

A contradictory proposition is:

$$A = Be$$

because it would cause the elimination of the letter A.

Two propositions are independent when both can be read in the Reasoning Frame and each eliminates an entirely different set of combinations.

666. Let us take the proposition:

$$(1) AB = ABC$$

Make an ABC diagram:

AB	Ab	aB	ab	
				c
1				c

Fig. 266.

Now, if $AB = ABC$, then the combination ABC is inconsistent, and we eliminate it by making a figure 1 in that section.

We can now read:

$$(1) c = ca \mid cAb$$

This is the negative equivalent of the given proposition.

$$(2) Bc = Bea$$

This is a consistent proposition but not an inference, because if the combination Bca was destroyed, it would not effect the expression of the given proposition.

$$(3) BC = BCa$$

is an inconsistent proposition, because it destroys the visible expression of the given proposition, that is, it eliminates a combination which is necessary to the expression of the given proposition.

$$(4) A = ABC$$

This is a contradictory proposition because it eliminates the letter A.

CHAPTER XXIV.

QUANTIFICATION.

667. The predicates of propositions generally have no such words as "all" or "some" affixed to them to denote the distribution or the non-distribution of the predicate name. And yet, of course, the predicate must always be meant to represent either "all" or "some." Whether expressed or not, it must really be either distributed or undistributed.

668. Sir William Hamilton is the writer on logic who has most strenuously insisted on the quantification of the predicate. By quantifying the predicate we expressly state whether the predicate term means "all" or "some."

Hamilton's rule was, "We must render explicit in the statement whatever is implicit in the thought." In nearly all arguments there are omissions and ellipses which must be supplied. Thus, for instance, the universal affirmative proposition,

$$\text{All } A = B$$

would be quantified and become,

$$\text{All } A \text{ is some } B$$

which we would state symbolically,

$$A = AB$$

669. In our system we always quantify the predicate, and I think Hamilton is clearly right in insisting that before we can have accurate reasoning the predicate must be quantified. By quantifying the predicate, the old logic obtains four new moods, symbolized by U, Y, n and w.

I have substituted n and w for the Greek letters which Prof. Hamilton used.

U stands for "All S is all P."

Y stands for "Some S is all P." (P meaning predicate and S meaning subject).

n stands for "No S is some P."

w stands for "Some S is not some P."

670. Some writers think that there should be recognized by the old logic, six forms of propositions, thus:

A All S is P

Y Only S is P, i. e., All P is some S

E No S is P

I Some S is P

n Not only S is P, i. e. No S is some P, Not S alone is P

O Some S is not P

671. Dr. Keynes says, "Formal Logic," p. 334: "By a rigid quantification of the predicate, however, the distinction between subject and predicate may be dispensed with; and such being the case, there is no ground left for distinction of figure, which depends upon the position of the middle term as subject or predicate."

672. The old logic uses the word "some" in the sense of "some and it may be all." In our system we use it in the sense of "some only."

673. Prof. Lotze in his work on Logic, expresses some very correct views on the quantification of the predicate. On p. 59, he says: "When we say 'Gold is yellow,' it is indisputable that in this judgment our idea of gold lies within the sphere of yellow, and that accordingly the predicate is of wider extent than the subject; but it certainly is not this that we intended to express by the judgment."

Logic, indeed, has already drawn attention to the fact that we are not quite right even in making this sentence; appealing from what we express to what we mean, it teaches that the subject also, from its side, limits the too extensive predicate; gold is not yellow simply, but golden yellow, the rose is rosy red, and this particular rose only this particular rosy red. The

relation which exists between them is primarily no more than this, that whenever or wherever, under certain conditions, the one idea, gold, is found, there the other idea, yellow, is also found, but that the former is not always present when the latter is.

We say, "Some men are black," and suppose ourselves to be making a synthetical judgment, because blackness is not contained in the concept of man. But the true subject of this sentence is not the universal concept man (for it is not that which is black), but certain individual men; these individuals, however, though they are expressed as merely an indefinite portion of the whole of humanity, are yet by no means understood to be such an indefinite portion; for it is not left to our choice what individuals we will take out of the whole mass of men; our selection, which makes them 'Some' men, does not make them black, if they are not so without it; we have, then, to choose these men, and we mean all along only those men who are black, in short, negroes; these are the true subjects of the judgment. That the predicate is not meant in its universality, that on the contrary, only the particular black is meant which is found on human bodies, is at once clear, and I shall follow out this remark later; here I will only observe that it is merely the want of inflection in the German expression which deceives us as to its proper sense; the Latin '*Non nulli homines sunt nigri*' shows at once by number and gender that 'homines' has to be supplied to 'nigri.' The full sense, then, of the judgment is, 'some men,' by whom, however, we are to understand only black men. 'are black men;' as regards the matter, it is perfectly identical, and as regards its form, it is only synthetical because one and the same subject is expressed from two different points of view, as 'black men' in the predicate, as a fragment of all men in the subject."

Again, we say, 'the dog drinks.' But the universal dog does not drink; only a single definite dog, or many, or all single dogs, are the subject of the sentence. In the predicate, too, we mean something different from what we express; we do not think of

the dog as an ever-running syphon; he does not drink simply, always, and unceasingly, but now and then, and this ‘now and then’ also, though expressed as an indefinite number of moments, is not so meant; the dog drinks only at definite moments, when he is thirsty, or, at any rate, inclined, when he finds something to drink, when nobody stops him; in short, the dog which we mean in this judgment, is really only the drinking dog, and the same drinking dog is also the predicate.”

Again, ‘Cæsar crossed the Rubicon,’ but not the Cæsar who lay in the cradle or was asleep, or was undecided what to do, but the Cæsar who came out of Gaul, who was awake, conscious of the situation, and who had made up his mind; in a word, the Cæsar whom the subject of this judgment means is that Cæsar only whom the predicate characterizes, the Cæsar who is crossing the Rubicon, and in no previous moment of his life was he the subject to whom this predicate could have been attached. It is obvious, moreover, to every capacity, that when he had crossed the river he could not go on crossing it, but was across, so that in no subsequent moment of his life either can he be the subject intended in this judgment.

I will give two more examples which Kant has made famous. It is said that the judgment, “A straight line is the shortest way between two points,” is synthetical, for neither in the concept “straight,” nor in that of “line,” is there any suggestion of longitudinal measure. But the actual geometrical judgment does not say of a straight line in general, that it is the shortest way, but only of that one which is included between these two points.

Now this fact, the fact that its extension is bounded by two points, (and it is only with this qualification that it forms the true subject of the sentence), is the ground, in this certainly the satisfactory ground, for assigning the predicate to it. It is easy to see that the concept of a straight line, ab, between the points a and b, is perfectly identical with the concept of the distance of the two points; for we cannot give any other idea of what we mean by “distance in space” than this, and that it is

the length of the straight line between a and b. There is not, therefore, a shorter and longer distance between a and b, but only the one distance, ab, which is always the same. On the other hand, we can speak of shorter and longer ways between a and b; the concept of "way" implies merely any sort of progression which leads from a to b; as this requires the getting over of the difference which separates b from a, there can be no way leading from a to b which leaves any part of this difference not got over; accordingly that the shortest of all possible ways is the distance, i. e., the straight line between the given points, is a judgment which as regards its matter is perfectly identical, and merely regards the same object from different aspects.

Nor, again, can the arithmetical judgment $7 + 5 = 12$, because 12 is not contained in either 7 or 5; the complete subject does not consist in either of the quantities singly, but in the combination of them required by the sign of addition; but in this combination, if the equation is correct, the predicate must be wholly contained; the equation would be false if some unknown quantity had to be added to 7 plus 5 in order to produce 12. Here, too, then, we have a perfectly identical judgment as regards its matter, and it is only synthetical formally, because it exhibits the same number 12 first, as the sum of 7 plus 5." (p. 85).

674. These remarks of Lotze tend very strongly to confirm our theory that the subject and the predicate are names of the same identical thing, and also to confirm our other theory that when we state propositions in symbolical language we should state them in the form of identical propositions, that is, propositions which are true when read either forward or backward.

675. The reader will have seen that in our system we have no difficulty in treating particular propositions. But in "Studies in Logic," p. 47, the writer makes the following criticism on Boole's system: "The plan of treating a set of universal premises as a command to exclude certain combinations

of the terms which enter them, is due to Boole; no adequate extension of his method, so as to take in particular propositions, is possible, without the use of some device which shall be equivalent to a particular copula."

676. I think that a particular copula is an impossibility. I am not familiar enough with Boole's system to say whether this criticism is just or not, but from what I have read of it, I cannot see any reason why it should not take in particular propositions.

677. On page 124, "Elements of Logic," Miss Jones says: "Quantification of the predicate in categorical propositions, seems to me to occupy an impregnable position in logic, a position, however, very different from that assigned to it by Sir William Hamilton, Dr. Thomson, Prof. Baynes, and others. My opinion is, that while the traditional form of A, E, I, O propositions is to be retained, quantification is an indispensable instrument of conversion, and therefore of reduction. The place of quantification in logic is very curious, its function being often as completely hidden from those processes of conversion which involve it as the subterranean train in one of the loop-tunnels of the Swiss Alps would be to an observer who only saw it rush into one opening and emerge again in a few minutes from another, just above or just below. My meaning will be best elucidated by taking an ordinary proposition and tracing the changes which it undergoes in conversion.

Let the proposition be:

(1) All human beings are rational.

The ordinary converse of this is:

(2) Some rational creatures are human beings, or

(3) Some rational creatures are human.

(3) is perhaps the more perfect converse, because (1) and (3) resemble each other in having an adjectival term for P, while (2) has a substantive term for P. (1) and (3) are adjectival propositions, (2) a coincidental proposition. Adjectival propositions cannot be converted

Again, she says: "If I alter the position of S and P in (1) as it stands, and say

Rational are all human beings

it is clear that conversion in the logical sense has not taken place; for rational is still the predicate and 'all human beings' is still the subject. The proposition has been merely turned round."

678. In our logic the subject and predicate being names for the same thing, the only difference is in their position. The subject can become the predicate and the predicate the subject.

679. Again, she says: "But it may be transformed to the equivalent coincidental proposition,

(4) All human beings are rational creatures,
and with this we can deal. It is not, however, any more than the adjectival (1) simply convertible. If altered into,

Rational creatures are all human beings,
the proposition thus obtained, besides being awkward, is ambiguous—it is by no means clear which term is to be taken as subject, and the 'all' might even be understood to qualify (or quantify) 'rational creatures.'

The first step towards real conversion is taken when we pass from the unquantified coincidental (4) to the quantified proposition,

(5) All human beings are some rational creatures.

From this we go on to the quantified converse,

(6) Some rational creatures are all human beings,
and from (6) to the unquantified converse of (5),

(7) Some rational creatures are human beings.

From (7) we pass to the equivalent adjectival proposition,

(8) Some rational creatures are human.

680. Contrast this process with our method.

Let us take the same example:

All human beings are rational.

Let A = human beings,

B = rational.

The proposition can be stated thus:

$$A = AB$$

Now, if $A = AB$, then the combination Ab is inconsistent, and we eliminate it by making a figure 1 in that section of an AB Reasoning Frame:

A	a	
		B
1		b

Fig. 267.

Now, with us conversion is merely reading the result of the statement of the proposition in the Reasoning Frame. We can get the following:

$$B = A | a, \text{ which we can translate,}$$

Rational beings are either human beings or not human beings.

The old logic would translate this:

Some rational beings are all human beings.

681. Again, Miss Jones says: "In converting an E proposition, we should, I think, proceed as follows: Let the proposition to be converted be,

$$(1) \text{ No } R \text{ is } Q$$

(2) Any R is not Q (by mere equivalence). Quantifying (2) we get: Any R is not any Q (3), (3) converts to: Any Q is not any R (4).

By disquantifying (4) we reach:

$$(5) \text{ Any } Q \text{ is not } R,$$

and (5) No Q is R (by equivalence)."

682. By our system an eliminated combination reads backward or forward by simply prefixing the word "No" to the combination; in other words, "No" represents the effect of the eliminating process. We can also insert it between the letters.

In the example given by Miss Jones we would at once read, "No Q is R," Q is no R.

683. It seems to me that Miss Jones does not believe in the Aristotelian theory of inclusion in a class, that is, that the subject is included in the predicate. She says, "When, for example, I say, 'The sky is blue,' my meaning, and my whole meaning is, that the sky has that particular color. I am not thinking of the class blue, as regards extension at all, I am not caring, not necessarily, what blue things there are, or if there is any blue thing except the sky. I am thinking only of the sensation of blue, and am judging that the sky produces this sensation in my sensitive faculty; or (to express the meaning in technical language) that the quality answering to the sensation of blue or the power of exciting the sensation of blue, is an attribute of the sky. When, again, I say, All oxen ruminate, I have nothing to do with the predicate considered in extension—the comprehension of the predicate—the attribute or set of attributes signified by it—are all that I have in mind; and the relation of this attribute, or these attributes, to the subject, is the entire matter of the judgment.

When we say Phillip is a man, or, A herring is a fish, do the words "man" and "fish" signify anything to us but the bundle of attributes connoted by them? Do the propositions mean anything except that Phillip has the human attributes and a herring the piscine ones? Assuredly not. Any notion of a multitude of other men among whom Phillip is ranked, or a variety of fishes besides herrings, is foreign to the proposition."

Again, Miss Jones says: "We have seen that propositions on their way to conversion, have to undergo the process of quantification. But the reason why O is pronounced inconvertible is not because there is not any more difficulty in quantifying it than in quantifying the other propositions, but because when the quantified converse of O has been reached, the quantification of its predicate cannot be dropped without an illegitimate alternation of signification. For the commonly accepted signification of the disquantified converse

of O involves a quantification different from that which has been dropped—the dropped P—indicator being some, the P-indicator understood as involved in the unquantified proposition reached by dropping it being any. And, as at the same time, ordinary thought and speech will not admit the explicitly quantified form, it is inevitable that a logic which deals with the forms of ordinary thought and speech, should regard O as inconvertible. To take an instance:

The proposition,

(1) Some blackbirds are not black birds
becomes by quantification,

(2) Some blackbirds are not any black birds.

This converts to,

(3) Any black birds are not some blackbirds.

Dropping the quantification of (3) we get:

(4) Any black birds are not blackbirds,

and this would be understood to mean,

(5) Any black birds are not any blackbirds,
(No black birds are blackbirds)." (p. 130.)

Now, in our system, the proposition,

Some blackbirds are not black birds means,
blackbirds which are not black birds are not black birds and
we state it symbolically thus:

$$Ab = Ab$$

and read it thus,

not black birds are blackbirds.

684. The quantification of the predicate led Sir William Hamilton to recognize eight different forms of propositions instead of the usual four:

All S is all P	U i.e. A = B. B = A
All S is some P	A i.e. A = AB
Some S is all P	Y i.e. BA = B
Some S is some P	I i.e. AB = AB
No S is any P	E i.e. No A = B
No S is some P	n i.e. No A = AB
Some S is not any P	O i.e. Ab = Ab
Some S is not some P	w i.e. Ab = Ab

685. In our system we are not compelled to have propositions given to us in any specified form, because our system is able to deal with every kind of proposition in every kind of form, excepting numerical propositions.

686. Prof. Hamilton's position that "In thought the predicate is always quantified," is correct, and hence it follows that, "in logic the quantity of the predicate must be expressed on demand, in language."

687. Dr. Bain says, "The quantity of the predicate is not expressed in common language, because common language is elliptical. Whatever is not really necessary to the clear comprehension of what is contained in thought, is usually elided in expression. But we must distinguish between the ends which are sought by common language and logic respectively. Whilst the former seeks to exhibit with clearness the matter of thought, the latter seeks to exhibit with exactness the form of thought. Therefore, in logic, the predicate must always be quantified."

688. Dr. Keynes says, "Formal Logic," p. 168, "'Predication is nothing more nor less than the expression of the relation of quantity in which a notion stands to an individual, or two notions to each other. If this relation were indeterminate—if we were uncertain whether it was a part, or whole, or none—there could be no predication.'

Amongst the practical advantages said to result from quantifying the predicate are the reduction of all species of the conversion of propositions to one, namely, simple conversion; and the simplification of the laws of syllogism. As regards the doctrine of the quantification of the predicate, the distinction between subject and predicate resolves itself into a difference in order of statement alone. Each propositional form can, without any alteration in meaning, be read either forwards or backwards, and every proposition may, therefore, rightly be said to be simply convertible. It is further argued that the new propositional forms resulting from the quantification of

the predicate are required in order to express relations that cannot otherwise be so simply expressed. Thus U alone serves to express the fact that two classes are co-extensive and even w is said to be needed in logical divisions, since if we divide (say) Europeans into Englishmen, Frenchmen, etc., this requires us to think that some Europeans are not some Europeans (e. g., Englishmen are not Frenchmen)."

689. On p. 172, Formal Logic, Dr. Keynes says, "It is altogether doubtful whether writers who have adopted the eight-fold scheme have themselves recognized the pitfalls surrounding the use of the word "some." Many passages might be quoted in which they distinctly adopt the meaning—some but not all. Thus Thomson ('Laws of Thought,' p. 150), makes U and A inconsistent."

690. Dr. Keynes says, p. 335, "(2) IUn in Fig. 1, is invalid, if some is used in its ordinary logical sense. The premises are:

Some M is some P,

All S is all M.

We may therefore obtain the legitimate conclusions by substituting S for M in the major premise. This yields,

Some S is some P."

Let us substitute A for S, B for M and C for P.

The premises can be stated thus:

$$(1) BC = BC$$

$$(2) A = B$$

$$(3) B = A$$

$BC = BC$ has no contradictions in the Reasoning Frame.

If $A = B$, then the combinations containing Ab are inconsistent, and we eliminate them by making a figure 1 in those sections of an ABC Reasoning Frame:

AB	Ab	aB	ab	
	1	2		C
	1	2		c

Fig. 268.

Again, if $B = A$, then all the combinations containing Ba are inconsistent, and we eliminate them by making a figure 2 in those sections.

From the combinations which remain we can get the following definition of A:

$AC \mid Ac = AC \mid Ac$, which can be translated, according to the old logic,

Some A is some C.

He says next: "If, however, "some" is here used in the sense of "some only." No S is some P follows from some S is some P, and the original syllogism is valid, although a negative conclusion is obtained from two affirmative premises."

From the eliminated combinations in the example given we can read:

No A = AbC | Abc

A translation of this would be, according to the old logic

No A is some C

691. Again, Dr. Keynes says:

"(3) AYI in Figure 1, some being used in its ordinary logical sense, is equivalent to AAI in Figure 3, in the ordinary syllogistic scheme, and it is therefore valid, but it is invalid if some is used in the sense of some only, for the conclusion now implies that S and P are partially excluded from each other as well as partially coincident, whereas, this is not implied by the premises.

692. AYI consists of three propositions in the form of,

All A is some B,

Some A is all B

Therefore some A is some B

Some A is all B can be read,

All B is some A

and the premises can be stated thus:

(1) $A = AB$

(2) $B = BA$

Now, if $A = AB$, then the combination Ab is inconsistent, and we eliminate it by making a figure 1 in that section of an AB Reasoning Frame:

AB	Ab	aB	ab	
	1	2		C
	1	2		e

Fig. 269.

Again, if $B = BA$, then the combination Ba is inconsistent, and we eliminate it by making a figure 2 in that section.

From the combinations which remain we can get the following definition of A:

$A = B$, i. e., All A = all B

The conclusion "I" does not logically follow from the premises A and Y.

693. Dr. Keynes says, p. 177, of the proposition n, "This proposition in the form of No S is some P, is not, I think, ever found in ordinary use. We may, however, recognize its possibility; and it must be pointed out that a form of proposition which we do meet with, namely, Not only S is P, or Not S alone is P, is practically n, provided that we do not regard this proposition as implying that any S is certainly P."

Let us work the example given,

Let $A = S$

$B = P$

then the proposition can be stated:

No $A = AB$

Now, if No $A = AB$, then the combination AB is inconsistent, and we eliminate it by making a figure 1 in that section of an AB Reasoning Frame:

A	a	
1		B
		b

Fig. 270.

The combinations which remain are,

Ab , i. e., All $A = \text{some } b$,

aB " Some $a = \text{all } B$, a only is B ,

ab " Some $a = \text{some } b$.

694. Dr. Keynes further says that Archbishop Thomson remarks that n 'has the resemblance only and not the power of a denial. True though it is, it does not prevent our making another judgment of the affirmative kind, from the same terms.'"

("Laws of Thought," p. 79).

695. When we allow n to be represented by symbols, as in the preceding example, it denies the existence of one combination, that is, eliminates it. He is right, however, when he says that it does not prevent our making another judgment of the affirmative kind,—that is, on our theory that all propositions are affirmative.

The proposition that we obtained was,

$A = Ab$

Archbishop Thomson would probably call this "another judgment of the negative kind," because the old logic calls any proposition "negative" which contains a negative term.

696. Dr. Keynes then says, "This is erroneous, for although A and n may be true together, U and n cannot, and Y and n are strictly contradictories."

Let us take examples of these propositions:

Let " $A = AB$ " represent the proposition A,

"No $A = AB$ " represent the proposition n.

Now, if $A = AB$, then the combination Ab is inconsistent, and we eliminate it by making a figure 1 in that section of an Ab diagram:

A	a	
2		B
1		b

Fig. 271.

Again, if No $A = AB$, then the combination AB is inconsistent, and we eliminate it by making a figure 2 in that section.

An examination of the Reasoning Frame now shows that all the A's are eliminated. This proves that the propositions A and n are contradictories, and that it is not possible for them to be true together.

697. Let us take the propositions Y and n:

Let $AB = B$ represent the proposition Y,

No $A = AB$ represent the proposition n.

Make an AB diagram:

A	a	
2	1	B
		b

Fig. 272.

Now, if $B = AB$, then the combination Ba is inconsistent, and we eliminate it by making a figure 1 in that section.

Again, if $No\ A = AB$, then the combination AB is inconsistent, and we eliminate it by making a figure 2 in that section.

The result proves that Y and n are contradictories because the letter B is eliminated.

698. Let us take the propositions U and n :

$Let\ A = B$ and $B = A$ represent U ,

$No\ A = AB$ represent n .

Make an AB diagram:

A	a	
3	2	B
1		b

Fig. 273.

Now, if $A = B$, then the combination Ab is inconsistent, and we eliminate it by making a figure 1 in that section.

Again, if $B = A$, then the combination Ba is inconsistent, and we eliminate it by making a figure 2 in that section.

Again, if $No\ A = AB$, then the combination AB is inconsistent, and we eliminate it by making a figure 3 in that section.

The result proves that **U** and **n** are contradictories because the letter terms **A** and **B** are eliminated.

699. Dr. Keynes says, (p. 178),

"The proposition **w** is absolutely of no importance." And I think the same can usually be said of all particular affirmative propositions.

700. The exclusive propositions,

Only **S** is **P**,

S alone is **P**,

are examples of the propositional form called **Y** in the eight-fold scheme. They usually mean,

Some **S** is all **P**

If we let **A** stand for **S** and **B** for **P**, then by repeating the predicate in the subject, thus:

AB = **B**, we can state the proposition **Y** so that it will be true when read backward.

In working it we can say, If **B** = **AB**, then the combinations which contain **aB** are inconsistent, and we eliminate them, etc.

701. According to our system, the eight propositional forms can be stated thus:

U	All A is all B ,	A = B , B = A
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A	All A is some B ,	A = AB
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Y	Some A is all B ,	AB = B
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I	Some A is some B ,	AB = AB
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E	No A is any B ,	No A = B
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n	No A is some B ,	No A = AB
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O	Some A is not any B	Ab = Ab
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w	Some A is not some B ,	Ab = Ab
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The proper stating of propositions is a very important matter in logic. We should always bear in mind Hamilton's rule, "Render explicit in the statement, whatever is implicit in the thought."

CHAPTER XXV.

INCONSISTENCY.

702. In our system the total elimination of any letter is a certain sign of contradictoriness in the premises.

703. According to the old logic, if a universal affirmative proposition is given, the universal negative is false, the particular affirmative is true and the particular negative is false.

If A is AB is true,
Then, No A is B is false,
and AB is AB is true,
and Ab is Ab is false.

704. If a universal negative proposition is given as true, then the universal affirmative proposition is false and the particular affirmative proposition is false and the particular negative proposition is true.

If No A is B,
then A is AB is false,
and AB is AB is false,
and Ab is Ab is true.

705. If the particular affirmative proposition is given as true, then, the universal affirmative proposition is unknown, the universal negative proposition is false and the particular negative proposition is unknown.

If AB is AB,
then, A is AB is unknown,
and No A is B is false,
and Ab is Ab is unknown. (?)

706. If the particular negative proposition is given as true, then the universal affirmative proposition is false, the universal negative proposition is unknown and the particular affirmative proposition is unknown.

If Ab is Ab ,
 then, A is AB is false, (?)
 and $No A$ is B is unknown,
 and AB is AB is unknown. (?)

707. If A is AB is false,
 then, $No A$ is B is unknown, (?)
 and AB is AB is unknown, (?)
 and Ab is Ab is true.
708. If $No A$ is B is false,
 then, A is AB is unknown, (?)
 and AB is AB is true,
 and Ab is Ab is unknown. (?)
709. If AB is AB is false,
 then, A is AB is false,
 and $No A$ is B is true,
 and Ab is Ab is true.
710. If Ab is Ab is false,
 then A is AB is true,
 and $No A$ is B is false,
 and AB is AB is true.

I doubt the correctness of the conclusions marked with a (?).

711. The denial of a truth of a proposition is equivalent to the affirmation of the truth of its contradictory, and vice versa. Of two contradictory propositions, one must be true and the other false.

712. In our system, given any proposition as true, we eliminate the inconsistent propositions. In order to do this there must be no ambiguity in the proposition which is given to us. If we do not fully understand the meaning of the given proposition, we cannot tell what it denies. The meaning of a proposition depends upon what it denies.

713. A compound proposition or a complex proposition, usually has a large number of inconsistent propositions. If any one of the inconsistents is true, then, the given proposition is not true.

714. If the contradictories of a given proposition contain a common term, they may be converted into a single alternative proposition, and then the single alternative proposition would be the contradictory of the given proposition.

715. When we have two propositions like those which follow composed of the same terms or of their opposites, the two propositions are equivalent, and, consequently, if one is true, the other is also true. Thus, if the proposition,

$$\text{All } A = \text{All } B$$

is true, then the proposition,

$$\text{All } a = \text{all } b$$

is also true.

716. If the proposition,

$$A = AB$$

is true, then the proposition,

$$b = ba$$

is also true.

717. If the proposition,

$$a = ab$$

is true, then the proposition,

$$B = BA$$

is also true.

718. Equivalent propositions are sometimes called equipollent, meaning equal in signification and logical force.

719. The old logic says that the particular affirmative proposition,

$$AB = \Lambda B$$

is inferrible from the universal affirmative proposition,

$$\text{"All } A \text{ is } B\text{"}$$

I think this is extra logical.

720. A proposition is inconsistent with a given proposition when it eliminates any combination which is necessary to the expression of the given proposition in the Reasoning Frame.

Let the given proposition be,

$$C = A \mid B$$

Make an ABC diagram:

AB	Ab	aB	ab	
1			1	C
				c

Fig. 274.

Now, if $C = A \mid B$, then the combinations CAB, Cab are inconsistent, and we eliminate them by making a figure 1 in those sections.

An examination of the Reasoning Frame now shows that either of the propositions,

- (1) $Ab = Abc$
- (2) $aB = aBc$
- (3) $A = Ac$
- (4) $a = ac$
- (5) $B = Bc$
- (6) $b = bc$

is inconsistent with the given proposition, because it would eliminate one or the other of the combinations AbC , aBC , and these combinations are necessary to the expression of the given proposition.

721. By the use of the Reasoning Frame I have discovered an easy method of finding propositions inconsistent with the given proposition. The method is as follows:

First, make the visible expression of the given proposition in the Reasoning Frame, by eliminating the inconsistent combinations.

Second, make a similar Reasoning Frame and eliminate in it the combinations which the given propositions saved, and which are necessary to its expression.

Third, from the uneliminated combinations in the Second Reasoning Frame, get definitions of the letter-terms and each of these definitions will be inconsistent with the given proposition, because it will eliminate a combination which is necessary to the expression of the given proposition.

722. Let the given proposition be,

$$C = A \mid B$$

Make an ABC diagram:

AB	Ab	aB	ab	
1			1	C
				c

Fig. 275.

Now, if $C = A$ or B , then the combinations ABC , abC , are inconsistent, and we eliminate them by making a figure 1 in those sections.

Make another ABC diagram:

AB	Ab	aB	ab	
	1	1		C
				c

Fig. 276.

Eliminate the combinations AbC , aBC , by making a figure 1 in those sections.

From the uneliminated combinations we can get these definitions of the different letter-terms:

- (1) $A = AB \mid Abc$
- (2) $B = BA \mid Bac$
- (3) $a = ab \mid aBe$
- (4) $b = ba \mid bAc$
- (5) $C = CAB \mid Cab.$

Each one of these definitions is inconsistent with the given proposition, because it will destroy a combination necessary to the expression of the given proposition, for example,

$$A = AB \mid Abc.$$

Make an ABC diagram:

AB	Ab	aB	ab	
	1			C
				c

Fig. 277.

Now, if $A = AB \mid Abc$, then the combination containing AbC is inconsistent and we eliminate it by making a figure 1 in that section.

The Reasoning Frame now shows that the combination AbC is eliminated. This was a necessary combination to the expression of the given proposition, hence, the definition $A = AB \mid Abc$, is inconsistent with the given proposition.

723. Let us take this example:

The powers not delegated to the United States by this Constitution nor prohibited by it to the states, are reserved to the states respectively, or to the people.

Let $A =$ the powers delegated to the U. S.,

$B =$ the powers prohibited to the states,

$C =$ the powers reserved to the states,

$D =$ the powers reserved to the people.

The proposition can be stated thus:

- (1) $ab = Cd \mid cD$
- (2) $Cd \mid cD = ab$
- (3) $C = Cd.$

Make an ABCD diagram:

AB	Ab	aB	ab	
2	2	2	21	CD
1	1	1		Cd
1	1	1		cD
			1	cd

Fig. 278.

Now, if $ab = Cd \mid cD$, and if $Cd \mid cD = ab$, then the combinations containing ACd , $aBCd$, $abCD$, AcD , $aBcD$, $abcd$, are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $C = Cd$, then the combinations containing CD are inconsistent and we eliminate them by making a figure 2 in those sections.

Make another ABCD diagram:

AB	Ab	aB	ab	
				CD
			1	Cd
			1	cD
1	1	1		cd

Fig. 279.

Eliminate the combinations containing Acd, aBcd, abCd, abcD, by making a figure 1 in those sections.

In this diagram we have eliminated the combinations which the given propositions saved.

From the uneliminated combinations we can get the following definitions of the different letter-terms:

- (1) $A = AC | A\bar{c}D$, which can be translated:

The powers delegated to the United States are either reserved to the states or reserved to the people.

- (2) $B = BC | B\bar{c}D$, which can be translated:

The powers prohibited to the states are either reserved to the states, or to the people.

- (3) $C = CD | CA | C\bar{a}B$, which can be translated:

The powers reserved to the states are either reserved to the people, or delegated to the U. S., or prohibited to the states.

- (4) $D = DC | DA | D\bar{a}B$, which can be translated:

The powers reserved to the people are either reserved to the states, or delegated to the United States, or prohibited to the states.

- (5) $a = aBC | aB\bar{c}D | abCD | ab\bar{c}d$, which can be translated:

The powers not delegated to the United States are either prohibited to the states and reserved to the states, or prohibited to the states and reserved to the people, or reserved to the states and to the people or are neither prohibited to the states nor reserved to the states nor reserved to the people.

- (6) $b = bAC | bA\bar{c}D | baCD | bac\bar{d}$, which can be translated:

The powers not prohibited to the states are either delegated to the United States and reserved to the states, or delegated to the United States, and reserved to the people or reserved to the states and to the people, or are neither delegated to the United States nor reserved to the states nor reserved to the people.

(7) $c = cAD \mid caBD \mid cabd$, which can be translated:

The powers not reserved to the states are either delegated to the United States and reserved to the people, or prohibited to the states and reserved to the people, or are neither delegated to the United States nor prohibited to the states nor reserved to the people.

(8) $d = dAC \mid daBC \mid dabc$, which can be translated:

The powers not reserved to the people are either delegated to the United States and reserved to the states, or are prohibited to the states and reserved to the states, or are neither delegated to the United States nor prohibited to the states nor reserved to the states.

Each and every one of these definitions is inconsistent with the given proposition, because it destroys a combination which is necessary to the expression of the given proposition.

Let us take the definition of b , $b = bAC \mid bAcD \mid baCD \mid baCd$:

Make an ABCD diagram:

AB	Ab	aB	ab	
				CD
			1	Cd
			1	cD
				cd

Fig. 280.

Now, if $b = bAC \mid bAcD \mid baCD \mid baCd$, then the combinations containing abCd, abcD, are inconsistent and we eliminate them by making a figure 1 in those sections.

The Reasoning Frame now shows that we have eliminated two combinations which were necessary to the expression of the given propositions. Hence, the definition of b is inconsistent with the given proposition.

724. Another method of obtaining inconsistent propositions is by prefixing the particle no to the subject, or predicate of an uneliminated combination, e. g., let the uneliminated combination be AB, i. e., $A = B$, then $No\ A = B$, $No\ B = A$, are inconsistent with $A = B$.

CHAPTER XXVI.

CONVERSION.

725. When the terms of a proposition are transposed, i. e., when the subject is made the predicate and the predicate the subject, the proposition is said to be converted. The old logic had a great many rules for the conversion of propositions.

In our system we state all propositions so that they will read backward as well as forward. Prof. Jevons calls propositions in this form, Identical propositions. An Identical proposition can be read either way without any rules.

If all A is all B, all B must be all A.

If twice two is four, then four is twice two.

726. In our system we read our combinations in any order we please, without stopping to think whether there are any rules for converting subjects into predicates and predicates into subjects. We do not recognize any real distinction between subject and predicate, except in the matter of position, one must come before the other, that is all there is to it. Neither do we recognize any difference between so-called positive and negative propositions. These are not logical but conversational distinctions.

727. In conversion, the original proposition is called the convertend, and the inferred proposition is called the converse.

728. Dr. Keynes gives the two following rules:

(1) The converse must be the same in quality as the convertend.

(2) No term must be distributed in the converse unless it was distributed in the convertend. These rules apply to what is called simple conversion. Thus, I, Some A is some B, can be converted simply into, Some B is some A. Stated symbolically, AB is AB can be read BA is BA.

729. The proposition E, no A is B, can also be converted simply into No B is A. In our system the logical effect of No A is B, is the elimination of the AB combination.

We translate the eliminating mark by "No" and then we can read the combination indifferently,

No A is B, No B is A, A is no B, B is no A.

In a similar way we can write the letter s in the section containing the combination AB, to indicate that the combination represents the proposition, Some A is some B, and that we can read it either way, Some A is some B and some B is some A. This way of indicating the particular affirmative proposition is logically equivalent to the form we commonly use, viz.:

AB is AB.

730. In the case of the universal affirmative proposition A,

All A is AB,

we cannot infer

All B is A,

because, in the first proposition the B means some B, and is therefore undistributed, while in the second proposition it is distributed. So that in this case the converse would be,

Some B is all A.

This is called conversion *per accidens* or conversion by limitation.

731. In the old logic the particular negative proposition, Some A is b, is said to be incapable of ordinary conversion, because some A is undistributed in the convertend, and if A became the predicate, it must be distributed, for it is a rule of the old logic, that the predicate of a negative proposition must be distributed.

732. Aristotle proved the conversion of the universal negative proposition, No A is B, as follows: No A is B, therefore, No B is A; for, if not, some individual B, say C is A; and hence, C is both A and B; but this is inconsistent with the given proposition, No A is B.

733. Dr. Keynes proves the conversion *per accidens*, of the universal affirmative proposition, A is AB, as follows: All A is B, therefore, some B is A; for, if not, no B is A, and, therefore, by conversion, No A is B, but this is inconsistent with the given proposition, All A is B.

734. In the simple conversion of particular affirmative propositions, unless the predicate and the subject are co-extensive, the word "some" has a different value in the two propositions, according to Dr. Bain. He says: "In the couple,

Some men are dark-haired,

Some dark-haired beings are men,

"Some men," as compared with "All men" is a larger fraction than some dark-haired beings as compared with all dark-haired beings."

In every true proposition the predicate and subject must be co-extensive, because they are merely names for the same thing.

735. Dr. Bain gives an example of the application of this process of contraposition to a universal affirmative proposition, thus:

All men are mortal,

No men are immortal,

No immortals are men,

or, by symbols, thus,

All X is Y

No X is not-Y

No not-Y is X.

736. In our system we would state the proposition, All men are mortal, by

$$A = AB,$$

This would cause us to eliminate the combination

Ab,

and then we can read the eliminated combination,

No A = b,
 A = no b
 No b = A
 b = no A.

A	a	
		B
1		b

Fig. 281.

737. This process of conversion by negation or contraposition, is applicable to universal affirmatives.

"A universal affirmative may be stated as a universal negative, thus,

Every A = B,
 No A = not-B, A is no not-B,

and this again may be converted into,

Not-B = not-A,

for instance,

Every true poet is a man of genius, which may be converted into,

No true poet is not-a-man-of-genius,
 which may be converted into:

No one who is not a man of genius is a true poet."

738. In our system, Every true poet is a man of genius, would be stated thus,

A = AB,

This would cause us to eliminate Ab, and we can read the eliminated combination,

No b = A, which can be translated,
 No not-genius is a true poet, thus,

A	a	
		B
1		b

Fig. 282.

The foregoing expression, No not-genius is a true poet, can be expressed in the following equivalent phrases,

“None but a man of genius can be a true poet.”

“A man of genius alone can be a true poet,”

“One cannot be a true poet without being a man of genius.”

739. In examples like the above, the words, “may,” “can,” “cannot,” etc., have no reference to power exercised by an agent; they refer merely to the confidence or doubtfulness we feel in respect to some supposition. To say, for instance, that “A man who has the plague may recover,” does not mean that it is in his power to recover if he chooses; but it is only a form of stating “Some who have the plague recover.” This is an ordinary particular affirmative proposition.

740. So also to say,

“A virtuous man cannot betray his country,” or,

“It is impossible that a virtuous man should betray his country,”

does not mean that he lacks the power to betray his country, but it is merely a different way of stating the universal negative proposition,

“No virtuous man betrays his country.”

741. In order to reason correctly, it is necessary, as we have remarked before, that propositions should be stated correctly, and to do this, we must perceive the exact logical force of the given proposition. The correct stating of propositions is of as

much importance as the working of them out in the Reasoning Frame.

The preceding examples are taken from Archbishop Whately.

742. Prof. Bain says, that when we affirm one thing, we must be prepared to deny its opposite. A fact can be stated in two different ways, thus,

"This road is level,"

"This road is not inclined."

These are not two facts, but the same facts stated in two different ways. This process is named Obversion.

743. He gives the following examples (the lettering is mine).

Every man is mortal, every A is B,

First obvert the predicate,

Every man is not mortal, every A is not B.

Next, prefix the sign of negation to the subject,

No man is immortal, no A is not B.

744. To obvert the particular affirmative proposition,

Some men are wise, some A is B,

first obvert the predicate, and then prefix the sign of negation to the predicate.

Some men are not-wise, i. e., foolish.

745. To obvert the universal negative proposition,

No men are gods, no A is B,

transfer the sign of negation from the subject to the predicate,

All men are no-gods,

All A is not-B.

746. To obvert the particular negative proposition,

Some men are not wise,

Some A is not B,

change the form of the proposition thus,

Some men are not-wise, i. e., foolish.

Some A is not-B.

747. Prof. Bain gives the rule for obversion thus,

"Obvert the predicate and change the quality of the proposition."

748. Prof. Lotze says, vol. 1, p. 107:

"We cannot infer from the negation of the universal proposition, either the truth or the untruth of the particular."

I think this is a mistake,

Let $A = AB$ represent the universal proposition; its negation would be represented by,

$$\text{No } A = B.$$

Make an AB diagram:

A	a	
1		B
		b

Fig. 283.

Now, if $\text{No } A = B$, then the combination AB is inconsistent, because it implies that $A = B$, and we eliminate it by making a figure 1 in that section.

Now, as the combination AB , is non-existent, the untruth of the particular proposition AB is AB , which means Some A is some B , follows necessarily.

749. Prof. Lotze says in "Logic" vol. 1, p. 108: "If we deny the proposition,

All S are P , i. e., A is AB ,

the denial is consistent with both the assumptions, E and O ,

No S is P (i. e., No A is B),

Some S are not P (i. e., $Ab = Ab$),

but the second which is included in the first is true in any case; consequently the truth of O follows certainly from the untruth of A ."

By our method we make an AB diagram, then we eliminate the AB combination, then we read the AB combination,

No A = B, this is E,
and we read the Ab combination,

All A is some b, thus,

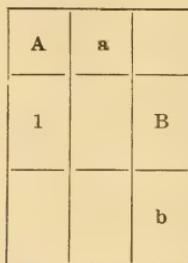


Fig. 284.

750. Again, Prof. Lotze says: "If we further deny O,
Some S are not P (i. e., Ab = Ab),

This means, according to what we said above,

"There is no such thing as Some S which are not P, and
this is equivalent to A,
All S are P, i. e., A = AB."

Make an AB diagram, eliminate the Ab combination:

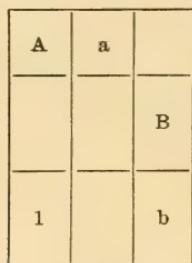


Fig. 285.

And then we can read the AB combination,
All A is some B.

751. Again Prof. Lotze says: "If we deny I, this means:
There is no such thing as Some S which are P, and is
equivalent to the affirmation of E,
No S is P.

Make an AB diagram:

A	a	
2		B
		b

Fig. 286.

Let $AB = AB$ represent the proposition I.

Now if we deny I, we can represent the denial of I by eliminating the AB combination.

Suppose we eliminate the AB combination by making a figure 2 in that section.

We can read the eliminated combination thus:

No $A = B$,

which is equivalent to the affirmation of E.

752. Again, Prof. Lotze says, that "when we have a proposition which can be stated:

$A = B$

and read backward as well as forward, we have a case of what is called pure conversion, but when the proposition is of the form

$A = AB$,

which means some B , which reads backward,

Some $B = A$,

we have a case of impure conversion.

It is a very common mistake and also a favorite means of deception to convert the proposition,

$A = AB$ into

$B = A$."

753. Prof. Lotze calls propositions of the form,

$A = B$,

$B = A$,

reciprocal judgments. He gives these examples:

"All men are naturally capable of language,"

"All equi-lateral triangles are equi-angular;"

They can be converted into,

"All that is naturally capable of language is man."

"Every equi-angular triangle is an equi-lateral one."

754. In speaking of the conversion of the particular affirmative proposition in the old logic, Lotze says: "But when S is the genus of which P is a species, as in the proposition, "Some dogs are pugs," the only logical admissible conversion, "Some pugs are dogs," will contrast unfavorably with the actually true one, "All pugs are dogs." The former is no doubt true also, but it expresses only a part of the truth which appears rather to deny than affirm the other part, that, "All other pugs are also dogs."

We feel this still more if we start with the judgment, "All pugs are dogs," and convert it twice over. From the first conversion, "Some dogs are pugs," we cannot get back again by the second to the original proposition, and thus the logical operations have here resulted in eliminating a part of the truth. This inconvenience could easily be avoided if the expressions of quantity were regarded, as the sense requires that they should be as inseparable from their substances, we should then formulate the proposition, in the first instance, as follows: "All pugs are some dogs;" then by conversion, "Some dogs are all pugs," and by a second conversion, "All pugs are some dogs." But it is not worth the trouble to improve what are, after all, barren formulæ."

755. Speaking of the particular negative judgment: "Some S are not P," Prof. Lotze says, "The pure conversion, therefore, "Some P are not S," does not hold good universally, but only of those P which are predicates common to different subjects and are not, therefore, exclusively dependent upon the nature of S for their occurrence. For this reason, the proposition, "Some men are not black," can be converted into, "Something black is not man;" but the judgments, "Some men are not pious," "Some are not Christians," would yield "Something

pious is not man," "Some Christians are not men," both inadmissible, because piety and Christianity, though not belonging to all men, belong only to men.

These disadvantages are in general only avoided by joining the negative to the predicate, and then converting the proposition, "Some S are non-P," like a particular affirmative, into "Some non-P are S," e. g., "Something not-black, something not-pious, some non-Christians are men. The process necessary in this case, has been extended to all judgments under the name of conversion by contraposition; in the affirmative judgments the negation of non-P takes the place of affirmation of P; in the negative the affirmation of non-P, takes that of the negation of P; the judgments thus changed are then converted according to ordinary rules. In this way we get the following results: First, for A, "All S are P," "No S is non-P," and so non-P is no S;" for I, on the other hand, "Some S are P," the transformation into "Some S are non-P," would not, after what has been said above, allow any conversion and transposition would therefore be impossible; for E again, "No S is P," we get, "All S are non-P," "Some non-P are S."

I would like to say right here, that Prof. Lotze is in some respects one of the clearest thinkers in logic that I know of.

756. Miss Jones, in "Elements of Logic," p. 143, proposes a new terminology for the different kinds of conversion. She suggests,

- Subversion for subalternation,
- Reversion for simple conversion,
- Intraversion for conversion *per accidens*,
- Contraversion for contraposition,
- Retroverse for obverted converse,
- Extraversions for added determinants,
- Extraversion for inference by complex conception.

757. She says that subversion is passing from a complete proposition to a partial proposition that has the same subject, the same predicate, and the same quality, thus:

“Every wind is ill to a broken ship,”
“Some winds are ill to a broken ship.”

758. In obversion, she says that the obverse has the same subject and the same quantity as the obvertend, but different quality; and the predicate of the obverse is the negative of the predicate of the obvertend, thus:

“All is fine that is fit,”
“Nothing is not fine that is fit.”

The principle of obverting is, that the affirmation (or denial) of any predicate, justifies the denial (or affirmation) of its negative.

759. In a reversion, she says that the educt has the predicate of the educend for its subject, and the subject of the educend for its predicate.

Reverse and Revertend do not differ in quantity or quality; only E and I can be reverted.

To take two simple examples,”

“Those plants are biennials,”
“Some biennials are all those plants,”
“No man is a free agent who cannot command himself,”
“No free agent is a man who cannot command himself.”

760. In intraversion she says, that we infer from an affirmative proposition, a partial proposition of the same quality, which has the predicate of the educend for its subject and the subject of the educend for its predicate, thus:

“An honest miller has a black thumb,”
“Some persons having a black thumb are honest millers.”

761. In contraversion she says, that the educt differs from the educend as follows:

- (1) The subject of the educt is the negative of the predicate of the educend.
- (2) The predicate of the educt is the subject of the educend.
- (3) Educt and educend differ in quality.
- (4) Every contraverse, except that of E, has the same quantity as the contravertend, thus:

“If is stiff,”
“Not-stiff is not-if.”

762. She says that O cannot be contraverted, because its obverse is O, which cannot be converted; and to reach the converse of any proposition, it has to be obverted, and then the obverse thus obtained has to be converted.

763. In retroversion she says the educt differs from the educend in quality, the predicate of the educend is the subject of the educt, and the negative of the subject of the educend is the predicate of the educt.

764. In the universal affirmative proposition only, the educt and the educend differ in quantity. She gives these examples:

- “All who love me keep my commandments,”
- “Some who keep my commandments are not those who do not love me,”
- “Some doctrines are universally accepted,”
- “Some things universally accepted are not doctrines which are not true,”
- “Some believers in Spiritualism are these well-known writers,
- “These well-known writers are not disbelievers in Spiritualism,”
- “These R’s are Q’s,”
- “Some Q’s are not-these-R’s.”

765. In inversion, she says, “We obtain from a given proposition a new proposition having the contradictory of the original subject for its subject and the original predicate for its predicate. Also the inverse of any proposition differs from the invertend in both quality and quantity. A and E are the only propositions which can be inverted. The following are examples:

"No sunshine is without shadow,"

"Some things that are not sunshine are without shadow,"

"A friend in need is a friend indeed,"

"Some who are not friends in need are not friends indeed."

766. "Only coincidental propositions can be converted, contraverted, retroverted, or inverted. Only A propositions can be intraverted. Adjectivals as well as coincidentals may be subverted, obverted and extraverted."

767. The reader will understand that in this chapter on Conversion I am trying to give a brief account of what the writers on the old logic have to say in regard to this process.

768. In our system, as heretofore pointed out, we quantify the terms when we state the proposition, so that the subject and predicate are equivalent terms. This does away at once with the necessity of conversion. The process for making the subject and predicate identical is exceedingly simple. If the subject is wider than the predicate we reduce it to an equivalence with the predicate by adding the predicate to the subject. Thus, given the propositions,

$$\text{Some } A = \text{All } B$$

we state it,

$$AB = B$$

This means that the A which is B is B, e. g.,

Some dogs are pugs,

The dogs which are pugs are pugs, or,

Pug dogs are pugs,

and of course this proposition can be read backward,

Pugs are pug dogs.

If the predicate is wider than the subject, we reduce the predicate to the limits of the subject by adding the subject to the predicate, thus:

If we have the proposition,

A is some B

we state it thus,

$A = AB$

This means that A is the B which is A, e. g.,

Men are animals,

Men are the animals which are men, or,

Men are animal men,

which is equally true when read backward,

Animal men are men.

I do not think it is possible to reason exactly in complicated cases, unless this process of making the terms equivalent is pursued.

769. Dr. Keynes in his work on "Formal Logic" gives a more detailed statement of conversion than any other writer. For obtaining a knowledge of the old logic his work is very satisfactory. On p. 99 he says, in speaking of the A, E, I, O propositions, "We have, therefore, the following table of propositions connecting any two terms S and P (the lettering is mine):

$A = AB$

$B = BA$

No A = B No B = A

AB = B BA = A

Ab = b

Ba = a

The translations are,

All A is some B,

All B is some A,

No A is B, No B is A,

Some A is B, Some B is A,

Some A is not-B,

Some B is not-A.

The pair of propositions, A is AB and B is BA, are independent, and the same is true of the pairs,

A not-B is not-B and B not-A is not-A,
 A is AB and B not-A is not-A,
 B is BA and A not-B is not-B."

Make an AB diagram:

A	a	
	2	B
	1	b

Fig. 287.

Now, if $A = AB$, then the combination Ab is inconsistent, and we eliminate it by making a figure 1 in that section.

Again, if $B = BA$, then the combination Ba is inconsistent and we eliminate it by making a figure 2 in that section.

The Reasoning Frame now shows that the two propositions neither concur nor conflict, that is, they are independent.

770. Make an AB diagram:

'A	a	
		B
	21	b

Fig. 288.

Now, if $Ab = b$, then the combination ab is inconsistent and we eliminate it by making a figure 1 in that section.

Again, if $Ba = a$, then the combination ab is inconsistent and we eliminate it by making a figure 2 in that section.

The Reasoning Frame now shows us that the propositions are equivalent, because they both remove the same combina-

tion. They are not independent in the sense that the propositions $A = AB$ and $B = BA$ are.

771. Make an AB diagram:

A	a	
		B
1	2	b

Fig. 289.

Now, if $A = AB$, then the combination Ab is inconsistent and we eliminate it by making a figure 1 in that section.

Again, if $Ba = a$, then the combination ab is inconsistent and we eliminate it by making a figure 2 in that section.

The Reasoning Frame now shows us that all the b's are eliminated. This means that the propositions are contradictory.

Make an AB diagram:

A	a	
		B
	1	b
	2	

Fig. 290.

Now, if $B = BA$, then the combination Ba is inconsistent, and we eliminate it by making a figure 1 in that section.

Again, if $Ba = a$, then the combination ba is inconsistent, and we eliminate it by making a figure 2 in that section.

The Reasoning Frame now shows that all the a's are eliminated. This proves that the propositions are contradictory.

772. Dr. Keynes says, that the first pair taken together may be called complementary propositions; the second pair, he says, are neither coextensive, nor either included within the other (?), and they may be called sub-complementary propositions.

The third pair may be called contra-complementary propositions. The fourth pair may also be called contra-complementary propositions.

773. Dr. Keynes says that obversion is a process of immediate inference, in which the inferred proposition, or obverse, whilst retaining the original subject, has for its predicate the contradictory of the predicate of the original proposition, or obvertend.

774. Make an AB diagram:

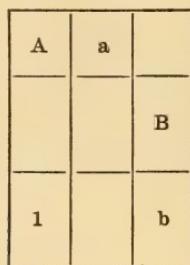


Fig. 291.

Now, if $A = AB$, then the combination Ab is inconsistent, and we eliminate it by making a figure 1 in that section.

We can read the eliminated combination,

No $A = b$

This is the obverse proposition.

775. Make an AB diagram:

A	a	
	1	B
		b

Fig. 292.

Now, if $AB = B$, then the combination Ba is inconsistent and we eliminate it by making a figure 1 in that section.

The Reasoning Frame now shows that the definition of A is,

$$A = B \mid b,$$

from which the old logic would infer,

$$\text{Some } A \text{ is } B,$$

which by the use of a double negative it would convert into,

$$\text{Some } A \text{ is not not-}B,$$

This is the obverse of $\text{No } A = B$.

$$\text{Some } A \text{ is } B.$$

776. Make an AB diagram:

A	a	
1		B
		b

Fig. 293.

Now, if $\text{No } A = B$, then the combination AB is inconsistent and we eliminate it by making a figure 1 in that section.

The Reasoning Frame now shows us that the definition of A is,

$$A = Ab.$$

777. Make an AB diagram:

A	a	
		B
	1	b

Fig. 294.

Now, if $Ab = b$, then the combination ab is inconsistent and we eliminate it by making a figure 1 in that section.

The Reasoning Fame now shows that the definition of A is,

$$A = B \mid b,$$

from which the old logic infers,

Some A is not-B.

This is the obverse of

Some A is not B.

778. Obversion is called Permutation by Prof. Fowler; Equipollence by Prof. Ueberweg; Infinitation by Prof. Bowen; Immediate Inference by Primitive Conception by Prof. Jevons; Contraversion by Prof. DeMorgan, and Contraposition by Prof. Spalding.

779. Dr. Keynes says, "Prof. Bain distinguishes between formal obversion and material obversion. By formal obversion is meant the kind of obversion discussed in the above section, and this is the only kind of obversion that can be properly recognized by the formal logician. Material obversion is described as the process of making 'obverse inferences which are justified only on an examination of the matter of the proposition, and the following are given as examples:

'Warmth is agreeable, therefore,

'Cold is disagreeable,

'War is productive of evil, therefore,

'Peace is productive of good,

'Knowledge is good, therefore,
'Ignorance is bad.'

It is very doubtful if these are legitimate inferences, formal or otherwise."

780. It seems to me improper to call material obversion a logical process. In a correct logical process it is impossible to get any term in the conclusion that is not given in the premises.

781. Dr. Keynes says that contraposition is a process of immediate inference, in which from a given proposition another proposition is inferred, having for its subject the contradictory of the original predicate.

Thus, given a proposition having A for its subject and B for its predicate, we seek to obtain a new proposition having not-B for its subject.

782. Every proposition which admits of contraposition will accordingly have two contrapositives, each of which is the obverse of the other, for example, in the case of,

$$\text{All } A = B$$

there will be two forms,

No not-B is A

All not-B is not-A.

783. Make an AB diagram:

A	a	
		B
1		b

Fig. 295.

Now, if $A = AB$, then the combination Ab is inconsistent and we eliminate it by making a figure 1 in that section.

We can now read the eliminated combination,

No not-B is A,

and we can read,

All not-B is not-A.

784. Dr. Keynes says, that so far as it is necessary to distinguish these forms, we may call that one in which A is the predicate, the contrapositive, and the one in which not-A is the predicate, the obverted contrapositive.

785. He gives this rule for obtaining the contrapositive: Obvert the original proposition and then convert the proposition thus obtained.

786. Make an AB diagram:

A	a	
1		B
		b

Fig. 296.

Now, if No A = B, then the combination AB is inconsistent and we eliminate it by making a figure 1 in that section.

The Reasoning Frame now shows us that we can get the following obverse proposition:

$$A = Ab$$

and the following contrapositive proposition,

$$Ab = A,$$

which means,

Some not-B is A.

787. Make an AB diagram:

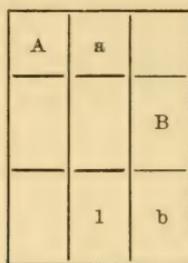


Fig. 297.

Now, if $Ab = b$, which means, Some A is all not-B, then the combination ab is inconsistent and we eliminate it by making a figure 1 in that section.

The Reasoning Frame now shows us that the definition of A is,

$$A = B \mid b,$$

from which the old logic infers the obverse proposition,

Some A is not-B.

We also get the following definition of not-B,

$$b = bA,$$

which means,

not-B is some A,

and this is the contrapositive of,

Some A is not-B.

788. Dr. Keynes quotes the following from DeMorgan, "Euclid may have been ignorant of the identity of 'Every X is Y and every not-Y is not-X,' for anything that appears in his writings he makes the one follow the other by a new proof each time."

789. Dr. Keynes says, "In most text books no definition of contraposition is given at all, and it may be pointed out that in the attempt to generalize from special examples, Jevons, in his 'Elementary Lessons in Logic,' involves himself in difficulties. For the contrapositive of A he gives, All not-A is not-B; O, he says, has no contrapositive (but only a converse

by negation, Some not-B is A); and for the contrapositive of E he gives, No B is A.

It is impossible to discover any definition of contraposition that can yield these results. Assuming that in contraposition the quality of the proposition is to remain unchanged, as in Jevons' contrapositive of A, then the contrapositive of both E and O is, "Some not-B is not not-A." (For S and P I have substituted A and B.)

790. Dr. Keynes says that Inversion is a process in which from a given proposition another proposition is inferred, having for its subject the contradictory of the original subject.

Given a proposition with A for subject and B for predicate, we obtain by inversion a new proposition with not-A for subject. The original proposition is called the invertend and the inferred proposition the inverse.

791. The predicate may be either B or not-B. The former is called the inverse and the latter the obverted inverse.

792. Make an AB diagram:

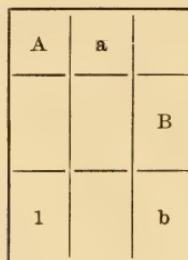


Fig. 298.

Now, if $A = AB$, then the combination Ab is inconsistent and we eliminate it by making a figure 1 in that section.

The Reasoning Frame now shows us that the definition of a is,

$$a = B \mid b,$$

from which the old logic infers,

Some not-A is not-B.

793. Make an AB diagram:

A	a	
1		B
		b

Fig. 299.

Now, if No A = B, then the combination AB is inconsistent and we eliminate it by making a figure 1 in that section.

The Reasoning Frame now shows us that the definition of not-A is,

$$a = B \mid b,$$

from which the old logic infers,

Some not-A is B.

794. This is the way in which Dr. Keynes proceeds with the proposition,

All A is B,

therefore (by obversion) No A is not-B,

therefore (by conversion) No not-B is A,

therefore (by obversion) All not-B is not-A,

therefore (by conversion) Some not-A is not-B,

therefore (by obversion) Some not A is not B.

Which last is the desired form.

795. Make an AB diagram:

A	a	
	1	B
		b

Fig. 300.

Now, if $AB = B$, which means, Some A is all B, then the combination aB is inconsistent and we eliminate it by making a figure 1 in that section.

The Reasoning Frame now shows us that the definition of not-A is,

$$a = ab,$$

which means,

All not-A is Some not-B.

and this is the inverse of the proposition,

Some A is B.

Now, if $Ab = b$, which means, Some A is all not B, then the combination ab , is inconsistent and we eliminate it by making a figure 1 in that section.

Make an AB diagram:

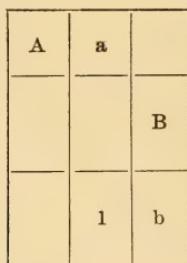


Fig. 301.

The Reasoning Frame now shows us that the definition of not-A is,

$$a = aB,$$

which means,

All not-A is some B,

and this is the inverse of the proposition,

Some A is not B.

796. Dr. Keynes says, "We may now inquire further in what cases it is possible to infer a proposition with not-A as subject," and he answers it, "The required proposition can be obtained only if the given proposition is universal."

797. Dr. Keynes says, "In passing from All A is B, to its inverse, Some not A is not B, there is an apparent illicit process, which is far from easy either to account for or explain away. For the term B, which is undistributed in the premise, is distributed in the conclusion, and yet, if the universal validity of obversion and conversion is granted, it is impossible to detect any flaw in the argument by which the conclusion is reached. On this ground Prof. Ray rejects the validity of the above inference. "If a term is not distributed in the premise, it cannot be distributed in the conclusion, that is, if a term is taken in the premise to mean at least one thing denoted by it, it cannot in the conclusion be taken to mean, all things denoted by it. The above conclusion is, therefore, inadmissible. It is obtained from the original premise by the processes of obversion and conversion, and the fallacy lies not in the process of conversion, but in that of obversion, which assumes that the term B has a contradictory and is therefore limited in its sphere, although in the original premise, its limitation is not implied and it may cover the whole sphere of thought and existence." ("Deductive Logic, p. 313.")

"Instead, however, of thus denying altogether the validity of the inference under consideration, it is better to investigate the conditions of its validity. There can be no question that it is valid under the existential assumption upon which we have been proceeding. By the aid of diagrams this can be shown directly and without the intervention of the processes of obversion and conversion. We must then deny that, (under our present assumption) any illicit process, whatever, is involved in the inference. In other words, admitting contradictory terms, and assuming that the original terms and their contradictories are all represented in the universe of discourse, it is not correct to say that a term not distributed in the premise of an immediate inference, may never be distributed in the conclusion. For, although a term (B), may be undistributed relatively, (A), it may nevertheless be distributed relatively to the contradictory of A. When we say All A is B, B is undistributed relatively to A, but implic-

itly it is at the same time, entirely excluded from some portion of not-A, and is, therefore, distributed relatively to that portion of not-A."

798. Make an AB diagram:

A	a	
		B
1		b

Fig. 302.

Let us take the proposition,

All A = B, which we can state thus:

$$A = AB.$$

Now, if $A = AB$, then the combination Ab is inconsistent and we eliminate it by making a figure 1 in that section.

Reading the eliminated combination we get,

$$\text{No } A = b.$$

This is the obverse.

We can also read,

$$B = A \mid a,$$

which the old logic would translate,

Some B is A.

This is the converse.

We can also read, according to the old logic,

Some B is not not-A,

This is the obverted converse.

We can also read the eliminated combination,

$$\text{No } b = A$$

This is the contrapositive.

We can also read the combination,

$$\text{All } b = a$$

This is the obverted contrapositive.

We can also read,

$$a = B \mid b$$

which the old logic would translate,

$$\text{Some } a = b$$

This is the inverse and the obverted inverse is the same.

799. The above is a list of the inferences which Dr. Keynes gives, but I do not think that he has exhausted the list according to the methods of the old logic, for instance, we can read according to the old logic,

Some B is not-A

Some not-B is no A

Some not-A is B,

and by the use of double negatives we can read, according to the old logic,

Some not-A is not not-B

Some A is no not-B

800. Make an AB diagram:

A	a	
1		B
		b

Fig. 303.

Let us take the proposition,

$$\text{No } A = B.$$

Now, if No $A = B$, then the combination AB is inconsistent and we eliminate it by making a figure 1 in that section.

We can read,

$$A = Ab,$$

which means,

$$\text{All } A \text{ is not-B}$$

This is the obverse.

We can also read,

$$\text{No } B = A,$$

This is the converse.

We can also read,

$$B = Ba,$$

which means,

$$\text{All } B \text{ is not-}A,$$

This is the obverted converse.

We can also read,

$$b = A \mid a,$$

which means, according to the old logic,

$$\text{Some not-}B \text{ is } A,$$

This is the contrapositive.

We can also read,

$$\text{Some not-}B \text{ is not not-}A,$$

This is the obverted contrapositive.

We can also read,

$$a = B \mid b,$$

which means, according to the old logic,

$$\text{Some not-}A \text{ is } B,$$

This is the inverse.

We can also read,

$$\text{Some not-}A \text{ is not-}B,$$

This is the obverted inverse.

801. The above are the inferences given by Dr. Keynes, but, according to the old logic, we can also read, it seems to me,

Some not-B is not A,

Some A is no B

Some not A is not-B,

Some B is no A.

802. Let us take the proposition,

Some dogs are all pugs.

Let A = dogs,

B = pugs.

We can state the proposition thus:

$$BA = B.$$

Make an AB diagram:

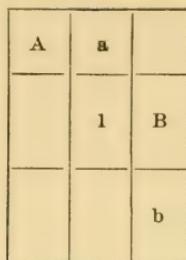


Fig. 304.

Now, if $B = BA$, then the combination Ba is inconsistent and we eliminate it by making a figure 1 in that section.

We can now make, according to the old logic, the following readings:

- (1) Some A is B, i. e., Some dogs are pugs,
- (2) Some A is not-B, i. e., Some dogs are not-pugs,
- (3) Some A is no not-B, i. e., Some dogs are no not-pugs,
- (4) No not-A is B, i. e., No not-dogs are pugs,
- (5) No B is not-A, i. e., No pugs are not-dogs,
- (6) All not-A is some not-B, i. e., All not-dogs are some not-pugs,
- (7) All B is some A, i. e., All pugs are some dogs,
- (8) Some not-B is some A, i. e., Some not-pugs are some dogs,
- (9) Some not-B is some not-A, i. e., Some not-pugs are some not-dogs,
- (10) Some a is no B, i. e., Some not-dogs are no pugs.
- (11) Some B is no a, i. e., Some pugs are no not-dogs.

803. Let us take the proposition,
Salt is chloride of sodium,

Let $A = \text{salt}$,

$B = \text{chloride of sodium}$.

The proposition can be stated thus:

- (1) $A = B$
- (2) $B = A$

Make an AB diagram:

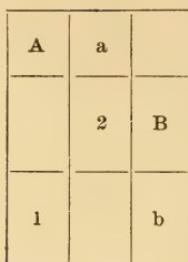


Fig. 305.

Now, if $A = B$, then the combination Ab , is inconsistent and we eliminate it by making a figure 1 in that section.

Again, if $B = A$, then the combination Ba is inconsistent and we eliminate it by making a figure 2 in that section.

We can make the following readings:

(1) All $A =$ all B ,

All salt is all chloride of sodium,

(2) All $B =$ All A ,

All chloride of sodium is all salt,

(3) All $a =$ all b ,

All not-salt is all not-chloride of sodium,

(4) All $b =$ all a ,

All not-chloride of sodium is all not-salt,

(5) All $A =$ no b ,

All salt is no not-chloride of sodium,

(6) All $B =$ no a ,

All chloride of sodium is no not-salt,

(7) All $b =$ no A ,

All not-chloride of sodium is no salt,

(8) All $a =$ no B ,

All not-salt is no chloride of sodium.

804. We will now give some examples taken from Archbishop Whately's "Elements of Reasoning," pp. 118, 119 and 120, to exhibit the application of conversion to the syllogism.

All wits are dreaded,

All wits are admired,

Some who are admired are dreaded.
We can state the propositions thus:

$$A = AB$$

$$A = AC$$

$$CB = CB$$

Make an ABC diagram:

AB	Ab	aB	ab	
	1			C
2	12			c

Fig. 306.

Now, if $A = AB$, then the combinations containing Ab are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $A = AC$, then the combinations containing Ac are inconsistent and we eliminate them by making a figure 2 in those sections.

The given propositions were reduced into Darii, by converting by limitation the minor premise.

All wits are dreaded,

Some who are admired are wits,

Some who are admired are dreaded,
which we can state,

$$A = AB$$

$$CA = A$$

$$CB = CB.$$

Make an ABC diagram:

AB	Ab	aB	ab	
	1			C
2	2 1			c

Fig. 307.

Now, if $A = AB$, then the combinations containing Ab are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $A = CA$, then the combinations containing cA are inconsistent and we eliminate them by making a figure 2 in those sections.

The appearance of the two Reasoning Frames proves the validity of the conversion.

The original example was in Darapti.

805. Let us take these examples in Camestres:

All true philosophers account virtue a good in itself,
The advocates of pleasure do not account, etc.,

Therefore they are not true philosophers,
which we can state,

$$A = AB$$

$$C = Cb$$

$$C = Ca$$

Make an ABC diagram:

AB	Ab	aB	ab	
2	1	2		C
	1			e

Fig. 308.

Now, if $A = AB$, then the combinations containing Ab are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $C = Cb$, then the combinations containing CB are inconsistent and we eliminate them by making a figure 2 in those sections.

This syllogism was reduced to Celerant by simply converting the minor, and then transposing the premises.

Those who account virtue a good in itself are not advocates of pleasure,

All true philosophers account virtue, etc, therefore,

No true philosophers are advocates of pleasure,
which we can state,

$$\begin{aligned} B &= Be \\ A &= AB \\ \text{No } A &= C \end{aligned}$$

Make an ABC diagram:

AB	Ab	aB	ab	
1	2	1		C
	2			c

Fig. 309.

Now, if $B = Bc$, then the combinations containing BC are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $A = AB$, then the combinations containing Ab are inconsistent and we eliminate them by making a figure 2 in those sections.

The appearance of the two Reasoning Frames proves the validity of the conversion.

806. Let us take this syllogism in Baroco:

Every true patriot is a friend to religion,

Some great statesmen are not friends to religion, therefore,

Some great statesmen are not true patriots.

We can state it thus:

$$A = AB$$

$$Cb = Cb.$$

• Make an ABC diagram:

AB	Ab	aB	ab	
	1			C
	1			c

Fig. 310.

Now, if $A = AB$, then the combinations containing Ab are inconsistent and we eliminate them by making a figure 1 in those sections.

This syllogism was converted into Ferio by converting the major premise by negation, i. e., contraposition.

He who is not a friend to religion is not a true patriot,

Some great statesmen are not friends to religion.

They can be stated thus:

$$b = ba$$

$$Cb = Cb.$$

Make an ABC diagram:

AB	Ab	aB	ab	
	1			C
	1			c

Fig. 311.

Now, if $b = ba$, then the combinations containing Ab are inconsistent and we eliminate them by making a figure 1 in those sections.

The appearance of the two Reasoning Frames proves the validity of the conversion.

807. Let us take this syllogism in Bocardo:

Some slaves are not discontented,

All slaves are wronged, therefore,

Some who are wronged are not discontented.

The premises may be stated thus:

$$Ab = Ab$$

$$A = AC$$

Make an ABC diagram:

AB	Ab	aB	ab	
				C
1	1			c

Fig. 312.

$Ab = Ab$ will not cause us to eliminate any combinations.

Now, if $A = AC$, then the combinations containing Ac are inconsistent and we eliminate them by making a figure 1 in those sections.

The major premise was converted by negation (contraposition) and the premises were transposed.

All slaves are wronged,

Some who are not discontented are slaves, therefore,

Some who are not discontented are wronged, therefore,

Some who are wronged are not discontented.

The premises may be stated thus:

$$A = AC$$

$$bA = bA$$

Therefore,

$$bC = bC,$$

Therefore,

$$Cb = Cb.$$

Make an ABC diagram:

AB	Ab	aB	ab	
				c
1	1			e

Fig. 313.

Now, if $A = AC$, then the combinations containing Ac are inconsistent and we eliminate them by making a figure 1 in those sections.

$bA = bA$ will not cause us to make any elimination.

The appearance of the two Reasoning Frames proves the validity of the conversion.

808. The old logic had another method called *Reductio ad impossible*. The following is an example:

All true patriots are friends to religion,

Some great statesmen are not friends to religion, there-

fore, Some great statesmen are not true patriots.

We can state the premises thus:

$$A = AB$$

$$Cb = Cb, \text{ therefore,}$$

$$Ca = Ca.$$

Make an ABC diagram:

AB	Ab	aB	ab	
	1			C
	1			c

Fig. 314.

Now, if $A = AB$, then the combinations containing Ab are inconsistent and we eliminate them by making a figure 1 in those sections.

If the conclusion $Ca = Ca$ be not true, then its contradictory must be true, viz.:

All great statesmen are true patriots.

This can be stated thus:

$$C = CA,$$

Substitute this for the minor premise and the syllogism will now stand thus:

All true patriots are friends to religion,

All great statesmen are true patriots, therefore,

All great statesmen are friends to religion.

They can be stated thus:

$$A = AB$$

$$C = CA,$$

Therefore,

$$C = CB.$$

Make an ABC diagram:

AB	Ab	aB	ab	
	1	2	2	C
	1			c

Fig. 315.

Now, if $A = AB$, then the combinations containing Ab are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $C = CA$, then the combinations containing Ca are inconsistent and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that $C = CB$.

All great statesmen are friends to religion.

But this conclusion is the contradictory of the original minor premise, therefore, it must be false, because the premises are always taken to be true. Therefore, one of the premises by which the conclusion has been correctly proved, must be false also, but, as the major premise is true, the falsity must be in the new minor premise, which is the contradictory of the original conclusion.

Therefore, the original conclusion must be true. This kind of reasoning was employed for Baroco and Bocardo in the old logic.

809. Miss Jones says in "Elements of Logic," p. 147, "With Inferentials (Hypotheticals), there seems to be only one kind of Eversion which corresponds pretty nearly to Contraversion e. g., from any hypothetical.

If A , then C ,

Another hypothetical,

If not C then not A ,

may be deduced.

If A then C means,

If $A = B$ then $C = D$.

If not C then not A , means,

If $C = d$ then $A = b$.

Make an ABCD diagram:

AB	Ab	aB	ab	
				CD
2 1		2	2	Cd
1				cD
1				cd

Fig. 316.

The propositions can be stated thus:

- (1) $AB = ABCD$
- (2) $Cd = CdAb$

Now, if $AB = CD$, then the combinations containing ABCd, ABc are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $Cd = Ab$, then the combinations containing ABCd, aCd, are inconsistent and we eliminate them by making a figure 2 in those sections.

The result shows that the proposition,

If not C then not A is consistent with the proposition, if A then C, but that it is not an inference from it.

810. Again Miss Jones says. "For instance, the conditional,

(1) If any flower is a Datura that flower is fragrant,
is equivalent to the categorical,

(2) Any flower that is Datura is fragrant.

Let A = flower,

B = Datura,

C = fragrant.

The premises can be stated thus:

- (1) $AB = ABC$
- (2) $AB = ABC$

So that Miss Jones' statement is correct.

811. Let us take this example:

- (1) If honesty is not the best policy, life is not worth living,
- (2) Life is not worth living or honesty is the best policy.

Let A = honesty,
 B = best policy,
 C = life,
 D = worth living.

The propositions can be stated thus:

- (1) $Ab = AbCd$
- (2) $C = Cd \mid A = AB$

Make an ABCD diagram:

AB	Ab	aB	ab	
2 1 3	2	2	CD	
3 2			Cd	
	3 1		cD	
	3 1		cd	

Fig. 317.

Now, if $Ab = Cd$, then the combinations containing $AbCD$, Abc , are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $C = Cd$, except where $A = AB$, then the combinations containing $ABCd$, $AbCD$, aCD , are inconsistent and we eliminate them by making a figure 2 in those sections.

Again, if $A = AB$, except where $C = Cd$, then the combinations containing $ABCd$, $AbCD$, Abc , are inconsistent and we eliminate them by making a figure 3 in those sections.

The Reasoning Frame now shows,

- (1) That the two propositions are not equivalent.
- (2) That they are consistent.
- (3) That we can infer (1) from (2).
- (4) We cannot infer (2) from (1).

CHAPTER XXVII.

THE ELIMINATION OF NEGATIVE TERMS.

812. Dr. Keynes says in "Formal Logic." p. 113, "The process of obversion enables us by the aid of negative terms, to reduce all propositions to the affirmative form * * *. It is of course clear that by means of obversion, we can get rid of a negative term occurring as the predicate of a proposition. The problem is more difficult when the negative term occurs as subject, but in this case, elimination may still be possible."

813. Dr. Keynes says, "We may even be able to get rid of two negative terms; for example,

All not A is some not B,

is equivalent to,

All B is some A.

The two propositions may be stated thus:

$$a = ab$$

$$B = BA$$

Make an AB diagram:

A	a	
		B
	1 2	
		b

Fig. 318.

Now, if $a = ab$, then the combination aB is inconsistent and we eliminate it by making a figure 1 in that section.

Again, if $B = BA$, then the combination Ba is inconsistent and we eliminate it by making a figure 2 in that section.

The result proves that the two propositions are equivalent,

because they both cause the elimination of the same combination.

814. Again Dr. Keynes says, p. 114, "No not-A is not-B is equivalent to the statement that Nothing is both not-A and not-B, and this becomes by obversion, Everything is either A or B."

The propositions may be stated thus:

- (1) No $a = b$
- (2) Everything is $A \neq B$.

Make an AB diagram:

A	a	
		B
	1	b

Fig. 319.

Now, if $No a = b$, then the combination ab is inconsistent and we eliminate it by making a figure 1 in that section.

Make another AB diagram:

A	a	
		B
1		
	1	b

Fig. 320.

Now, if Everything, (i. e., every combination) $= A \neq B$, then the combinations AB , ab , are inconsistent and we eliminate them by making a figure 1 in those sections. We assume that or is exclusive.

An examination of the two Reasoning Frames shows us that the two propositions are not equivalent.

815. Let us take this example,

Are the following propositions equivalents?

- (1) All A is some B
- (2) All not-B is some not-A
- (3) Nothing is A not-B
- (4) Everything is not-A | AB

The propositions may be stated thus:

- (1) $A = AB$
- (2) $b = ba$
- (3) $\text{No } A = b$
- (4) $\text{Everything} = a | AB.$

Make an AB diagram:

A	a	
		B
2 1		b
4 3		

Fig. 321.

Now, if $A = AB$, then the combination Ab, is inconsistent and we eliminate it by making a figure 1 in that section.

Again, if $b = ba$, then the combination Ab is inconsistent and we eliminate it by making a figure 2 in that section.

Again, if $\text{No } A = b$, then the combination Ab, is inconsistent and we eliminate it by making a figure 3 in that section.

Again, if Everything (i. e., every combination), $= a | AB$, then the combination Ab is inconsistent and we eliminate it by making a figure 4 in that section.

The result proves that the four propositions are equivalents.

816. Let us take this example,

Are the following propositions equivalents?

- (1) All not-A is some not-B
- (2) All B is some A
- (3) Nothing is not-A, B
- (4) Everything is A or not-A, not-B.

The propositions may be stated thus:

- (1) $a = ab$
- (2) $B = BA$
- (3) $\text{No } a = B$
- (4) $\text{Everything} = A \mid ab.$

Make an AB diagram:

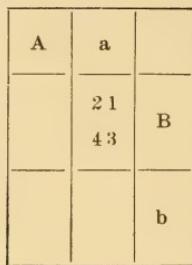


Fig. 322.

Now, if $a = ab$, then the combination aB is inconsistent and we eliminate it by making a figure 1 in that section.

Again, if $B = BA$, then the combination aB is inconsistent and we eliminate it by making a figure 2 in that section.

Again, if $\text{No } a = B$, then the combination aB is inconsistent and we eliminate it by making a figure 3 in that section.

Again, if Everything (i. e., every combination), $= A \mid ab$, then the combination aB is inconsistent and we eliminate it by making a figure 4 in that section.

The result now proves that the given propositions are equivalent.

817. Let us take this example,

Are the following propositions equivalents?

- (1) All A is some not-B
- (2) All B is some not-A
- (3) Nothing is AB
- (4) Everything is not-A or A not-B.

The propositions may be stated thus:

- (1) $A = Ab$
- (2) $B = Ba$
- (3) $\text{No } A = B$
- (4) $\text{Everything} = a \mid Ab.$

Make an AB diagram:

A	a	
2 1		
4 3		B
		b

Fig. 323.

Now, if $A = Ab$, then the combination AB is inconsistent and we eliminate it by making a figure 1 in that section.

Again, if $B = Ba$, then the combination AB is inconsistent and we eliminate it by making a figure 2 in that section.

Again, if $\text{No } A = B$, then the combination AB is inconsistent and we eliminate it by making a figure 3 in that section.

Again, if Everything (i. e., every combination), $= a \mid Ab$, then the combination AB is inconsistent and we eliminate it by making a figure 4 in that section.

The result proves that the propositions are equivalent.

818. Let us take this example,

- (1) All not-A is some B
- (2) All not-B is some A
- (3) Nothing is not-A, not-B.
- (4) Everything is A or not-A, B.

The propositions may be stated thus:

- (1) $a = aB$
- (2) $b = bA$
- (3) $\text{No } a = b$
- (4) $\text{Everything} = A \mid aB$

Make an AB diagram:

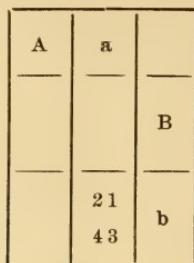


Fig. 324.

Now, if $a = aB$, then the combination ab is inconsistent, and we eliminate it by making a figure 1 in that section.

Again, if $b = bA$, then the combination ab is inconsistent, and we eliminate it by making a figure 2 in that section.

Again, if $\text{No } a = b$, then the combination ab is inconsistent, and we eliminate it by making a figure 3 in that section.

Again, if Everything (i. e. every combination) $= A | aB$, then the combination ab is inconsistent, and we eliminate it by making a figure 4 in that section.

The Reasoning Frame now shows that the given propositions are equivalent.

819. Let us take this example:

Are the following propositions equivalents:

- (1) Every A is B
- (2) Every not-B is not-A
- (3) Nothing (i. e., no combination) is both A and not-B
- (4) Everything (i. e., every combination) is either B or not-A, not-B.

The propositions can be stated thus:

- (1) $A = AB$
- (2) $b = ba$
- (3) $\text{No } A = b$
- (4) $\text{Everything} = B | ab$

Make an AB diagram:

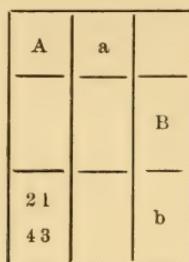


Fig. 325.

Now, if $A = AB$, then the combination Ab is inconsistent, and we eliminate it by making a figure 1 in that section.

Again, if $b = ba$, then the combination Ab is inconsistent, and we eliminate it by making a figure 2 in that section.

Again, if $\text{No } A = b$, then the combination Ab is inconsistent, and we eliminate it by making a figure 3 in that section.

Again, if $\text{Everything} = B | ab$, then the combination Ab is inconsistent, and we eliminate it by making a figure 4 in that section.

The Reasoning Frame now shows that the given propositions are equivalents.

820. This diagram illustrates the fundamental laws of thought.

The propositions (1) and (2) are opposites. If one is true the other is true. Each is a necessary inference from the other, and each is the equivalent of the other.

We might call them the two extremes.

If either one is true, then every proposition composed of half of one and half of the other would be false, thus:

- (1) Every A is not-B is false
- (2) Every not-B is A is false
- (3) Every not-A is B is false
- (4) Every B is not-A is false

Whenever we have any proposition given to us, if we will find its negative equivalent, then we can tell at once that any proposition which is composed partly of the given proposition and partly of the opposite is false.

821. Dr. Keynes relates that one of the old Greek logicians, Alexander of Aphrodisias, established the conversion of E by means of a syllogism in Ferio.

No A is B

therefore,

No B is A

for, if not, then by the law of contradiction

Some B is A

and we have this syllogism,

No A is B

Some B is A

therefore,

Some B is not B

A reductio ad absurdum.

The premises can be stated thus:

(1) No A = B

(2) BA = A

Make an AB diagram:

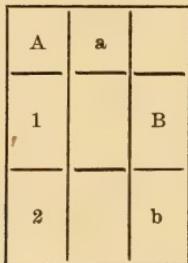


Fig. 326.

Now, if No A = B, then the combination AB, is inconsistent and we eliminate it by making a figure 1 in that section.

Again, if A = BA, then the combination Ab is inconsistent and we eliminate it by making a figure 2 in that section.

The Reasoning Frame now shows that all the A's are eliminated and that the premises are inconsistent. The note at the foot of the page says, "The conversion of A and the conversion of I may be established similarly."

822. Dr. Keynes says on page 121, of Formal Logic,
“The contraposition of A may also be established indirectly
by means of a syllogism in Darii.”

All A is B

Therefore,

No not-B is A

for, if not,

Some not-B is A

and we have the following syllogism,

(1) All A is B

(2) Some not-B is A

therefore,

(3) Some not-B is B

which is absurd.

Make an AB diagram:

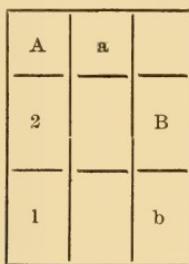


Fig. 327.

The premises can be stated:

(1) $A = AB$

(2) $bA = A$.

Now, if $A = AB$, then the combination Ab is inconsistent and we eliminate it by making a figure 1 in that section.

Again, if $A = bA$, then the combination AB is inconsistent and we eliminate it by making a figure 2 in that section.

The Reasoning Frame now shows us that all the A's are eliminated. This proves that the premises are inconsistent, i. e., absurd.

CHAPTER XXVIII.

LOGICAL EXISTENCE.

823. Some writers on the old logic have a good deal to say on the question whether the existence of A or of B is necessarily implied by their use in a proposition.

We may suppose, says Dr. Keynes:

- (1) That every proposition implies the existence of both subject and predicate and their contradictories.
- (2) That every proposition implies simply the existence of its subject.
- (3) That no proposition implies the existence of its subject or of its predicate.
- (4) That particulars imply the existence of their subjects and universals do not.

824. It is a fundamental proposition in this system, that subject and predicate are names for the same thing, consequently, both subject and predicate and their contradictories, imply the existence in thought of every object of which they are names. We believe that negative terms are names just the same as affirmative terms are.

All A is all B, means that the same thing has the names A and B. It also means that there are no things called A, which are also called a.

The proposition No A = B, means that the things which are called A, are also called b.

The proposition Some A = B, means that there are some things which are called A and B.

The proposition Some A = b, means that there are some things which have the names of A and b.

825. As to whether the terms used in logic imply the existence of the objects of which they are names, outside of thought, is a question with which logic has nothing to do. Logic is only

concerned with words and with the thoughts which the words represent, that is, with words and their meanings. The logician must understand the meanings of the terms which are used in the proposition given to him. His object, then, is to find out all the equivalent meanings of the given proposition.

826. I agree with Prof. Jevons, that in deductive logic there cannot be any question about existence. Words and their meanings alone, concern the logician. He has no interest in the things which they represent.

827. We have seen, in working out our examples, that whenever any letter-term, affirmative or negative, was totally eliminated, that we could draw no conclusions. This, it seems to me, tends to prove that the existence in the Universe of Discourse of every term used in the given proposition, is absolutely necessary to correct reasoning.

828. No predication, it seems to me, can be made about things which do not exist. Even contradictory propositions must be made about things which exist in thought.

829. In our system, the Universe of Discourse is represented by a square, which is divided into a certain number of sections, determined by the number of terms and their contradictories. Our Universe of Discourse is limited to the terms given us. Our propositions are limited to a given Universe of Discourse. In that Universe of Discourse they are either consistent or inconsistent. Outside of that Universe of Discourse we do not know what they are.

Under other circumstances, that is, in another Universe of Discourse, propositions which were once consistent, may now be inconsistent, not because of any change in existence, but because of a change in names.

Suppose I say, there are no such things as unicorns. Whether unicorns have an actual material existence, is not a logical question; it is a question of fact which science must answer.

830. In my judgment, logic has no more to do with actual existences in solving its problems, than arithmetic has. If a

boy were asked: How much nine unicorns would come to at nine dollars apiece, how utterly absurd it would be for him to insist on knowing before he proceeded to solve the problem whether there were any real, material unicorns.

Take this example: "No person condemned for witchcraft, in the reign of Queen Anne, was executed." This means: Every person condemned for witchcraft, in the reign of Queen Anne, was not executed, and any person who was executed in the reign of Queen Anne, was not condemned for witchcraft.

Both of these propositions imply the existence in thought of persons who have been condemned for witchcraft and of persons who have been executed. Neither of them is a proposition about a non-existing subject.

831. Dr. Keyne's position seems to me to be that particulars imply the actual existence of their subjects, and universals do not. His tables of the equivalences of propositions, seem to be based on that theory.

832. Miss Jones makes a very good point when she says: "A further point is, that unless the very positing of a term signifies the existence of something named by the term, we could never say, S is P, since the mere symbol S is certainly not the symbol P."

CHAPTER XXIX.

NUMERICAL REASONING.

833. I have already said that I considered numerical reasoning as distinct from logical reasoning. The time may come, however, when they will be unified by the discovery of a system which will enable us to state logical propositions in numbers, so that we can proceed to solve logical problems in the same way in which we now solve arithmetical problems, or, by the discovery of a logical system which will enable us to state arithmetical problems in logical symbols and solve arithmetical problems by means of Reasoning Frames, or some other logical invention, but at present I consider it useless to try to solve numerical problems by the processes of Formal Logic. Logic is the explanation of the meanings of names; arithmetic deals with the properties of numbers; numbers are not merely names, though a number may be used for a name, and when so used logic can interpret its meanings.

834. We may, however, consider some of the problems in numerical logic which have engaged the attention of other writers on logic, for the purpose of ascertaining what success they have met with. On page 333, Formal Logic, Prof. Keynes gives this example of numerically definite reasoning:

"If 70 per cent of M are P, and 60 per cent are S, then at least 30 per cent are both S and P."

The argument may be put as follows: On the average, of 100 M's, 70 are P and 60 are S. Suppose that the 30 M's which are not P are S, still, 30 S's are to be found in the remaining 70 M's which are P's; and this is the desired conclusion."

Of course this reasoning is correct but I am unable to demonstrate its validity by the Reasoning Frame.

835. Dr. Keynes on p. 332, gives the following example of valid reasoning:

Most M is P

Most M is S

Therefore, Some S is P.

Let A = Most M

B = P

C = S.

The propositions can be stated thus:

A = AB

A = AC

Make an ABC diagram:

AB	Ab	aB	ab	
	1			C
2	12			c

Fig. 328.

Now, if A = AB, then the combinations containing Ab are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if A = AC, then the combinations containing Ac are inconsistent and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that the definition of C is:

C = AB + aB + ab,

which the old logic translates, some C is B, i. e., Some S is P.

836. Dr. Keynes gives the following examples on p. 366, of Formal Logic:

All M's are P's,

At least n S's are M's,

Therefore,

At least n S's are P's,

Let B = M

C = P

A = at least n S's,

The propositions can be stated thus:

$$B = BC,$$

$$A = AB$$

Make an ABC diagram:

AB	Ab	aB	ab	
	2			C
1	2	1		e

Fig. 329.

Now, if $B = BC$, then the combinations containing Bc are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $A = AB$, then the combinations containing Ab are inconsistent and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that $A = ABC$, which we can translate:

At least n S's are P's.

837. "All P's are M's,

Less than n S's are M's

Therefore,

Less than n S's are P's."

Let $C = P$

$B = M$

$A = \text{less than } n \text{ S's}$,

The premises may be stated thus:

$$C = CB$$

$$A = AB.$$

Make an ABC diagram:

AB	Ab	aB	ab	
	1 2		1	C
	2			c

Fig. 330.

Now if $C = CB$, then the combinations containing Cb are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $A = AB$, then the combinations containing Ab are inconsistent and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that the definition of A is:

$$A = ABC \mid ABC,$$

which the old logic would translate,

Some less than n S's are P's.

838. Dr. Keynes gives several other examples of numerical syllogisms, but they all involve more than three terms.

Let us state the second one for an example:

Less than n M's are P's,

All S's are M's,

Therefore,

Less than n S's are P's.

Let B = less than n M's

C = P

A = S

D = M

E = less than n S's.

The premises can be stated thus:

$$B = BC$$

$$A = AD$$

Therefore,

$$E = EC.$$

Make an ABCDE diagram:

AB	Ab	aB	ab	
				CDE
				CDe
2	2			CdE
2	2			Cde
1		1		cDE
1		1		cDe
2 1	2	1		cdE
2 1	2	1		cde

Fig. 331.

Now, if $B = BC$, then the combinations containing Bc are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $A = AD$, then the combinations containing Ad are inconsistent and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that the definition of E is:

$$E = EC \mid Ec,$$

which the old logic would translate,

Some less than n S's are P's.

CHAPTER XXX.

COMPLEX PROPOSITIONS.

839. A complex proposition has a complex term in its subject or in its predicate or in both.

The following are examples:

$$A = B \mid C = D$$

$$A \mid B = C \mid D$$

$$\text{Some } AB = \text{All } CD$$

$$\text{All } AB = \text{Some } CD$$

$$\text{All } AB = \text{All } CD.$$

840. Dr. Keynes says, on page 392, Formal Logic (the lettering is mine): "Thus, taking A and C as symbols representing propositions, and a and c as their contradictories, the hypothetical proposition, If A then C, expresses an alternative between a and C and is therefore equivalent to the alternative proposition a or C.

The propositions may be stated thus:

$$(1) AB = ABCD$$

$$(2) A = b \mid C = D.$$

Make an ABCD diagram:

AB	Ab	aB	ab	
	2			CD
1 2				Cd
1 2				cD
1 2				cd

Fig. 332.

Now, if $AB = ABCD$, then the combinations containing ABCd, ABC, are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $A = b$, except where $C = D$, then the combinations containing $AbCD$, $ABCd$, ABC , are inconsistent and we eliminate them by making a figure 2 in those sections.

I think that the proposition, $A = b \mid C = D$, can be read and worked backwards, but on the supposition that such is not the case, the result shows that the propositions are not equivalent. But (1) is an inference from (2).

841. Let us take this example:

(1) Some B and $C =$ all A , or all B is either C or both D and E

(2) No A is both B and C and no B is either C or DE .

Are these propositions contradictions?

The propositions can be stated thus:

(1) $ABC = A, \mid B = C \mid DE$

(2) No $A = BC$ and no $B = C \mid DE$.

Make an ABCDE diagram:

AB	Ab	aB	ab	
2		1		CDE
2 1 3		3		CDe
2 1 3		3		CdE
2 1 3		3		Cde
3		3		cDE
1		1		cDe
1		1		cdE
1		1		cde

Fig. 333.

Now, if $ABC = A, \mid B = C \mid DE$, then the combinations containing $ABCDe$, $ABCd$, $ABCDe$, $AB \sqcap aBeDe$, $aBCDE$, $aBcd$, are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $No\ A = BC$, then the combinations containing ABC are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if $No\ B = C \mid DE$, then the combinations containing ABCDe, ABCd, ABcDE, aBCDe, aBCdE, aBcDE, are inconsistent and we eliminate them by making a figure 3 in those sections.

The Reasoning Frame now shows that the given propositions are contradictories, because all the B's are eliminated.

It is very easy to get the contradictory of a complex proposition by framing an alternative proposition which has "every combination" for its subject and the different eliminated combinations stated in the alternative for its predicate, or, by framing an alternative proposition with "no combination" for its subject and the different uneliminated combinations stated as alternants, for its predicate.

842. Let us take this proposition:

All $AB = AC \mid DE$,

and ascertain whether it is equivalent to,

All $AB = C \mid DE$.

Now, if $AB = AC \mid DE$, then the combination ABCDE is inconsistent because it implies $AB = AC$ and DE , and we therefore eliminate it by making a figure 1 in that section.

Again, if $AB = AC \mid DE$, then the following combinations are inconsistent:

$ABcDe$,

$ABedE$,

$ABcde$,

and we therefore eliminate them by making a figure 1 in those sections.

Make an ABCDE diagram:

AB	Ab	aB	ab	
1 2				CDE
				cDE
1 2				cDe
1 2				cdE
1 2				cde

Fig. 334.

Now, if $AB = C | DE$, then the combination ABCDE is inconsistent because it implies that $AB = C$ and DE , and we therefore eliminate it by making a figure 2 in that section.

Again, if $AB = C | DE$, then the following combinations are inconsistent:

ABcDe,
ABcdE,
ABcde,

and we therefore eliminate them by making a figure 2 in those sections.

The appearance of the Reasoning Frame shows that the two propositions are equivalent. This example is taken from Dr. Keynes' "Formal Logic," p. 396.

843. Let us take the three following propositions and ascertain whether they are equivalent to each other:

- (1) All $AB = a | C$
- (2) All $B = a | C$
- (3) $AB = C$.

Make an ABC diagram:

AB	Ab	aB	ab	
				C
1 2				c

Fig. 335.

Now, if $AB = a \mid C$, then the combination ABC is inconsistent and we eliminate it by making a figure 1 in that section.

Again, if $AB = C$, then the combination ABC is inconsistent and we eliminate it by making a figure 2 in that section.

The result shows that (1) and (3) are equivalent.

Make another ABC diagram:

AB	Ab	aB	ab	
				C
1				c

Fig. 336.

Now, if $B = a \mid C$, then the combination ABC , is inconsistent because it contains neither a nor C , and we therefore eliminate it by making a figure 1 in that section.

Again, if $B = a \mid C$, then the combination aBC is inconsistent because it means $B = a$, and C , and we therefore eliminate it by making a figure 2 in that section.

The result shows that (2) is not equivalent to either (1) or (3). (1) and (3) can be inferred from (2).

844. Let us take this example:

'Are the following propositions equivalent?

$$(1) A \mid B = C$$

$$(2) A = AC \text{ and } B = BC.$$

Make an ABC diagram:

AB	Ab	aB	ab	
				C
2 3	1 2	1 3		c

Fig. 337.

Now, if $A | B = C$, then the combinations Abc , aBc , are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $A = AC$, then the combinations containing Ac are inconsistent and we eliminate them by making a figure 2 in those sections.

Again, if $B = BC$, then the combinations containing Bc are inconsistent and we eliminate them by making a figure 3 in those sections.

The Reasoning Frame now shows that the given propositions are not equivalents.

845. Let us take this example:

Are the following propositions equivalents?

- (1) $A = B | C$
- (2) $A = B | A = C$.

Make an ABC diagram:

AB	Ab	aB	ab	
2 1 3				C
	2 1 3			c

Fig. 338

Now, if $A = B \mid C$, then the combinations ABC, Abc, are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $A = B$, except where $A = C$, then the combinations ABC, Abc, are inconsistent and we eliminate them by making a figure 2 in those sections.

Again if $A = C$, except where $A = B$, then the combinations ABC, Abc, are inconsistent and we eliminate them by making a figure 3 in those sections.

The Reasoning Frame now shows that the given propositions are equivalent.

(Keynes, "Formal Logic," p. 398.)

846. Let us take this example:

Are the following propositions equivalent?

- (1) No $A = C \mid$ No $B = C$
- (2) No $AB = C$.

Make an ABC diagram,

AB	Ab	aB	ab	
3	1	2		C
				c

Fig. 339.

Now, if No $A = C$, except where No $B = C$, then the combination AbC is inconsistent and we eliminate it by making a figure 1 in that section.

Again, if No $B = C$, except where No $A = C$, then the combination aBC is inconsistent and we eliminate it by making a figure 2 in that section.

Again, if No $AB = C$, then the combination ABC, is inconsistent and we eliminate it by making a figure 3 in that section.

The Reasoning Frame now shows that the given propositions are not equivalents.

847. Let us take this example:

Given the proposition $A \mid B = CD$, can we infer that all $A = C$?

The propositions can be stated thus:

$$(1) Ab \mid aB = CD$$

$$(2) A = AC.$$

Make an ABCD diagram:

AB	Ab	aB	ab	
				CD
	1	1		Cd
	1	1		cD
	1	1		cd

Fig. 340.

Now, if $A \mid B = CD$, then the combinations containing $AbCd$, Abc , $aBCd$, aBc , are inconsistent and we eliminate them by making a figure 1 in those sections.

From the uneliminated combinations we can get this definition of A:

$$A = AB \mid AbC.$$

This shows that we cannot infer that $A = C$.

848. Let us take this example:

From the proposition,

$$(1) CD = ABCD$$

can we infer the proposition,

$$(2) C = AC?$$

Make an ABCD diagram:

AB	Ab	aB	ab	
	1	1 2	1 2	CD
		2	2	Cd
				cD
				cd

Fig. 341.

Now, if $CD = ABCD$, then the combinations containing 'AbCD, aCD, are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $C = AC$, then the combinations containing aC are inconsistent and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that:

- (1) The given propositions are not equivalents.
- (2) They are consistent.
- (3) Neither can be inferred from the other.

849. Let us take the following example:

Is the proposition,

No A = C, equivalent to the following proposition?

$$\text{No Ab} \mid \text{aB} = \text{Cd} \mid \text{cD}.$$

Make an ABCD diagram:

AB	Ab	aB	ab	
1	1			CD
1	1 2	2		Cd
	2	2		cD
				cd

Fig. 342.

Now, if No A = C, then the combinations containing AC are

inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $No\ Ab\mid aB = Cd\mid cD$, then the combinations containing $AbCd$, $AbcD$, $aBCd$, $aBcD$, are inconsistent and we eliminate them by making a figure 2 in those sections.

The result now shows that the given propositions are not equivalent.

850. Let us take this example:

Given the proposition,

$$(1) \text{ All } A = BC \mid DE,$$

is it equivalent to the proposition,

$$(2) \text{ No } A = bd \mid be \mid cd \mid ce?$$

Make an ABCDE diagram:

AB	Ab	aB	ab	
1				CDE
	21			CDe
	21			CdE
	1			Cde
				cDE
21	21			cDe
21	1			cdE
1	1			cde

Fig. 343.

Now, if $A = BC \mid DE$, then the combinations containing $ABCDE$, $ABCDe$, $ABcDe$, $AbCDe$, $AbCd$, $AbcDe$, $Abcd$, are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $No\ A = bd \mid be \mid cd \mid ce$, then the combinations containing $AbCDe$, $AbCdE$, $AbcDe$, $ABcDe$, $ABcdE$ are inconsistent and we eliminate them by making a figure 2 in those sections.

The result proves that the given propositions are not equivalent, but (2) can be inferred from (1).

851. Let us take this example:

Given the following proposition,

(1) AB is not either C or D,
can we infer that,

(2) A = Ac?

Make an ABCD diagram:

AB	Ab	aB	ab	
2	2			CD
1 2	2			Cd
1				cD
				cd

Fig. 344.

Now, if $AB =$ not either $C \mid D$, then the combinations containing $ABCd$, $ABcD$, are inconsistent and we eliminate them by making a figure 1 in those sections. I assume that or is exclusive.

Again, if $A = Ac$, then the combinations containing AC are inconsistent and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that the given propositions are inconsistent, because the combination $ABCD$ is eliminated, and this combination is necessary to the expression of the given proposition, AB is not either C or D .

852. Dr. Keynes gives on p. 400, "Formal Logic," the following examples:

- (1) All A is CD, therefore, All AB is C,
- (2) No A is C, therefore, No AB is CD.
- (3) Given All A is C, then, All AB is C by rule (1) above;
and from this we obtain All AB is BC by rule (2) of
section 338.

Make an ABCD diagram:

AB	Ab	aB	ab	
—	—	—	—	CD
1	1	—	—	Cd
1	1	—	—	cD
1	1	—	—	cd

Fig. 345.

Now, if All A = CD, then the following combinations are inconsistent, ABCd, ABcD, ABcd, AbCd, AbcD, Abcd, and we eliminate them by making a figure 1 in those sections.

The Reasoning Frame now shows that All AB = C, therefore, (1) is correct.

853. Make another ABCD diagram:

AB	Ab	aB	ab	
1	1	—	—	CD
1	1	—	—	Cd
—	—	—	—	cD
—	—	—	—	cd

Fig. 346.

Now, if No A = C, then the following combinations are inconsistent, ABCD, ABCd, AbCD, AbCd, and we eliminate them by making a figure 1 in those sections.

The Reasoning Frame now shows that No AB = CD, therefore, (2) is correct.

854. The proposition, Given All A is C, may be stated thus:

$$A = AC.$$

Now, if A = AC, then the combinations containing Ac are

inconsistent and we eliminate them by making a figure 1 in those sections.

We can now read in the Reasoning Frame, All $AB = C$, and All $AB = BC$, therefore, (3) is correct.

Make an ABC diagram and eliminate as above directed.

AB	Ab	aB	ab	
				C
1	1			c

Fig. 347.

855. Dr. Keynes also gives the following examples on p. 401, of Formal Logic:

- (1) Given No A is C, then No AB is C, and, therefore, by rule (5) of section 338, No AB is b or C.
- (2) Given No A is C, then No A is BC, and, therefore, by rule (6) of section 338, No A or b is BC.

Make an ABC diagram:

AB	Ab	aB	ab	
1	1			C
				c

Fig. 348.

Now, if No A = C, then the combinations containing AC are inconsistent and we eliminate them by making a figure 1 in those sections.

We can now read in the Reasoning Frame,

No $AB = C$.

The term b or C, means b without C or C without b. It can be expressed thus:

$$bc \mid BC$$

Now, if No AB = bc | BC, then the combination ABC is inconsistent and we eliminate it by making a figure 1 in that section.

Make an ABC diagram:

AB	Ab	aB	ab	
1				C
				c

Fig. 349.

We can now read in the Reasoning Frame the proposition, Given No A = C, then, No AB = b | C.

Make an ABC diagram:

AB	Ab	aB	ab	
1	1			C
				c

Fig. 350.

Now, if No A = C, then the combinations containing AC are inconsistent and we eliminate them by making a figure 1 in those sections.

The term A or b means A without b or b without A, and can be expressed thus:

$$AB \mid ab.$$

We can now read in the Reasoning Frame,

No A = BC, and also No AB | ab = BC. (1) and (2) are correct.

856. In solving problems, I use stiff pieces of cardboard on which are drawn the different diagrams, and two kinds of counters to place in the sections. One kind signifies, when placed in a section, that the section is eliminated. The other kind, when placed in a section, signifies that the section is saved.

When I have a disjunctive proposition with a number of alternants, for example, A is B or C or D, I put a counter which signifies that a section is saved in every combination which contains A, and also contains B or C or D, or a combination of them.

In the remaining A sections I put a counter which signifies that the combination in that section is eliminated.

Next, I proceed to eliminate the combinations containing A, which also contain B and C, and I put in the sections containing those combinations, counters which signify that the combinations are eliminated.

Next, in a combination containing A, which also contains B and D, I take up the saving counter and put in its place an eliminating counter.

Next, in a section containing A, which also contains C and D, I take up the saving counter and put in its place an eliminating counter.

My Reasoning Frame now shows me all of the A combinations which are eliminated and all which are saved.

The two kinds together make all of the A combinations.

This method of getting the logical expression of a proposition is very simple and easy.

857. Let us take this example:

Can we infer from the proposition,

- (1) $BC = ABC$, that,
- (2) $AC = ABC$?

Make an ABC diagram:

AB	Ab	aB	ab	
2		1		C
				c

Fig. 351.

Now, if $BC = ABC$, then the combination aBC , is inconsistent and we eliminate it by making a figure 1 in that section.

Again, if $AC = ABC$, then the combination AbC , is inconsistent and we eliminate it by making a figure 2 in that section.

The Reasoning Frame now shows:

- (1) The given propositions are not equivalents,
- (2) They are consistent.
- (3) Neither can be inferred from the other.

858. Let us take this example:

Are the following propositions equivalents?

- (1) $ABC = BC$
- (2) $ABC = AC$.

Make an ABC diagram:

AB	Ab	aB	ab	
2		1		C
				c

Fig. 352.

Now, if $BC = ABC$, then the combination aBC is inconsistent and we eliminate it by making a figure 1 in that section.

Again, if $AC = ABC$, then the combination AbC , is inconsistent and we eliminate it by making a figure 2 in that section.

The result proves that the two propositions are not equivalent.

859. Let us take this example:

What propositions are consistent with the following proposition?

$$BC \mid BD = A.$$

Make an ABCD diagram:

AB	Ab	aB	ab	
				CD
		1		Cd
		1		cd

Fig. 353

Now, if $BC \mid BD = A$, then the combinations $aBCd$, $aBcd$, are inconsistent and we eliminate them by making a figure 1 in those sections.

We can now read:

- (1) $aBC = aBCD$
- (2) $aBc = aBed$
- (3) $Cd = CdA \mid Cdab$
- (4) $cD = cDA \mid cDab$
- (5) No $a = BCd$
- (6) No $a = Bcd$
- (7) No $B = aCd$
- (8) No $B = acD$
- (9) No $C = aBd$
- (10) No $c = aBD$
- (11) No $D = aBe$
- (12) No $d = aBC$
- (13) $aB = CD \mid cd$
- (14) $C \mid D = A \mid ab.$

Many other readings could be given but they would be simply trifling variations of those already given.

860. Let us take the following example:

From the proposition,

$$(1) A = ABC$$

can we infer the proposition,

$$(2) a = b \mid c?$$

Make an ABC diagram:

AB	Ab	aB	ab	
	1	2		C
1	1		2	c

Fig. 354.

Now, if $A = ABC$, then the combinations containing ABC , Ab , are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $a = b \mid c$, then the combinations containing aBC , abc , are inconsistent, and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows:

- (1) The given propositions are not equivalents
- (2) They are consistent
- (3) Neither can be inferred from the other

861. Let us take this example:

Are the following propositions equivalents?

$$(1) \text{No } A = B \mid C$$

$$(2) A = Abc$$

Make an ABC diagram:

AB	Ab	aB	ab	
2	1 2			C
2 1				c

Fig. 355.

Now, if $\text{No } A = B \mid C$, then the combinations ABC , AbC , are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $A = Abc$, then the combinations containing AB , AbC , are inconsistent, and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that the given propositions are not equivalents.

862. Let us take the following example:

From the proposition,

$$(1) AB = ABC$$

can we infer the proposition,

$$(2) A = b \mid C?$$

Make an ABC diagram:

AB	Ab	aB	ab	
				C
2				c

Fig. 356.

Now, if $AB = ABC$, then the combination ABC is inconsistent, and we eliminate it by making a figure 1 in that section.

Again, if $A = b \mid C$, then the combinations containing AbC ,

A_Bc , are inconsistent, and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows:

- (1) The given propositions are not equivalents
- (2) They are consistent
- (3) (1) can be inferred from (2)
- (4) (2) cannot be inferred from (1)

863. Let us take the following example:

From the proposition,

$$(1) A = B \mid C$$

can we infer the proposition,

$$(2) Ab = AbC?$$

Make an ABC diagram:

AB	Ab	aB	ab	
1				C
	21			c

Fig. 357.

Now, if $A = B \mid C$, then the combinations ABC , Abc , are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $Ab = AbC$, then the combination Abc is inconsistent, and we eliminate it by making a figure 2 in that section.

The Reasoning Frame now shows:

- (1) The given propositions are not equivalents
 - (2) They are consistent
 - (3) (2) can be inferred from (1)
 - (4) (1) cannot be inferred from (2)
- (Keynes' "Formal Logic," p. 407).

864. Let us take the following example:

Are the following propositions equivalents?

- (1) $AB = CD \mid de$
- (2) $cD \mid dE = a \mid b$

Make an ABCDE diagram:

AB	Ab	aB	ab	
				CDE
				CDe
1 2			2	CdE
				Cde
1 2			2	cDE
1 2			2	cDe
1 2			2	cdE
				cde

Fig. 358.

Now, if $AB = CD \mid de$, then the combinations containing ABCdE, ABcD, ABcdE, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if cD or $dE = a$ or b , then the combinations containing ABCdE, ABcD, ABcdE, abCdE, abcD, abcdE, are inconsistent, and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that the given propositions are not equivalents.

We can infer (1) from (2), but not conversely.

865. Let us take the following example:

Are the following propositions equivalents?

- (1) No $AB = CD \mid EF$
- (2) No $A = BCD \mid BEF$

Make an ABCDEF diagram:

ABC								
1								DEF
1								DEf
1								DeF
1								Def
1	1							dEF
								dEf
								deF
								def

Fig. 359.

Now, if $\text{No } AB = CD \mid EF$, then the combinations containing ABCDEF, ABCDe, ABCdEF, ABcEF, are inconsistent, and we eliminate them by making a figure 1 in those sections.

The Reasoning Frame now shows that,

$\text{No } A = BCD \mid BEF$,

and the given propositions are equivalents.

(Keynes' "Formal Logic," p. 409.)

866. Dr. Keynes gives the following example on p. 409 of "Formal Logic":

- (1) No AB is CD or EF, therefore,
 No A is BCD or BEF
 No C is ABD or ABEF
 No BD is AC or AEF

The proposition No AB is CD or EF, can be stated thus:

$\text{No } AB = CD \mid EF$

Make an ABCDEF diagram:

ABC								
	1							DEF
1								DEF
1								DeF
1								Def
1	1							dEF
								dEf
								deF
								def

Fig. 360.

Now, if $\text{No } AB = CD \mid EF$, then the following combinations ABCDEF, ABCDef, ABCDef, ABCdEF, ABcDEF, ABcdEF, are inconsistent, and we eliminate them by making a figure 1 in those sections.

The Reasoning Frame now shows the logical expression of $\text{No } AB = CD \mid EF$.

We can also read in it, $\text{No } A = BCD \mid BEF$, and $\text{No } C = ABD \mid ABEF$, and $\text{No } BD = AC \mid AEF$.

867. Let us take the following example:

Can we infer the proposition,

All F = ABDE

from the propositions,

(1) F = AB | bce,

(2) F = aBC | DE?

Make an ABCDEF diagram:

ABC								
		1 2	2 1	1	1 2	2 1	2 1	DEF
								DEF
2	2	1 2	2	1	1 2	2 1	2	DeF
								Def
2	2	1 2	1 2	1	1 2	2 1	2 1	dEF
								dEf
2	2	1 2	2	1	1 2	2 1	2	deF
								def

Fig. 361.

Now, if $F = AB \mid bce$, then the combinations containing $AbCF$, $AbcEF$, aBF , $abCF$, $abcEF$, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $F = ABC \mid DE$, then the combinations containing abF , $aBcF$, AbF , $ABCDef$, $ABCdF$, $ABcDeF$, $ABcdF$, are inconsistent, and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that,

$$\text{All } F = ABDE$$

868. Let us take this example:

Can the proposition,

$$\text{No } A = bc \mid Cd$$

be inferred from the propositions,

$$(1) \text{ No } A = bc$$

$$(2) \text{ No } A = Cd?$$

Make an ABCD diagram:

AB	Ab	aB	ab	
				CD
2	2			Cd
	1			cD
	1			cd

Fig. 362.

Now, if $\text{No } A = bc$, then the combinations containing $A\bar{b}c$ are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $\text{No } A = Cd$, then the combinations containing $A\bar{C}d$ are inconsistent and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that $\text{No } A = bc \mid Cd$, i. e., $bcD \mid BCd$.

869. Let us take the following example:

Can we infer the proposition,

$$D = A \mid Bc$$

from the propositions,

- (1) $D = A \mid B$
- (2) $\text{No } D = aC?$

Make an ABCD diagram:

AB	Ab	aB	ab	
1		3 2	3 2 1	CD
				Cd
1 3			3 1	cD
				cd

Fig. 363.

Now, if $D = A \mid B$, then the combinations containing ABD, abD, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $No D = aC$, then the combinations containing aCD are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if $D = A \mid Bc$, then the combinations containing ABCD, aCD, abcD, are inconsistent, and we eliminate them by making a figure 3 in those sections.

Now, as the given inference can be read in the Reasoning Frame, and it does not eliminate any combinations which the given propositions did not eliminate, this proves that the given inference can be inferred from the given propositions.

870. Propositions can be divided into two groups, viz.:

- (1) Those which are consistent,
- (2) Those which are inconsistent.

871. Consistent propositions may be again divided into,

- (1) Those which are consistent only,
- (2) Those which stand in the relation of inferend and inference,
- (3) Those which are equivalent.

872. Inconsistent propositions may be divided into three groups, viz.:

- (1) Those which are inconsistent only,
- (2) Those which are contradictions,
- (3) Those which are perfect contradictions.

873. An Inferend proposition is a proposition from which another can be inferred, which is called the Inference.

874. To ascertain whether two propositions are consistent, we get the visible expression of both in the proper Reasoning Frame.

875. If both can now be read in the Reasoning Frame, then they are consistent.

876. If both eliminate exactly the same combinations, then they are equivalents.

877. If one of them eliminates more combinations than the other, and if the one which eliminates the fewer combinations

does not eliminate any combinations excepting those which the other eliminated, and both can be read in the Reasoning Frame, then these two propositions stand to each other in the relation of inferend and inference.

The one which eliminated the greater number of combinations is the Inferend.

The one which eliminated the lesser number is the Inference.

878. If two given propositions eliminate entirely different combinations, and can both be read in the Reasoning Frame, then they are merely consistent, but one is not an inference from the other.

879. To ascertain whether two propositions are inconsistent, we get the visible expression of both in the Reasoning Frame.

880. If both cannot now be read in the Reasoning Frame, then they are inconsistent.

881. If a letter-term has been eliminated, then they are contradictories.

882. If one eliminates the combinations which the other saved and saves the combinations which the other eliminated, then they are perfect contradictories.

883. Let us take the following example:

Can we infer the proposition,

No A = BD | BE | CD | CE

from the propositions,

- (1) No A = B | C
- (2) No A = D | E?

Make an ABCDE diagram:

AB	Ab	aB	ab	
	1	1		CDE
2	3 2 1	1		CDe
2	3 2 1	1		CdE
	1	1		Cde
				cDE
3	2			cDe
3	2			cdE
				cde

Fig. 364.

Now, if $No\ A = B \mid C$, then the combinations containing AbC, aBC, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $No\ A = D \mid E$, then the combinations containing ACDe, ACdE, AcDe, AcdE, are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if $No\ A = BD \mid BE \mid CD \mid CE$, then the combinations containing ABcDe, ABedE, AbCDe, AbCdE, are inconsistent, and we eliminate them by making a figure 3 in those sections.

The Reasoning Frame now shows that we can infer the given inference from the given propositions, because the given inference can be read, and it eliminates combinations only which have already been eliminated, and does not eliminate any combinations which the given propositions did not eliminate.

884. Let us take the following example. Can we infer the proposition,

$$A = B \mid C$$

from the proposition,

$$(1) A = B \mid A = C?$$

Make an ABC diagram:

AB	Ab	aB	ab	
1 2				C
	1 2			c

Fig. 365.

Now, if $A = B \mid A = C$, then the combinations ABC, Abc, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $A = B \mid C$, then the combinations ABC, Abc, are inconsistent, and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows:

- (1) That the given inference can be inferred from the given inferend, and conversely, that the given inferend can be inferred from the given inference.
- (2) That the given inferend and the given inference are equivalents.

885. Let us take the following example: Can we infer the proposition

$$\text{No } A = BC \mid bc$$

from the proposition

$$(1) A = b \mid A = c?$$

Make an ABC diagram:

AB	Ab	aB	ab	
—	—	—	—	—
2 3				C
—	—	—	—	—
	1 3			c

Fig. 366.

Now, if $A = b$, except where $A = c$, then the combination Abc is inconsistent, and we eliminate it by making a figure 1 in that section.

Again, if $A = c$, except where $A = b$, then the combination ABC is inconsistent, and we eliminate it by making a figure 2 in that section.

Again, if $No A = BC | bc$, then the combinations ABC , Abc , are inconsistent, and we eliminate them by making a figure 3 in those sections.

The Reasoning Frame now shows:

- (1) The given propositions are equivalent.
- (2) Each can be inferred from the other.
- (3) They are consistent.

886. Let us take the following example: Can we infer the proposition,

Something (i. e. some combinations) $= AB | CD$
from the proposition,

$$(1) A = B | C = D?$$

Make an ABCD diagram:

AB	Ab	aB	ab	
1 2				CD
	1 2	2	2	Cd
	1			cD
	1			cd

Fig. 367.

Now, if $A = B$, except where $C = D$, then the combinations containing $ABCD$, $AbCd$, Abc , are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $C = D$, except where $A = B$, then the combinations containing $ABCD$, $AbCd$, aCd , are inconsistent, and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that we can read in it,

- (1) Something = AB | CD.
- (2) Something = ac.

887. The following example is taken from Keynes' "Formal Logic," p. 425: "Given,

1st. That wherever the properties A and B are combined, either the property C, or the property D, is present also, but they are not jointly present.

2d. That wherever the properties B and C are combined, the properties A and D are either both present or both absent.

3d. That wherever the properties A and B are both absent, the properties C and D are both absent also; and vice versa, where the properties C and D are both absent, A and B are both absent also.

Find what can be inferred from the presence of A with regard to the presence or absence of B, C, and D."

The premises may be stated thus:

- (1) AB = Cd | cD
- (2) BC = AD | ad
- (3) ab = cd
- (4) cd = ab

Make an ABCD diagram:

AB	A _b	aB	ab	
1		2	3	CD
2			3	Cd
			3	cD
14	4	4		cd

Fig. 368.

Now, if AB = Cd | cD, then the combinations ABCD, AB_cd, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if BC = AD | ad, then the combinations ABCd,

$aBCD$, are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if $ab = cd$, then the combinations $abCD$, $abCd$, $abcD$, are inconsistent, and we eliminate them by making a figure 3 in those sections.

Again, if $cd = ab$, then the combinations $ABcd$, $Abcd$, $aBcd$, are inconsistent, and we eliminate them by making a figure 4 in those sections.

The Reasoning Frame now gives us the following combinations containing A, B, C, D:

(1) $ABcD$

which can be translated,

When B is present with A, C is absent and D is present.

(2) $AbCD$

Which can be translated,

When C and D are present with A, B is absent; or

(3) $AbCd$

which can be translated,

When B and D are both absent from A, C is present; or

(4) $AbcD$

which can be translated,

When B and C are both absent from A, D is present.

This information can be summed up thus:

Where A is, B is absent and C is present, or C is absent and D is present.

888. Let us take the following example:

Can we eliminate B, together with b, from the propositions,

All $A = BC \mid bD$

Whatever is $B \mid D = a \mid BCD$?

Make an ABCD diagram:

AB	Ab	aB	ab	
	2			c D
2	1			Cd
1	2			cD
2 1	1			cd

Fig. 369.

Now, if $A = BC \mid bD$, then the combinations ABCD, AB \bar{c} D, A \bar{b} D, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $B \mid D = a \mid BCD$, then the combinations ABCd, A \bar{B} d, AbCD, Ab \bar{C} D, are inconsistent, because they imply that $B \mid D = A$, and we therefore eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows us that B is not eliminated from the given proposition, but that b is eliminated. But if B were eliminated, then all the A's would be eliminated, and this would prove the contradictoriness of the premises.

889. The following example is adapted from an example given by Dr. Keynes, on p. 429:

$$\text{Nothing} = ac \mid bC,$$

therefore,

$$\text{Nothing} = ab.$$

Make an ABC diagram:

AB	Ab	aB	ab	
	1		1	C
		1	1	e

Fig. 370.

Now, if Nothing (i. e. no combinations) = ac | bC, then the combinations containing ac, bC, are inconsistent, and we eliminate them by making a figure 1 in those sections.

The Reasoning Frame now shows that we can read in it,

$$\text{Nothing} = ab$$

Therefore, the given conclusion is correct.

I understand that in this connection nothing means no combinations.

CHAPTER XXXI.

EXAMPLES.

890. Let us suppose the following state of facts:

One Sunday morning five thieves went to the town of L—, and stayed there one week. They committed a burglary each night. Three, and three only, went out together. We will call them A, B, C, D, E. Commencing Sunday night, they went out in the inverse order of their names. B always went out with C, D, or E. On Saturday night, just before leaving town, the three who went out that night committed a murder.

From these facts, tell which three committed the murder.

Make an ABCDE diagram:

AB	Ab	aB	ab	
1	1	1		CDE
1			1	CD e
1			1	CdE
2	1	1	1	Cde
1			1	cDE
2	1	1	1	cDe
2	1	1	1	cdE
1	1	1	1	cde

Fig. 371.

Now, if three, and three only, went out together, then all the combinations which contain either more or less than three capital letters will be inconsistent, and we eliminate them by making a figure 1 in those sections.

An uneliminated combination containing three capital letters may contain the names of the three who went out together.

Again, if B went out only with C, D, or E, then all the remaining uneliminated combinations containing A and B are inconsistent, and we eliminate them by making a figure 2 in those sections.

The uneliminated combinations, taking them in inverse order, are CDE, BDE, BCE, BCD, ADE, ACE, ACD. Therefore the three who committed the murder were A, C, and D.

891. Let us take the following supposed state of facts:

Six rich men, whom we will call A, B, C, D, E, F, chartered a steam yacht for not to exceed seven trips, for two thousand dollars, and they agreed to divide the expenses according to the number of rides each one should take. But they kept no record of their trips, and some time afterward, when they came to have a final settlement, they hopelessly disagreed as to the number of trips which had been taken, and as to who went on each trip. A lawsuit resulted. On the trial the testimony was very conflicting, and in addition to the facts above given, the following facts only were proven:

- (1) Four, and four only, went on each trip.
- (2) Taken together in couples, A and B, A and C, B and C, never went on the same trip with D and E or D and F or E and F, and C and D never went with E and F.

From these facts, tell what each man's share of the expenses was.

Make an ABCDEF diagram:

ABC								
1	1	1		1		5	1	DEF
1	2	3	1	4	1	1	1	DEF
1	2	3	1	4	1	1	1	DeF
	1	1	1	1	1	1	1	Def
1	2	3	1	4	1	1	1	dEF
	1	1	1	1	1	1	1	dEf
	1	1	1	1	1	1	1	deF
1	i	1	1	1	1	1	1	def

Fig. 372.

Let capital letters represent the men on a trip. Now if four, and only four, went on a trip, then all the combinations containing either more or less than four capital letters are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, as A and B never went with D and E, or D and F, or E and F, we eliminate the remaining ABDE, ABDF, ABEF combinations by making a figure 2 in those sections.

Again, as A and C never went with D and E, or D and F, or E and F, we eliminate the combinations remaining which contain ACDE, ACDF, ACEF, by making a figure 3 in those sections.

Again, as B and C never went with D and E, or D and F, or E and F, we eliminate the remaining combinations which contain BCDE, BCDF, BCEF, by making a figure 4 in those sections.

Again, as C and D never went with E and F, we eliminate the remaining CDEF combination by making a figure 5 in that section.

The uneliminated combinations will give us the number of the trips taken and the names of the men who went on each trip.

They are ABCD, ABCE, ABCF, ADEF, BDEF.

An examination of these combinations shows that A went four times, B four times, C three times, D three times, E three times, and F three times. Therefore, A's share was four hundred dollars, B's four hundred dollars, and each of the others three hundred dollars.

892. Suppose the following facts to be granted:

Four young ladies, whom we will call A, B, C, and D, were in the habit of taking long walks for exercise. It was noticed that C and D never went out with A or B, and that they never stayed in with A or B; that C without D never went out, or stayed in with A and B; and that D without C never went out, or stayed in with A and B. One or more of them went out every day of the week, but the same ones never went together twice in the same week.

The problem is to tell which ones took walks together during the week.

Let capital letters represent those who went out walking.

Let small letters represent those who stayed in the house.

Make an ABCD diagram:

AB	Ab	aB	ab	
	1	1		CD
2			5	Cd
3			5	cD
	4	4		cd

Fig. 373.

Now, if C and D never went out with A or B, then the combinations AbCD, aBCD, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Now, if C without D, never went out with A and B, then the

combination ABCd is inconsistent, and we eliminate it by making a figure 2 in that section.

Again, if D without C never went out with A and B, then the combinations ABcD is inconsistent, and we eliminate it by making a figure 3 in that section.

Again, if C and D never stayed in with A or B, then the combinations Abcd, aBcd, are inconsistent, and we eliminate them by making a figure 4 in those sections.

Again, if A and B never stayed in with C or D, then the combinations abCd, abcD, are inconsistent, and we eliminate them by making a figure 5 in those sections.

The uneliminated combinations show who went out together.

Omitting the small letters, they are: A, B, C, and D, A and B, A and C, A and D, B and C, B and D, C and D.

893. Let us suppose the following facts:

A party of gentlemen whom we will call A, B, C, and D, went a-fishing on four different occasions. C and D never fished with A and B, or without A or B, C and D fished together or did not fish at all. The same ones never fished together more than once.

The problem is to tell who fished on the four different occasions.

Let capital letters represent the men when they went out a-fishing.

Let small letters represent the men when they stayed in.

Make an ABCD diagram:

AB	Ab	aB	ab	
1			1	CD
3	3	3	3	Cd
3	3	3	3	cD
2			2	cd

Fig. 374.

Now, if C and D never fished with A and B, or without A or B, then the combinations ABCD, abCD are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if when C and D stayed in, either A or B stayed in, then the combinations AB_d, ab_d, are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if C and D fished together or stayed in together, then all the combinations containing Cd, and also those containing cD, are inconsistent, and we eliminate them by making a figure 3 in those sections.

The uneliminated combinations are, omitting the small letters, ACD, BCD, A, B.

These combinations show who went a-fishing on the four different occasions.

894. Let us suppose the following state of facts:

A party of four hunters, whom we will call A, B, C, and D, went into the woods to hunt deer. One or more went out hunting on seven different days. A and C always went out together or stayed in together, and the same ones never went out twice, that is, a different party went out hunting each day.

The problem is to tell who went together and who went alone on the seven different days.

Let capital letters represent those who went out hunting.

Let small letters represent the men who stayed in camp.

Make an ABCD diagram:

AB	Ab	aB	ab	
		1	1	CD
		1	1	Cd
1	1			cD
1	1			cd

Fig. 375.

Now, if A and C always went out together, or stayed in together, then all the combinations containing Ac, aC, are

inconsistent, and we eliminate them by making a figure 1 in those sections.

The uneliminated combinations are, omitting the small letters, ABCD, ABC, ACD, AC, BD, B, D.

These combinations show who went out hunting each day.

895. Suppose the following state of facts:

A common council was composed of five members, whom we will call A, B, C, D, and E.

At a certain meeting of the council when they were all present, eleven roll calls were had on motions and the roll calls showed,

- (1) When A and C voted No, E voted Aye with either B or D.
- (2) When A and B voted Aye and E voted No, B and C both voted Aye or both voted No.
- (3) When A and B, or A and E, or A, B, and E, voted Aye, C voted Aye and D voted No, or C voted No and D voted Aye, and conversely.

Tell which four voted Aye on three ballots.

Tell how A, B, and C each voted.

Let capital letters represent those voting Aye.

Let small letters represent those voting No.

The premises can be stated thus:

- (1) $ac = acBdE \mid acbDE$,
- (2) $ADe = ADeBC \mid ADebc$,
- (3) $AB \mid AE \mid ABE = Cd \mid cD$, and conversely.

Make an ABCDE diagram:

AB	Ab	aB	ab	
3	3			CDE
3	2			CDe
		3	3	CdE
	3	3	3	de
		1	3	cDE
2	3	31	31	cDe
3	3		1	cdE
3		1	1	cde

Fig. 376.

Now, if $ac = BdE \mid bDE$, then the combinations $aBcde$, $abcde$, $aBcDe$, $abcDe$, $abcdE$, $aBeDE$, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $ADe = BC \mid bc$, then the combinations $AbCDe$, $AbcDe$, are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if $AB \mid AE \mid ABE = Cd \mid cD$, and conversely, If $Cd \mid cD = AB \mid AE \mid ABE$, then the following combinations $ABede$, $ABCde$; $ABedE$, $ABCDE$, $AbedE$, $AbCDE$, $AbCde$, $AbcDe$, $aBCde$, $aBcDe$, $aBCdE$, $aBeDE$, $abCde$, $abcDe$, $abCdE$, $abcDE$, are inconsistent, and we eliminate them by making a figure 3 in those sections.

The uneliminated combinations are,

$Abcde$, $AbCdE$, $AbcDE$, $ABCde$, $ABeDE$, $ABCde$, $abCDe$, $abCDE$, $aBCde$, $aBedE$, $aBCDE$.

These combinations show that A, B, C, and E voted Aye together once, and B, C, D, and E voted Aye together once, and A, B, D, and E voted Aye together once.

A voted with C or D, except when B, C, and D voted No.

C voted with A or D,
D voted with A or C.

896. Let us take the Tenth Amendment to the Constitution of the United States, which reads as follows:

The powers not delegated to the United States by the Constitution, nor prohibited by it to the States, are reserved to the States respectively, or to the people, and ascertain its latent meanings.

Let a = the powers not delegated to the United States,
b = the powers not prohibited to the states,
C = the powers reserved to the states,
D = the powers reserved to the people.

The propositions contained in the amendment may be stated thus:

- (1) ab = Cd | cD
- (2) Cd | cD = ab
- (3) C = Cd

Make an ABCD diagram:

AB	Ab	aB	ab	
3	3	3	3 1	cD
2	2	2		Cd
2	2	2		cD
			1	cd

Fig. 377.

Now, if ~~ab = Cd | cD~~, then the combinations abCD, abcd, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if Cd | cD = ab, then the combinations ABCd, ABcd, aBCd, ABcD, AbcD, aBcD, are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if C = Cd, then the combinations ABCD, AbCD, aBCD, abCD, are inconsistent, and we eliminate them by making a figure 3 in those sections.

We can now obtain the following definitions from the Reasoning Frame:

(1) AB = ABCd, which we can translate thus:

The powers delegated to the United States by the Constitution and prohibited by it to the States, are not reserved to the States, and are not reserved to the people.

(2) Ab = Abcd, which we can translate:

The powers delegated to the United States by the Constitution and not prohibited by it to the States, are not reserved to the States and are not reserved to the people.

(3) C = abd, which we can translate:

The powers reserved to the States are not delegated to the United States, and are not prohibited to the States and are not reserved to the people.

(4) D = abc, which we can translate:

The powers reserved to the people are not delegated to the United States, and are not prohibited to the States, and are not reserved to the States.

(5) B = Bc, which we can translate:

The powers prohibited to the States are not reserved to the States.

(6) No A = C, which can be translated:

No powers delegated to the United States are reserved to the States.

(7) No A = D, which can be translated:

No powers delegated to the United States are reserved to the people.

(8) No C = D, which can be translated:

No powers reserved to the States are reserved to the people.

The second definition which reads,

The powers delegated to the United States and not prohibited to the States, are not reserved to the States, and are not reserved to the people, is a very important latent meaning of the amendment.

The spirit of it seems to me to conflict with the spirit of certain decisions of the supreme court of the United States where it has been held.

(1) In the absence of congressional legislation on the subject of bankruptcies, the States may pass insolvent laws, if they do not violate the obligation of contracts. It is not the mere existence of the power, but its exercise, which is incompatible with the existence of the same power by the States.

(2) The power to fix the standard of weights and measures is exclusive in Congress when exercised.

(3) The power of Congress is not exclusive to provide for the punishment of counterfeiting the securities and current coin of the United States, etc.

The extraordinary power of our system which is exemplified above, to develop every latent meaning of a clause in a constitution, statute, ordinance, will, contract, etc., ought to render it very useful to judges and lawyers.

With one operation it will tell us everything which a clause affirms, everything which it denies and everything which it leaves in doubt.

Some of the examples already given show its remarkable ability to interpret the implied meanings contained in given statements of facts. This power ought to be very useful in cases depending on circumstantial evidence.

897. This example is from Prof. Jevons:

Where A is present, B and C are either both present at once, or absent at once, and where C is present A is present.

Describe the class not-B under these conditions.

The premises can be stated thus:

$$(1) A = BC \mid bc$$

$$(2) C = CA$$

Make an ABC diagram:

AB	Ab	aB	ab	
	i	2	2	C
1				c

Fig. 378.

Now, if $A = BC \mid bc$, then the combinations ABC , AbC , are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $C = A$, then the combinations containing aC are inconsistent, and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows us that,

$$b = c$$

898. This example is from Prof. Jevons:

It is known of certain things that,

- (1) Where the quality A is, B is not,
- (2) Where B is, and only where B is, C and D are.

Derive from these conditions a description of the class of things in which A is not present but C is.

The premises can be stated as follows:

- (1) $A = Ab$
- (2) $B = CD$
- (3) $CD = B$

Make an ABCD diagram:

AB	Ab	aB	ab	
1	3		3	CD
1 2		2		Cd
1 2		2		cD
1 2		2		cd

Fig. 379.

Now, if $A = b$, then the combinations containing AB are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $B = CD$, then all the B combinations, excepting those containing CD , are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if $CD = B$, then the combinations $AbCD$, $abCD$, are

inconsistent, and we eliminate them by making a figure 3 in those sections.

The Reasoning Frame now shows that the definition of aC is,

$$aC = BD \mid bd$$

899. Taking the same premises as in the previous section, draw descriptions of the classes A_c , ab and cD .

The Reasoning Frame shows that the definitions are,

$$(1) A_c = A_{cb}$$

$$(2) ab = Cd \mid cD \mid cd$$

$$(3) cD = cDb$$

900. The following example is from Prof. DeMorgan:

Every A is one only of the two, B or C ; D is both B and C , except when D is E , and then it is neither, therefore.

No A is D

The premises can be stated thus:

$$(1) A = Bc \mid bC$$

$$(2) D = BCe \mid bcE$$

Make an ABCDE diagram:

AB	a_b	aB	ab	
1	2	2	2	CDe
1	2		2	CDe
1				Cde
1				Cde
2	1	2		cDE
2	1	2	2	cDe
	1			cdE
	1			cde

Fig. 380.

Now, if $A = Bc \mid bC$, then the combinations containing

ABC, Abc, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $D = BCe \mid beE$, then in the remaining combinations, all the D combinations, excepting aBCDe, abcDE, are inconsistent and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that,

$$\text{No } A = D$$

901. The following example is taken from * * * (the lettering is mine):

"There is a certain class of things (D) from which E picks out the A that is B and the C that is not B, and F picks out from the remainder the B which is C and the A that is not C. It is then found that nothing is left but the class B which is not A. The whole of this class is, however, left. What can be determined about the class originally?

Make an ABCD diagram:

AB	Ab	aB	ab	
1	2	3	2	CD
				Cd
1	4			cD
				ed

Fig. 381.

Now, if E picks out of the D things, the A that is B, then we eliminate the combinations containing DAB by making a figure 1 in those sections.

Again, if E picks out of the D things the C that is b, then we eliminate the combinations CABD, CabD, by making a figure 2 in those sections.

Again, if F picks out from the remainder of the D things the B which is C, then we eliminate the DaBC combination by making a figure 3 in that section.

Again, if F picks out of the remainder of the D things the A

which is c, then we eliminate the combination DAbc by making a figure 4 in that section.

The Reasoning Frame now shows that there are two D combinations left, viz.: aBcD, abcD.

This proves that the answer given in the text is wrong. Originally the class D contained the following classes of things: ABC, ABc, AbC, Abc, aBC, aBc, abC, abc.

902. The following example is from Dr. Keynes, p. 433:

“Show what may be inferred as a possible description of warm-blooded vertebrates from the following, and state whether any of the information there given is superfluous for the purpose:

(1) All vertebrates may be divided into warm-blooded and cold-blooded, and all produce their young in but one of the two ways, i. e., are either viviparous or oviparous.

(2) No feathered vertebrate is both viviparous and warm-blooded.

(3) No oviparous vertebrate that is cold-blooded has feathers.

(4) Every viviparous vertebrate is either feathered or warm-blooded.

Let A = vertebrates,

B = warm-blooded,

C = cold-blooded,

D = viviparous,

E = oviparous,

F = feathered.

The premises can be stated thus:

- (1) $A = Bc \mid bC$
- (2) $\text{No } B = C$
- (3) $A = De \mid dE$
- (4) $\text{No } D = E$
- (5) $\text{No } AF = BD$
- (6) $\text{No } ACE = F$
- (7) $AD = bF \mid Bf$

Make an ABCDEF diagram:

ABC	ABc	AbC	Abc	aBC	aBc	abC	abc	
5 3 6 2 1 7 4	3 5 4 7	3 6 4	3 4 1	4 2	4	4	4	DEF
2 1 4 3	3 4	3 7 4	7 3 1 4	4 2	4	4	4	DEf
2 1 7 5	5 7		1	2				DeF
2 1		7	1 7	2				Def
2 1 6		6	1	2				dEF
2 1			1	2				dEf
2 1 3	3	3	3 1	2				deF
2 1 3	3	3	3 1	2				def

Fig. 382.

Now, if $A = Bc \mid bC$, then all the A combinations containing BC and bc are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $No B = C$, then all the combinations containing BC are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if $A = De \mid dE$, then all the combinations containing ADE, Ade, are inconsistent, and we eliminate them by making a figure 3 in those sections.

Again, if $No D = E$, then all the DE combinations are inconsistent, and we eliminate them by making a figure 4 in those sections.

Again, if $No AF = BD$, then the combinations containing ABDF are inconsistent, and we eliminate them by making a figure 5 in those sections.

Again, if $No ACE = F$, then the combinations containing ACEF are inconsistent, and we eliminate them by making a figure 6 in those sections.

Again, if $AD = bF \mid Bf$, then the combinations ABDF,

ADbf, are inconsistent, and we eliminate them by making a figure 7 in those sections.

The Reasoning Frame now gives us the following definition of warm-blooded vertebrates:

$AB = cDef \mid cdEF \mid cdEf$, which can be translated,

Warm-blooded vertebrates are viviparous, and featherless or oviparous.

I think that the information, No feathered vertebrate is both viviparous and warm-blooded is superfluous, for the reason that the combinations which it causes us to eliminate are eliminated by one or more of the other premises.

The reasoning Frame also gives us this definition:

$AC = bDeF \mid bDef \mid bdEf$,

which can be translated,

Cold-blooded vertebrates are viviparous and feathered, or oviparous and featherless.

903. (1) In a certain town the old buildings are either ecclesiastical and built entirely of stone, or, if not ecclesiastical, are built entirely of brick.

(2) The brick and stone buildings are all modern as well as secular, or they are neither.

(3) But there are no modern buildings at once secular and built entirely of stone.

State what assumptions you make in interpreting the above and determine,

(a) In what cases brick may be found in the buildings of this town and in what cases it cannot be.

(b) What old buildings it would be useless to look for."

(Keynes Formal Logic, p. 434).

Let $A = \text{buildings}$,

$B = \text{ecclesiastical}$,

$C = \text{built of stone}$,

$D = \text{built of brick}$,

$E = \text{modern}$,

$F = \text{secular}$,

$G = \text{old}$.

The premises can be stated thus:

- (1) AG = BCd | bcD,
- (2) ACD = EF | ef
- (3) No AE = CF
- (4) No E = G
- (5) No B = F
- (6) No b = f
- (7) No e = g

Make an ABCDEFG diagram:

ABC	ABc	AbC	Abc	aBC	aBc	abC	abc	
5134	514	341	4	54	54	4	4	DEFG
53	5	3		5	5			DEFg
241	41	2416	64	4	4	64	64	DEfG
2		62	6			6	6	DEfg
521	51	21		5	5			DeFG
752	75	72	7	75	75	7	7	DeFg
1	1	61	6			6	6	DefG
7	7	76	76	7	7	76	76	Defg
543	541	41	431	54	54	4	4	dEFG
53	5	3		5	5			dEFg
4	41	641	641	4	4	64	64	dE G
		6	6			6	6	dEfg
5	51	1	1	5	5			deFG
75	75	7	7	75	75	7	7	defg
		1	61	61		6	6	defG
7	7	76	76	7	7	76	76	defg

Fig. 383.

Now, if $AG = BCd \mid bcD$, then all the combinations containing $AGBc$, $AGbC$, $AGBCD$, $AGbcd$, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $ACD = EF \mid ef$, then the combinations containing $ACDEf$, $ACDeF$, are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if $No AE = CF$, then the combinations containing $AECF$ are inconsistent, and we eliminate them by making a figure 3 in those sections.

Again, if $No E = G$, then all the combinations containing EG are inconsistent, and we eliminate them by making a figure 4 in those sections.

Again, if $No B = F$, then all the combinations containing BF are inconsistent, and we eliminate them by making a figure 5 in those sections.

Again, if $No b = f$, then the combinations containing bf are inconsistent, and we eliminate them by making a figure 6 in those sections.

Again, if $No e = g$, then the combinations containing eg are inconsistent, and we eliminate them by making a figure 7 in those sections.

We have made the following assumptions, viz:

(1) Old and modern, not old and not modern, ecclesiastical and secular, not ecclesiastical and not secular, are inconsistent combinations.

The Reasoning Frame now gives us the following definitions:

$$AD = BcEfg \mid bcEFg \mid bceFG,$$

which can be translated,

The brick buildings are ecclesiastical and modern, or modern and secular, or secular and old.

$$No D = ABG,$$

which can be translated,

No brick can be found in old ecclesiastical buildings.

904. (1) If a nation has natural resources and a good government, it will be prosperous.

(2) If it has natural resources without a good government, or a good government without natural resources, it will be contented but not prosperous.

(3) If it has neither natural resources nor a good government, it will be neither contented nor prosperous.

Show that these statements may be reduced to two propositions of the form of Hamilton's U.

Let A = nation

B = natural resources

C = good government

D = prosperous

E = contented.

The premises can be stated as follows:

$$(1) \text{ABC} = \text{ABCD}$$

$$(2) \text{ABC} + \text{AbC} = \text{AdE}$$

$$(3) \text{Abc} = \text{Ade}$$

Make an ABCDE diagram:

AB	Ab	aB	ab	
	2			CDE
	2			De
1				CdE
1	2			Cde
2	3			cDE
2	3			cDe
	3			cdE
2				cde

Fig. 384.

Now, if $\text{ABC} = \text{ABCD}$, then the combinations containing ABCd are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $ABC \mid AbC = AdE$, then the combinations $ABcDE$, $ABCDe$, $ABcde$, $AbCDE$, $AbCDe$, $AbCde$, are inconsistent and we eliminate them by making a figure 2 in those sections.

Again, if $Abc = Ade$, then the combinations $AbcDE$, $AbcDe$, $AbcdE$, are inconsistent and we eliminate them by making a figure 3 in those sections.

This Reasoning Frame now shows the logical expression of the given premises but I think that the premises cannot be reduced to two propositions of the form of Hamilton's U.

We can get $ABC = AD$ (1); $Abc = Ade$ (2); and $AbC \mid ABC = AdE$ (3); but it will be seen that (3) prevents us from reducing the statements to two U propositions.

905. Let the observation of a class of natural productions be supposed to have led to the following general results:

- (1) That in whichever of these productions the properties A and C are missing, the property E is found, together with one of the properties B and D, but not with both.
- (2) That wherever the properties A and D are found, while E is missing, the properties B and C will either both be found or both be missing.
- (3) That wherever the property A is found in conjunction with either B or E, or both of them, there, either the property C or the property D will be found, but not both of them.
- (4) Wherever the property C or D is found, there the property A will be found in conjunction with either B or E, or both of them.

The premises can be stated as follows:

- (1) $ac = BdE \mid bDE$
- (2) $ADe = BC \mid bc$
- (3) $ABE \mid AbE \mid ABe = Cd \mid cD$
- (4) $Cd \mid cD = ABe \mid AbE \mid ABE.$

Make an ABCDE diagram:

AB	Ab	aB	ab	
3	3			CDE
3	2			CDe
		4	4	CdE
	4	4	4	Cde
		4 1	4	cDE
2	4	4 1	4 1	cDe
3	3		1	cdE
3		1	1	cde

Fig. 385.

Now, if $ac = BdE \mid bDE$, then the combinations $acBDE$, $acBDe$, $acBde$, $acbDe$, $abcdE$, $acbde$, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $ADe = BC \mid bc$, then the combinations $ABcDe$, $AbCDe$, are inconsistent and we eliminate them by making a figure 2 in those sections.

Again, if $ABe \mid AbE \mid ABE = Cd \mid cD$, then the combinations $ABCDE$, $ABCDe$, $ABedE$, $ABede$, $AbCDE$, $Abede$, are inconsistent and we eliminate them by making a figure 3 in those sections.

Again, if $Cd \mid cD = ABe \mid AbE \mid ABE$, then the combinations $AbCde$, $AbcDe$, $aBCde$, $aBCde$, $aBeDE$, $aBeDe$, $abCdE$, $abCde$, $abcDE$, $abcDe$, are inconsistent and we eliminate them by making a figure 4 in those sections.

We can now get the following definition of A:

$A = BCde \mid BCde \mid BeDE \mid bCdE \mid bcDE \mid bcede$, and the following definition of aC:

$$aC = D,$$

and the following definition of Cd \mid Dc,

$Cd \mid cD = ABE \mid ABe \mid AbE$,
and the following definition of bcd ,

$$bcd = Ae.$$

These definitions show that where A is found, there also C or D are found, or else B, C and D are absent, and where C or D is found, or B, C and D are together absent, A is found; and if A is absent and C present, D is present.

906. Given the same premises as in the preceding section, show that,

- (1) If the property B be present in one of the productions, either the properties A, C and D are all absent, or some one alone of them is absent, and conversely, if they are all absent, it may be concluded that the property B is present.
- (2) If A and C are both present or both absent, D will be absent quite independently of the presence or absence of B.

We can get the following definition of B:

$$B = ACdE \mid ACde \mid AeDE \mid aCDE \mid aCDe \mid acdE.$$

This definition shows that if B be present, A, C and D are all absent, or some one alone of them is absent, and it also shows that if A, C and D are all absent, B is present.

We can also get the following definition of $AC \mid ac$:

$$AC \mid ac = d.$$

This shows that if A and C are both present, or both absent, D is also absent.

907. The following is adapted from an example in Prof. Venn's "Symbolic Logic;"

$$\text{Given } BD = A$$

$$DE = C$$

Find BE in terms of A and C.

Make an ABCDE diagram:

AB	A b	aB	ab	
		1		CDE
		1		CDe
				CdE
				Cde
2	2	2 1	2	cDE
		1		cDe
				cdE
				cde

Fig. 386.

Now, if $BD = A$, then the combinations containing BDA are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $DE = C$, then the combinations containing DEC are inconsistent and we eliminate them by making a figure 2 in those sections.

We can now get the following definition of BE:

$$BE = ACD \mid ACd \mid Acd \mid acd \mid aCd.$$

From this we can get,

$$BE = AC \mid cd \mid aCd.$$

908. Are the three following systems of propositions equivalent?

- (1) $Ab = cd$
 $aB = Ce$
 $D = E$
- (2) $A = B \mid c \mid D$
 $BE = A$
 $Be = Ad \mid Cd$
 $bD = aE$

$$(3) A \mid e = B \mid d$$

$$a = bE \mid bd \mid BCe$$

$$bc = a$$

$$D = E.$$

Make an ABCDE diagram:

AB	Ab	aB	ab	
	1	2		CDE
3	13	3	3	CDe
	1	2		CdE
	1			Cde
	1	2		cDE
3	13	23	3	cDe
		2		cdE
		2		cde

Fig. 387.

Now, if $Ab = cd$, then the combinations containing $AbCD$, $AbCd$, $AbcD$ are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $aB = Ce$, then the combinations containing aBE , aBe , are inconsistent and we eliminate them by making a figure 2 in those sections.

Again, if $D = E$, then all the combinations containing De are inconsistent and we eliminate them by making a figure 3 in those sections.

This Reasoning Frame shows the logical expression of the first system of propositions.

Make an ABCDE diagram:

AB	Ab	aB	ab	
1	4	2		CDE
1 3	4	3	4	CDe
	1	2		CdE
3	1			Cde
1	1 4	2		cDE
1 3	1 4	3	4	cDe
1		2		cdE
1		3		cde

Fig. 388.

Now, if $A = B \mid c \mid D$, then the combinations containing ABCD, AB \bar{c} D, A $\bar{B}c$ D, A $\bar{B}\bar{c}D$, are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $BE = A$, then the combinations containing BEa, are inconsistent and we eliminate them by making a figure 2 in those sections.

Again, if $Be = Ad \mid Cd$, then the combinations ABCDe, ABCde, AB $\bar{c}De$, aBCDe, aB $\bar{c}De$, aBcde, are inconsistent and we eliminate them by making a figure 3 in those sections.

Again, if $bD = aE$, then the combinations containing AbD, abDe, are inconsistent and we eliminate them by making a figure 4 in those sections.

This Reasoning Frame shows the logical expression of the second system of propositions and it also shows that the first and second systems are not equivalents.

Make an ABCDE diagram:

AB	Ab	aB	ab	
	3 1	2		CDE
4	4 3	4	4 2 1	CD \bar{e}
1	3	2	2	CdE
	3	1		Cde
	1	2		cDE
4	4	4 2	4 2 1	cD \bar{e}
1		2	2	cdE
		2 1		cde

Fig. 389.

Now, if $A \mid e = B \mid d$, then the combinations containing ABdE, AbDE, aBde, abDe, are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $a = bE \mid bd \mid BCe$, then the following combinations containing aBCE, aBe, abCDe, abCdE, abcDe, abcdE, are inconsistent and we eliminate them by making a figure 2 in those sections.

Again, if $bC = a$, then the combinations containing bCA are inconsistent and we eliminate them by making a figure 3 in those sections.

Again, if $D = E$, then all the combinations containing De are inconsistent and we eliminate them by making a figure 4 in those sections.

This Reasoning Frame shows the logical expression of the third system of propositions and it also shows that it is not equivalent to either one of the first and second systems.

909. The following example is adapted from Dr. Keynes' "Formal Logic," p. 438.

- (1) $Abc = DE$
- (2) $B = C$

- (3) $b = c$
- (4) $BCD = AE \mid ae$
- (5) $BCd = Ae$
- (6) $abcD = E$

Then it follows that,

- (1) $Ab = cDE$
- (2) $Ad = BCe$
- (3) $aB = CDe$
- (4) $aE = bc$
- (5) $Bd = ACe$
- (6) $BE = ACD$
- (7) $bd = cE$
- (8) $bd = ac$
- (9) $be = acd$
- (10) $dE = abc.$

Make an ABCDE diagram:

AB	Ab	aB	ab	
	3	4	3	CDE
4	3		3	CDe
5	3	5	3	CdE
	3	5	3	Cde
2		2		cDE
2	1	2	6	cDe
2	1	2		cdE
2	1	2		cde

Fig. 390.

Now, if $Abc = DE$, then the combinations containing $AbcDe$, $AbcdE$, $Abcde$, are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $B = C$, then the combinations containing Bc are

inconsistent and we eliminate them by making a figure 2 in those sections.

Again, if $b = c$, then the combinations containing bC are inconsistent and we eliminate them by making a figure 3 in those sections.

Again, if $BCD = AE | ae$, then the combinations $ABCDe$, $aBCDE$, are inconsistent and we eliminate them by making a figure 4 in those sections.

Again, if $BCd = Ae$, then the combinations $ABCdE$, $aBCdE$, $aBCde$, are inconsistent and we eliminate them by making a figure 5 in those sections.

Again, if $abcD = E$, then the combination $abcDe$ is inconsistent and we eliminate it by making a figure 6 in that section.

We can now get the following definitions:

- (1) $Ab = cDE$
- (2) $Ad = BCe$
- (3) $aB = CDe$
- (4) $aE = bc$
- (5) $Bd = ACe$
- (6) $BE = ACD$
- (7) $bD = cE$
- (8) $bd = ac$
- (9) $be = aed$
- (10) $dE = abc$.

910. The members of a scientific society are divided into three sections, which are denoted by A, B, C. Every member must join one at least, of these sections, subject to the following conditions:

- (1) Any one who is a member of A but not of B, of B but not of C, or of C but not of A, may deliver a lecture to the members if he has paid his subscription, but, otherwise, not.
- (2) Any one who is a member of A, but not of C, of C but not of A, or of B but not of A, may exhibit an experiment to the members if he has paid his subscription, but, otherwise, not.

(3) But every member must either deliver a lecture or perform an experiment annually, before the other members.

Find the least addition to these rules which will compel every member to pay his subscription or forfeit his membership.

Let D = members of the society,

A = member of section A,

B = member of section B,

C = member of section C,

E = one who delivers a lecture,

F = one who has paid his subscription,

G = one who performs an experiment.

The premises are,

$$(1) \text{ Abc} \mid \text{aBc} \mid \text{abC} = \text{EF} \mid \text{ef}$$

$$(2) \text{ Abc} \mid \text{aBc} \mid \text{abC} = \text{FG} \mid \text{fg}$$

$$(3) \text{ D} = \text{E} \mid \text{G}$$

$$(4) \text{ No A} = \text{d}$$

$$(5) \text{ No B} = \text{d}$$

$$(6) \text{ No C} = \text{d}.$$

Make an ABCDEFG diagram:

ABC	A _B c	A _b C	A _{bc}	a _B C	a _B c	a _b C	a _{bc}	
3	3	3	3	3	3	3	3	DEFG
			2		2	2		DEFg
3	3	3	3 1 2	3	2 3 1	1 3 2	3	DEfG
			1		1	1		DEfg
			1		1	1		DeFG
3	3	3	3 1 2	3	1 2 3	1 2 3	3	DeFg
			2		2	2		DefG
3	3	3	3	3	3	3	3	Defg
5 4 6	5 4	6 4	4	6 5	5	6		dEFG
5 4 6	5 4	6 4	4 2	6 5	5 2	6 2		dEFg
5 4 6	5 4	6 4	4 1 2	6 5	5 2 1	2 6 1		dEfG
5 4 6	5 4	6 4	4 1	6 5	5 1	6 1		dEfG
5 4 6	5 4	6 4	4 1	6 5	5 1	6 1		deFG
5 4 6	5 4	6 4	4 1 2	6 5	5 1 2	6 1 2		deFg
5 4 6	5 4	6 4	4 2	6 5	5 2	6 2		defG
5 4 6	5 4	6 4	4	6 5	5	6		defg

Fig. 391.

Now, if $A_{bc} | a_{Bc} | abC = EF | ef$, then the combinations containing $A_{bc}Ef$, $A_{bc}eF$, $a_{Bc}Ef$, $a_{Bc}eF$, $abCEf$, $abCeF$, are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $A_{bc} | a_{Bc} | abC = FG | fg$, then the combinations containing $A_{bc}Fg$, $A_{bc}fG$, $a_{Bc}Fg$, $a_{Bc}fG$, $abCFg$, $abCfG$,

are inconsistent and we eliminate them by making a figure 2 in those sections.

Again, if $D = E | G$, then the combinations containing DEG , Deg , are inconsistent and we eliminate them* by making a figure 3 in those sections.

Again, if $No\ A = d$, then all the combinations containing Ad are inconsistent and we eliminate them by making a figure 4 in those sections.

Again, if $No\ B = d$, then all the combinations containing Bd are inconsistent and we eliminate them by making a figure 5 in those sections.

Again, if $No\ C = d$, then all the combinations containing Cd , are inconsistent and we eliminate them by making a figure 6 in those sections.

The Reasoning Frame now shows that if we add either one of the following rules, every member must pay his subscription or forfeit his membership:

(1) $D = F$, which can be translated,

Every member is one who has paid his subscription.

(2) $No\ D = f$, which can be translated,

No one is a member who has not paid his subscription.

(3) $No\ f = E | G$, which can be translated,

No one who has not paid his subscription can deliver a lecture or perform an experiment annually before the other members.

The first one seems to make the least addition to the rules.

911. Let us take this example:

“He that believeth and is baptized shall be saved but he that believeth not shall be damned.”

I assume that those who shall be saved shall not be damned.

Let A = those who believe,

B = those who are baptized,

C = saved,

D = damned.

The premises can be stated thus:

- (1) $AB = ABC$
- (2) $a = aD$
- (3) $\text{No } C = D$

Make an ABCD diagram:

AB	Ab	aB	ab	
3	3	3	3	CD
			2	Cd
1				cD
1		2	2	cd

Fig. 392.

Now, if $AB = ABC$, then the combinations containing ABC , are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $a = aD$. then the combinations containing ad are inconsistent and we eliminate them by making a figure 2 in those sections.

Again, if $\text{No } C = D$. then the combinations containing CD are inconsistent and we eliminate them by making a figure 3 in those sections.

From the uneliminated combinations we can now get the following consistent definitions:

$$(1) AB = ABC$$

which can be translated,

Those who believe and are baptized shall be saved.

$$(2) Ab = Cd \mid cD \mid cd,$$

which can be translated,

Those who believe and are not baptized shall be saved or damned or neither.

$$(3) C = AB \mid Ab,$$

which may be translated,

Those who shall be saved are believers, and baptized or not baptized.

$$(4) D = Ab \mid aB \mid ab,$$

which can be translated,

Those who shall be damned are either believers who are not baptized or not believers, whether baptized or not baptized.

$$(5) a = D,$$

which can be translated,

Those who do not believe shall be damned.

$$(6) c = D \mid d,$$

which can be translated,

Those who shall not be saved shall be damned, or not damned.

$$(7) d = AB \mid Ab,$$

which can be translated,

Those who shall not be damned are either those who believe and are baptized or those who believe and are not baptized.

912. Let us take this example.

"Except a man be born of water and of the spirit he cannot enter into the kingdom of God."

The logical meaning of this passage is,

If a man is not born of water and of the spirit he cannot enter into the kingdom of God.

Let $A = \text{man}$,

$B = \text{born of water}$,

$C = \text{born of the spirit}$,

$D = \text{enter into the kingdom of God}$.

(1) If $A = bc$, then $A = d$,

which can be reduced to,

$$Abc = Abcd.$$

Make an ABCD diagram:

AB	Ab	aB	ab	
			*	CD
				Cd
				cD
	1			cd

Fig. 393.

Now, if $A B c = A B c d$, then the combination $A B c D$ is inconsistent and we eliminate it by making a figure 1 in that section.

From the uneliminated combinations we can get the following consistent definitions:

$$(1) \quad A B c = d,$$

which can be translated,

A man not born of water and of the spirit shall not enter into the kingdom of God.

$$(2) \quad A B C = D \mid d,$$

which can be translated,

A man born of water and of the spirit shall or shall not enter into the kingdom of God.

$$(3) \quad A B c = D \mid d,$$

which can be translated,

A man born of water and not of the spirit, shall or shall not enter into the kingdom of God.

$$(4) \quad A b C = D \mid d,$$

which can be translated,

A man born of the spirit and not of water shall or shall not enter into the kingdom of God.

$$(5) \quad D = A B C \mid A B c \mid A b C \mid a,$$

which can be translated,

Those who shall enter into the kingdom of God are men born of water and of the spirit, or men born of water and not of the spirit, or men born of the spirit and not of water, or not-men.

CHAPTER XXXII.

INDUCTIVE EXAMPLES.

913. Given the combinations ABC or Abc or aBC or abC, we are to find a set of propositions not involving alternative combinations which shall produce them.

Make an ABC diagram:

AB	Ab	aB	ab	
S	1	S	S	C
1	S	1	1	c

Fig. 394.

In the sections which contain the given alternants, viz.: ABC, Abc, aBC, abC, make a letter S to indicate that these combinations are saved, that is, they are not to be eliminated.

The Reasoning Frame now shows us that the proposition, $bC = a$, will cause us to eliminate the combination AbC, and the proposition,

$c = Ab$, will cause us to eliminate the combinations ABC, aBc , abc.

Mark the eliminated combinations with a figure 1. The given combinations are all saved. This proves that the two propositions,

$$bC = a$$

$$c = Ab$$

will produce the given combinations, ABC or Abc or aBC or abC.

914. The given alternants are ACe, aBce, aBedE, abCe, abcE.

Make an ABCDE diagram:

AB	Ab	aB	ab	
				CDE
S	S	S	S	CDe
				CdE
S	S	S	S	Cde
			S	cDE
				cDe
		S	S	cdE
				cde

Fig. 395.

Now make a letter S in the sections containing the given alternants.

By reading the definitions which we can get from the combinations which are in the sections marked with the letter S, we can get the following definitions:

- (1) A = CeA
- (2) BD = CeBD
- (3) e = C
- (4) C = e

These definitions will produce the combinations ACE | aBCE | aBcdE | abCe | abcE.

Make an ABCDE diagram:

AB	Ab	aB	ab	
4 1 2	4 1	4 2	4	CDE
				CDe
4 1	4 1	4	4	CdE
				Cde
1 2	1	2		cDE
3 1 2	3 1	3 2	3	cDe
1	1			cdE
3 1	3 1	3	3	cede

Fig. 396.

Now, if $A = Ce$, then the combinations containing ACE, AcE, Ace, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $BD = Ce$, then the combinations containing BDCE, BDcE, BDce, are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if $e = C$, then the combinations containing ec are inconsistent, and we eliminate them by making a figure 3 in those sections.

Again, if $C = e$, then the combinations containing CE are inconsistent, and we eliminate them by making a figure 4 in those sections.

An examination of the two Reasoning Frames now shows us that the propositions,

$$\begin{aligned} A &= CeA \\ BD &= CeBD \\ e &= C \\ C &= e \end{aligned}$$

will produce the given combinations.

The propositions which we have given are not the only propositions which will produce the given combinations.

We could find several sets of propositions which taken together would produce the given combinations.

We can proceed in a tentative manner by taking one definition which the given combinations will yield, as a new premise, and then proceed to eliminate the combinations which are inconsistent with it. Then get another definition from the given combinations and eliminate the inconsistent combinations. And so continue this process of getting definitions from the given combinations and eliminating the inconsistent combinations, until all the inconsistent combinations are eliminated, being careful not to get any definition from the given combinations which would cause the elimination of any alternant in the given combinations.

915. The given combinations are ABCD | abCd.

Make an ABCD diagram:

AB	Ab	aB	ab	
S	S	S		CD
S			S	Cd
	S	S		cD
S		S		cd

Fig. 397.

Mark the given alternants with the letter S.

It will be understood by the reader that all the combinations not marked with the letter S are eliminated.

An examination of the Reasoning Frame shows that we can get the following definitions:

- (1) cd = Bcd
- (2) ab = Cdab
- (3) AbC = DAbC
- (4) aBC = DaBC
- (5) ABD = CABD

Make an ABCD diagram:

AB	Ab	aB	ab	
			2	CD
	3	4		Cd
5			2	cD
	1		21	cd

Fig. 398.

Now, if $cd = B$, then the combinations containing Acd , $abcd$, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $ab = Cd$, then the combinations containing abc , $abCD$, are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if $AbC = D$, then the combination $AbCd$ is inconsistent, and we eliminate it by making a figure 3 in that section.

Again, if $aBC = D$, then the combination $aBCd$ is inconsistent, and we eliminate it by making a figure 4 in that section.

Again, if $ABD = C$, then the combination $ABcD$ is inconsistent, and we eliminate it by making a figure 5 in that section.

The result proves that we have found propositions which will produce the given combinations.

916. The given combinations are $ABCDE$ | $abcdE$.

Make an ABCDE diagram:

AB	Ab	aB	ab	
S	S			CDE
S		S	S	CD _e
			S	CdE
S		S		Cde
				cDE
		S	S	cDe
	S		S	cdE
S	S			cde

Fig. 399.

Mark the sections containing the given alternants with a letter S.

From the given alternants we can obtain the following definitions:

- (1) $AbC = DEAbC$
- (2) $ABC = deABC$
- (3) $aBc = DeaBc$
- (4) $CdE = abCdE$
- (5) $DE = ADE$
- (6) $Abc = dAbc$
- (7) $abcD = EabCd$
- (8) $abcd = Eabcd$

Make an ABCDE diagram:

AB	Ab	aB	ab	
		5	5	CDE
	1			CDe
4	4 1	4		CdE
	1		7	Cde
2	6	5 3	5	cDE
2	6			cDe
2		3		cdE
		3	8	cde

Fig. 400.

Now, if $AbC = DE$, then the combinations containing $AbCd$, $AbCDe$, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $ABC = de$, then the combinations containing $ABCd$, $ABCdE$, are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if $aBc = De$, then the combinations containing $aBcd$, $aBcDE$, are inconsistent, and we eliminate them by making a figure 3 in those sections.

Again, if $CdE = ab$, then the combinations containing $ACdE$, $aBCdE$, are inconsistent, and we eliminate them by making a figure 4 in those sections.

Again, if $DE = A$, then the combinations containing DEa are inconsistent, and we eliminate them by making a figure 5 in those sections.

Again, if $Abc = d$, then the combinations containing $AbcD$ are inconsistent, and we eliminate them by making a figure 6 in those sections.

Again, if $abCd = E$, then the combination containing $abCde$ is inconsistent, and we eliminate it by making a figure 7 in that section.

Again, if $abcd = E$, then the combination containing $abcde$ is inconsistent, and we eliminate it by making a figure 8 in that section.

The result proves that we have obtained a set of propositions which will produce the given combinations. By the same method we can obtain other sets of propositions which will produce the given combinations.

CHAPTER XXXIII.

EQUIVALENTS FOR PROPOSITIONS.

917. By the use of the Reasoning Frame I have discovered a new method of finding a set of propositions which shall be equivalent to a complex categorical proposition, or to a combination of complex categorical propositions.

The method is as follows:

- (1) State the given proposition,
- (2) Eliminate the inconsistent combinations,
- (3) From the consistent combinations which remain, get definitions of any letter-term and its negative.

These definitions will be equivalent propositions to the given proposition, or,

- (4) From the consistent combinations get definitions by the method described in the preceding chapter. The definitions thus obtained will make a set of propositions equivalent to the given proposition.

Let the given propositions be,

$$aB = cD$$

$$cD = aB$$

Make an ABCD diagram:

AB	Ab	aB	ab	
		1		CD
		1		Cd
2	2		2	cD
		1		cd

Fig. 401.

Now, if $aB = cD$, then the combinations containing aBC ,

$aBcd$, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $cD = aB$, then the combinations containing AcD , $abcD$, are inconsistent, and we eliminate them by making a figure 2 in those sections.

From the consistent combinations we can get the following definitions of a letter-term and its negative:

$$\begin{aligned} A &= cd \mid C \\ a &= bed \mid bC \mid BeD \end{aligned}$$

Make an ABCD diagram:

AB	Ab	aB	ab	
		2		CD
		2		Cd
1	1		2	cD
		2		cd

Fig. 402.

Now, if $A = Cd \mid C$, then the combinations containing AcD are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $a = bed \mid bC \mid BeD$, then the combinations containing $aBed$, aBC , $abcD$, are inconsistent, and we eliminate them by making a figure 2 in those sections.

The result proves that we have obtained a pair of propositions which are equivalent to the pair of given propositions.

918. Suppose now that we wish to obtain a set of categorical propositions which shall be equivalent to the pair of propositions,

- (1) $aB = cD$
- (2) $cD = aB$

Make an ABCD diagram:

AB	Ab	aB	ab	
		1		CD
		1		Cd
2	2		2	cD
		1		cd

Fig. 403.

Now, if $aB = cD$, then the combinations containing aBC , $aBcd$, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $cD = aB$, then the combinations containing AcD , $abcD$, are inconsistent, and we eliminate them by making a figure 2 in those sections.

From the consistent combinations we can get the following categorical definitions:

- (1) $Ac = dAc$
- (2) $BC = ABC$
- (3) $abc = dabc$
- (4) $aBc = DaBc$

Make an ABCD diagram:

AB	Ab	aB	ab	
		2		CD
		2		Cd
1	1		3	cD
		4		cd

Fig. 404.

Now, if $Ac = d$, then the combinations containing AcD are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $BC = A$, then the combinations containing aBC are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if $abc = d$, then the combination $abcd$ is inconsistent, and we eliminate it by making a figure 3 in that section.

Again, if $aBc = D$, then the combination $aBcd$ is inconsistent, and we eliminate it by making a figure 4 in that section.

The result proves that we have obtained a set of propositions which are equivalent to the given propositions. They are equivalent to the given propositions for the reason that they cause the elimination of exactly the same combinations which the given pair of propositions caused us to eliminate, and they save exactly the same combinations which the given pair of propositions save.

919. Let the given proposition be,

$$A \mid C = B \mid D, \text{ and conversely.}$$

Make an ABCD diagram:

AB	Ab	aB	ab	
	4	2		CD
3			2	Cd
1			4	cD
	1	3		cd

Fig. 405.

The proposition $A \mid C = B \mid D$ means,

$$Ac \mid aC = Bd \mid bD.$$

Now, if $Ac = Bd \mid bD$, then the combinations $ABCd$, $Abcd$, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $aC = Bd \mid Db$, then the combinations $aBCD$, $abCd$ are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if $Bd = Ac \mid aC$, then the combinations $ABCd$, $aBcd$,

are inconsistent, and we eliminate them by making a figure 3 in those sections.

Again, if $bD = Ac \mid aC$, then the combinations $AbCD$, $abcD$, are inconsistent, and we eliminate them by making a figure 4 in those sections.

The Reasoning Frame now shows the logical expression of the proposition $A \mid C = B \mid D$, and conversely.

From the consistent combinations which remain we can get the following definitions:

- (1) $ABC = D$
- (2) $ABc = d$
- (3) $AbC = d$
- (4) $Abc = D$
- (5) $aBC = d$
- (6) $aBc = D$
- (7) $abC = D$
- (8) $abc = d$

This set of propositions is equivalent to the given proposition,

$$A \mid C = B \mid D$$

Make an ABCD diagram:

AB	Ab	aB	ab	
	3	5		CD
1			7	Cd
2			8	cD
	4	6		cd

Fig. 406.

Now, if $ABC = D$, then the combination $ABCd$ is inconsistent, and we eliminate it by making a figure 1 in that section.

Again, if $ABc = d$, then the combination $ABcD$ is inconsistent, and we eliminate it by making a figure 2 in that section.

Again, if $AbC = d$, then the combination $AbCD$ is inconsistent, and we eliminate it by making a figure 3 in that section.

Again, if $A B c = D$, then the combination $A b e d$ is inconsistent, and we eliminate it by making a figure 4 in that section.

Again, if $a B C = d$, then the combination $a B C D$ is inconsistent, and we eliminate it by making a figure 5 in that section.

Again, if $a B c = D$, then the combination $a B e d$ is inconsistent, and we eliminate it by making a figure 6 in that section.

Again, if $a b C = D$, then the combination $a b C d$ is inconsistent, and we eliminate it by making a figure 7 in that section.

Again, if $a b c = d$, then the combination $a b c d$ is inconsistent, and we eliminate it by making a figure 8 in that section.

The result proves that this set of eight propositions is equivalent to the given proposition.

920. Find the categorical equivalents for this set of propositions:

- (1) No $C D = a \mid A b$
- (2) No $C d = A \mid a B$
- (3) No $c D = a \mid A B$
- (4) No $c d = A \mid a b$.

Make an ABCD diagram:

$A B$	$A b$	$a B$	$a b$	
	1	1	1	$C D$
2	2	2		$C d$
3		3	3	$c D$
4	4		4	$c d$

Fig. 407.

Now, if $\text{No } C D = a \mid A b$, then the combinations containing $C D a$, $C D A b$, are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $\text{No } C d = A \mid a B$, then the combinations containing $A C d$, $a B C d$, are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if $\text{No } c D = a \mid A B$, then the combinations contain-

ing aCD , $ABcD$, are inconsistent and we eliminate them by making a figure 3 in those sections.

Again, if $No\ cd = A \mid ab$, then the combinations containing Acd , $abcd$ are inconsistent and we eliminate them by making a figure 4 in those sections.

The Reasoning Frame now shows us that we can get among others the following definitions:

$$AB = CD$$

$$Ab = cD$$

$$aB = cd$$

$$ab = Cd.$$

This set of propositions is equivalent to the given set of propositions.

Make an ABCD diagram:

AB	Ab	aB	ab	
	2	3	4	CD
1	2	3		Cd
1		3	4	cD
1	2		4	cd

Fig. 408.

Now, if $AB = CD$, then the combinations containing $ABCd$, ABc , are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $Ab = cD$, then the combinations containing AbC , $Abcd$, are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if $aB = cd$, then the combinations containing aBC , $aBcD$, are inconsistent, and we eliminate them by making a figure 3 in those sections.

Again, if $ab = Cd$, then the combinations containing $abCD$, abc , are inconsistent, and we eliminate them by making a figure 4 in those sections.

The result proves that we have found a set of propositions which is equivalent to the given set of propositions.

921. Let the pair of given propositions be,

$$(1) \ C = A \mid B$$

$$(2) \ A = C.$$

Make an ABCD diagram:

AB	Ab	aB	ab	
1			1	CD
1			1	Cd
2	2			cD
2	2			cd

Fig. 409.

Now, if $C = A \mid B$, then the combinations containing CAB, Cab, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $A = C$, then the combinations containing Ac are inconsistent, and we eliminate them by making a figure 2 in those sections.

This Reasoning Frame shows that we can get among others the following definitions:

$$(1) \ No \quad A = c$$

$$(2) \ No \ AB = C$$

$$(3) \ No \ ab = C.$$

Make an ABCD diagram:

AB	Ab	aB	ab	
2			3	CD
2			3	Cd
1	1			cD
1	1			cd

Fig. 410.

Now, if $No\ AB = c$, then the combinations containing Ac are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $No\ AB = C$, then the combinations containing ABC , are inconsistent and we eliminate them by making a figure 2 in those sections.

Again, if $No\ ab = C$, then the combinations containing abC , are inconsistent, and we eliminate them by making a figure 3 in those sections.

The result proves that we have obtained a triplet of propositions which is equivalent to the given pair of propositions.

922. Find four negative compound propositions, which, taken together, shall be equivalent to the following four disjunctive propositions taken together.

- (1) $AB = Cd \mid c$
- (2) $Cd = A \mid ab$
- (3) $Ab = C \mid cD$
- (4) $cD = A \mid aB$.

Make an ABCD diagram:

AB	Ab	aB	ab	
1				CD
				Cd
				cD
	3			cd

Fig. 411.

Now, if $AB = Cd \mid c$, then the combination $ABCD$, is inconsistent, and we eliminate it by making a figure 1 in that section.

Again, if $Cd = A \mid ab$, then the combination containing $aBCd$, is inconsistent and we eliminate it by making a figure 2 in that section.

Again, if $Ab = C \mid cD$, then the combination containing $Abcd$ is inconsistent and we eliminate it by making a figure 3 in that section.

Again, if $cD = A \mid aB$, then the combination containing $abcD$, is inconsistent and we eliminate it by making a figure 4 in that section.

This Reasoning Frame shows that we can get among others the following universal negative compound propositions:

- (1) No $AB = CD$
- (2) No $Ab = cd$
- (3) No $aB = Cd$
- (4) No $ab = cD$.

This quartet of propositions is equivalent to the given quartet of propositions.

Make an ABCD diagram:

AB	Ab	aB	ab	
1				CD
		3		Cd
			4	cD
2				cd

Fig. 412.

Now, if $\text{No } AB = CD$, then the combination $ABCD$ is inconsistent and we eliminate it by making a figure 1 in that section.

Again, if $\text{No } Ab = cd$, then the combination $Abcd$ is inconsistent and we eliminate it by making a figure 2 in that section.

Again, if $\text{No } aB = Cd$, then the combination $aBCd$, is inconsistent and we eliminate it by making a figure 3 in that section.

Again, if $\text{No } ab = cD$, then the combination $abcD$ is inconsistent and we eliminate it by making a figure 4 in that section.

The result proves that we have found a quartet of universal negative compound propositions equivalent to the given quartet of disjunctive propositions.

923. Find the categorical equivalents for the following propositions:

- (1) $AB \mid ab = CD \mid cd$
- (2) $CD \mid cd = AB \mid ab$.

Make an ABCD diagram:

AB	Ab	aB	ab	
	3	3		CD
1			2	Cd
1			2	cD
	4	4		cd

Fig. 413.

Now, if $AB = CD \mid cd$, then the combinations ABCd, ABCD, are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $ab = CD \mid cd$, then the combinations abCd, abcD, are inconsistent and we eliminate them by making a figure 2 in those sections.

Again, if $CD = AB \mid ab$, then the combinations AbCD, aBCD, are inconsistent and we eliminate them by making a figure 3 in those sections.

Again, if $cd = AB \mid ab$, then the combinations Abcd, aBed, are inconsistent and we eliminate them by making a figure 4 in those sections.

We can now get among others, the following definitions:

- (1) $ABC = D$
- (2) $ABc = d$
- (3) $AbC = d$
- (4) $Abc = D$
- (5) $aBC = d$
- (6) $aBc = D$
- (7) $abC = D$
- (8) $abc = d$.

These eight propositions taken together are the equivalents of the given pair of propositions.

Make an ABCD diagram:

AB	Ab	aB	ab	
	3	5		CD
1			7	Cd
2			8	cD
	4	6		cd

Fig. 414.

Now, if $ABC = D$, then the combination $ABCd$, is inconsistent and we eliminate it by making a figure 1 in that section.

Again, if $ABC = d$, then the combination $ABCd$, is inconsistent and we eliminate it by making a figure 2 in that section.

Again, if $AbC = d$, then the combination $AbCd$, is inconsistent and we eliminate it by making a figure 3 in that section.

Again, if $Abc = D$, then the combination $Abcd$, is inconsistent and we eliminate it by making a figure 4 in that section.

Again, if $aBC = d$, then the combination $aBCd$, is inconsistent and we eliminate it by making a figure 5 in that section.

Again, if $aBc = D$, then the combination $aBcd$, is inconsistent and we eliminate it by making a figure 6 in that section.

Again, if $abC = D$, then the combination $abCd$, is inconsistent and we eliminate it by making a figure 7 in that section.

Again, if $abc = d$, then the combination $abcd$, is inconsistent, and we eliminate it by making a figure 8 in that section.

The result proves that we have obtained an octave of propositions equal to the given pair of propositions.

924. Let the given propositions be:

- (1) $cd = AB \mid Ab \mid aB$
- (2) $ba = dC \mid Dc$
- (3) $No A = C$
- (4) $aC = bd$
- (5) $cD = ab$.

Find the equivalents.

Make an ABCD diagram:

AB	Ab	aB	ab	
3	3	4	4 2	CD
3	3	4		Cd
5	5	5		cD
			2 1	cd

Fig. 415.

Now, if $cd = AB \mid Ab \mid aB$, then the combination abcd is inconsistent and we eliminate it by making a figure 1 in that section.

Again, if $ba = dC \mid Dc$, then the combinations abCD, abcd, are inconsistent and we eliminate them by making a figure 2 in those sections.

Again, if $No A = C$, then the combinations containing AC are inconsistent and we eliminate them by making a figure 3 in those sections.

Again, if $aC = bd$, then the combinations containing aCD, aBCd, are inconsistent and we eliminate them by making a figure 4 in those sections.

Again, if $cD = ab$, then the combinations containing AcD, aBcD, are inconsistent and we eliminate them by making a figure 5 in those sections.

We can now get the following definitions:

- (1) $No C = D$
- (2) $abc = D$
- (3) $Cd = ab$
- (4) $Dc = ab$.

This quartet of propositions is equivalent to the set of given propositions.

Make an ABCD diagram:

AB	Ab	aB	ab	
1	1	1	1	CD
3	3	3		Cd
4	4	4		cD
			2	cd

Fig. 416.

Now, if $No\ C = D$, then the combinations containing CD are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $abc = D$, then the combination abcd is inconsistent and we eliminate it by making a figure 2 in that section.

Again, if $Cd = ab$, then the combinations containing ACd, aBCd, are inconsistent and we eliminate them by making a figure 3 in those sections.

Again, if $Dc = ab$, then the combinations containing AcD, aBcD, are inconsistent and we eliminate them by making a figure 4 in those sections.

Both of these sets of propositions are deductions from the tenth amendment to the Constitution of the United States. The result proves that we have found a set of propositions equivalent to the given set of propositions.

I think we have now given a sufficient number of examples to convince the reader that by this method we can find the equivalents for any set of complex propositions.

925. I consider this method for finding the equivalents for given propositions to be one of the most important logical discoveries which I have made.

By the use of the Reasoning Frame I have discovered another easy method of finding equivalent propositions for given universal categorical propositions, and propositions of the form of Hamilton's Y, i. e.,

Some A is all B.

926. When the proposition is in the form of All A is some B, we change the quality and quantity of B and make all b the subject of the new proposition. Then we change the quantity and quality of the subject All A, and make Some a, the predicate.

Thus, let the proposition be stated:

$$A = AB.$$

then its equivalent will be,

$$b = ba.$$

Make an AB diagram:

A	a	
		B
21		b

Fig. 417.

Now, if $A = AB$, then the combination Ab is inconsistent and we eliminate it by making a figure 1 in that section.

Again, if $b = ba$, then the combination Ab is inconsistent and we eliminate it by making a figure 2 in that section.

The result proves that the two propositions are equivalent.

927. Let the given proposition be:

Some A is all B,

which can be stated thus:

$$BA = B.$$

Then we change the quantity and quality of the predicate all B into some b, and take it for the subject.

Then we change the quantity and quality of the subject some A into all a, and take it for the predicate.

The proposition can be stated thus:

$$ab = a.$$

Make an AB diagram:

A	a	
	1 2	B
		b

Fig. 418.

Now, if $B = BA$, then the combination Ba is inconsistent, and we eliminate it by making a figure 1 in that section.

Again, if $a = ab$, then the combination Ba is inconsistent, and we eliminate it by making a figure 2 in that section.

The result proves the equivalence of the two propositions.

928. Let the given proposition be:

$$\text{All } A = \text{some } BC.$$

In this case we shall have to find two propositions which shall be together equivalent to the given proposition. The given proposition can be stated thus:

$$A = ABC.$$

Change the quantity and quality of B and of C and make b and c the subjects of the new propositions.

Change the quantity and quality of the subject A and make a the predicate of each new proposition.

The new propositions can be stated thus:

$$b = ba$$

$$c = ca.$$

Make an ABC diagram:

AB	Ab	aB	ab	
	2			C
	1			
1	2			
3	1			c
3	3			

Fig. 419.

Now, if $A = ABC$, then the combinations ABC , AbC , Abc , are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $b = ba$, then the combinations containing Ab are inconsistent and we eliminate them by making a figure 2 in those sections.

Again, if $c = ca$, then the combinations containing Ac are inconsistent and we eliminate them by making a figure 3 in those sections.

The result proves that the pair of propositions which we found are equivalent to the given proposition.

929. Let the given proposition be:

$$C = CAB,$$

The pair of equivalent propositions will be:

$$a = ac$$

$$b = bc.$$

Make an ABC diagram:

AB	Ab	aB	ab	
1	1		3	
3	2		1	C
			2	
				c

Fig. 420.

Now, if $C = CAB$, then the combinations containing CAb , Ca are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $a = ac$, then the combinations containing aC are inconsistent and we eliminate them by making a figure 2 in those sections.

Again, if $b = bc$, then the combinations containing bC are inconsistent and we eliminate them by making a figure 3 in those sections.

The result proves that we have found a pair of propositions equivalent to the given proposition.

930. Let the given proposition be:

All AB is some CD.

It can be stated thus:

$AB = ABCD$.

In this case the subjects of our two new propositions will be c and d, and the predicates will be the complete opposite of AB, which are cAb, | caB, | cab, and dAb | daB | dab.

The new propositions can be stated thus:

c = cAb | caB | cab

d = dAb | daB | dab.

Make an ABCD diagram:

AB	Ab	aB	ab	
				CD
3 1				Cd
2 1				cD
2 1 3				cd

Fig. 421.

Now, if $AB = ABCD$, then the combinations containing ABCd, ABC, are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $c = CAB | caB | cab$, then the combinations containing ABC are inconsistent and we eliminate them by making a figure 2 in those sections.

Again, if $d = dAb | daB | dab$, then the combinations containing Abd are inconsistent and we eliminate them by making a figure 3 in those sections.

The result proves that the pair of propositions which we have found are equivalent to the given proposition.

931. Let the given proposition be:

$AB = ABCDE$.

The equivalent propositions will be:

$$\begin{aligned}c &= cAb \mid caB \mid cab \\d &= dAb \mid daB \mid dab \\e &= eAb \mid eaB \mid eab.\end{aligned}$$

Make an ABCDE diagram:

AB	Ab	aB	ab	
				CDE
1 4				CDe
1 3				CdE
4 1 3				Cde
1 2				cDE
4 1 2				cDe
3 1 2				cdE
4 3 1 2				cde

Fig. 422.

Now, if $AB = ABCDE$, then all the combinations containing AB, excepting ABCDE, are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $c = cAb \mid caB \mid cab$, then the combinations containing ABC, are inconsistent and we eliminate them by making a figure 2 in those sections.

Again, if $d = dAb \mid daB \mid dab$, then the combinations containing ABD are inconsistent and we eliminate them by making a figure 3 in those sections.

Again, if $e = eAb \mid eaB \mid eab$, then the combinations containing ABe are inconsistent and we eliminate them by making a figure 4 in those sections.

The result proves that we have found a triplet of propositions equivalent to the given proposition.

CHAPTER XXXIV.

CONTRADICTORIES OF PROPOSITIONS.

932. By the use of the Reasoning Frame I have discovered an easy method of finding propositions contradictory to a given proposition.

It is as follows:

First, Obtain the visible expression of the given proposition in the Reasoning Frame.

Second, Observe what combinations it would be necessary to eliminate in order to produce the total elimination of a letter-term.

Third, Make a similar Reasoning Frame to the one first made, and eliminate in it the combinations which were necessary to cause the elimination of a letter-term in the first Reasoning Frame.

Fourth, From the uneliminated combinations in the second Reasoning Frame, get definitions which would cause the elimination of the eliminated combinations.

Let us take this example:

$$A = B \mid C = D,$$

and ascertain by the method above described, what contradictions to it we can find.

Make an ABCD diagram:

AB	A b	aB	ab	
1				CD
	1	1	1	Cd
	1			cD
	1			cd

Fig. 423.

Now, if $A = B$, except where $C = D$, and $C = D$, except where $A = B$, then the combinations containing ABCD, AbCd, ABC, aCd, are inconsistent and we eliminate them by making a figure 1 in those sections.

The Reasoning Frame now shows the visible expression of the proposition:

$$A = B \mid C = D.$$

By observation we now learn that if the combinations containing ABCd, ABC, AbCD were eliminated, then all the A's would be eliminated.

Make an ABCD diagram and eliminate the combinations containing ABCd, ABC, AbCD, by making a figure 1 in those sections.

Make an ABCD diagram:

AB	Ab	aB	ab	
	1			CD
1				Cd
1				eD
1				ed

Fig. 424.

From the uneliminated combinations we can get these definitions, and they will cause the elimination of the eliminated combinations.

- (1) $AB = ABCD$
- (2) $CD = AB \mid a.$

This pair of propositions is contradictory to the given proposition, because, with the given proposition it would cause the total elimination of the letter A.

Again, by observation of the Reasoning Frame, No. 423, we see that if the combinations containing ABCd, ABC, aBCD, aBc, were eliminated, then all the B's would be eliminated.

Make an ABCD diagram:

AB	Ab	aB	ab	
		2		CD
2				Cd
2		2		cD
2		2		cd

Fig. 425.

Eliminate the combinations containing ABCd, ABe, aBCD, aBe, by making a figure 2 in those sections.

From the uneliminated combinations we can get these definitions, and they will cause the elimination of the eliminated combinations:

- (1) $AB = ABCD$
- (2) $aB = aBCd.$

This pair of propositions is contradictory to the given proposition, because it would cause the total elimination of the letter B.

Again, by observation of the Reasoning Frame, No. 423, we can see that if the combinations containing ABCd, AbCD, aCD were eliminated, then all the C's would be eliminated.

Make an ABCD diagram:

AB	Ab	aB	ab	
	3	3	3	CD
3				Cd
				cD
				cd

Fig. 426.

Eliminate the combinations containing ABCd, AbCD, aCD, by making a figure 3 in those sections.

From the uneliminated combinations we can get these definitions and they will cause the elimination of the eliminated combinations,

$$(1) \text{ CD} = \text{CDAB}$$

$$(2) \text{ AB} = \text{ABCD} \mid \text{ABC}.$$

This pair of propositions is contradictory to the given proposition, because it would cause the elimination of the letter C.

By the same method we can learn that the following pair of propositions is contradictory to the given proposition, because it will cause the elimination of the letter D,

$$(1) \text{ AB} = \text{ABC} \mid \text{ABed}$$

$$(2) \text{ cD} = \text{cDAb},$$

Also the following proposition is contradictory to the given proposition, because it will cause the elimination of the letter a,

$$(1) \text{ a} = \text{aCd}.$$

Also the following pair of propositions is contradictory to the given proposition, because it will cause the elimination of the letter b,

$$(1) \text{ ab} = \text{abCd}$$

$$(2) \text{ CD} = \text{CDB}.$$

Also the following proposition is contradictory to the given proposition, because it will cause the elimination of the letter c,

$$(1) \text{ c} = \text{cAb}.$$

Also the following pair of propositions is contradictory to the given proposition, because it will cause the elimination of the letter d,

$$(1) \text{ cd} = \text{cdAb}$$

$$(2) \text{ AB} = \text{ABD}.$$

933. I have discovered another easy method of finding by our system the perfect contradictions of any given proposition.

The method is as follows:

(1) Eliminate the inconsistent combinations.

(2) Make a letter S in the eliminated combinations, for the purpose of indicating that those combinations are to be read.

(3) From these combinations which are to be read, get definitions according to the method hitherto pursued.

These definitions thus obtained, will eliminate all the combinations which are consistent with the given proposition, and will save all the combinations which are inconsistent with the given proposition. Thus, we will have found a set of propositions which will be complete contradictories to the given propositions.

934. Find a set of propositions which will be contradictories to the following propositions:

$$(1) A = AB$$

$$(2) D = DC$$

Make an ABCD diagram:

AB	Ab	aB	ab	
S				CD
1				
S				Cd
1				
S	S	S	S	cD
2	1	2	2	
S				cd
1				

Fig. 427.

Now, if $A = B$, then the combinations containing Ab are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $D = C$, then the combinations containing Dc are inconsistent and we eliminate them by making a figure 2 in those sections.

Make a letter S in the eliminated combinations.

From the eliminated combinations we can get among others, these definitions:

$$(1) AB = cD$$

$$(2) aB = cD$$

$$(3) ab = cD.$$

This set of definitions will eliminate what the given pair of

propositions saved and will save what the given pair of propositions eliminated.

Make an ABCD diagram:

AB	Ab	aB	ab	
1		2	3	CD
1		2	3	Cd
				cD
1		2	3	cd

Fig. 428.

Now, if $AB = cD$, then the combinations containing ABC, ABcd, are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $aB = cD$, then the combinations containing aBC, aBcd, are inconsistent and we eliminate them by making a figure 2 in those sections.

Again, if $ab = cD$, then the combinations containing abC, abed, are inconsistent and we eliminate them by making a figure 3 in those sections.

The result proves that we have found a set of propositions completely contradictory to the given propositions.

935. Let the given propositions be:

$$(1) AB = CD$$

$$(2) CD = AB.$$

Make an ABCD diagram:

AB	Ab	aB	ab	
	S 2	S 2	S 2	CD
S 1				Cd
S 1				cD
S 1				cd

Fig. 429.

Now, if $AB = CD$, then the combinations containing ABC , $ABCd$, are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $CD = AB$, then the combinations containing aCD , $AbCD$, are inconsistent and we eliminate them by making a figure 2 in those sections.

Make a letter S in the eliminated combinations. From the eliminated combinations we can get the following definitions:

- (1) $Cd = ABCd$
- (2) $cD = ABcD$
- (3) $cd = ABcd$
- (4) $CD = Ab | a$.

This set of propositions is contradictory to the given pair of propositions.

Make an ABCD diagram:

AB	Ab	aB	ab	
4				CD
	1	1	1	Cd
	2	2	2	cD
	3	3	3	cd

Fig. 430.

Now, if $Cd = AB$, then the combinations containing aCd , $AbCd$, are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $cD = AB$, then the combinations containing acD , $AbcD$, are inconsistent and we eliminate them by making a figure 2 in those sections.

Again, if $cd = AB$, then the combinations containing acd , $Abcd$, are inconsistent and we eliminate them by making a figure 3 in those sections.

Again, if $CD = Ab | a$, then the combination $ABCD$ is inconsistent and we eliminate it by making a figure 4 in that section.

The result proves that we have found a set of propositions completely contradictory to the given propositions.

936. The propositions which we found in the preceding example, are contradictory to the given propositions, but they are not the only ones which can be found. We can find several different sets of propositions contradictory to the given propositions.

937. Let it be required to find a set of universal affirmative propositions which shall be contradictory to the following propositions.

- (1) $BC = A$, or $B = C \mid DE$
- (2) $B = C \mid DE$, or $BC = A$.

Make an ABCDE diagram:

AB	Ab	aB	ab	
		S 1		CDE
S 12				CDe
S 21				CdE
S 21				Cde
S 21				cDE
S 2		S 2		cDe
S 2		S 2		cdE
S 2		S 2		cde

Fig. 431.

Now, if $BC = A$, or $B = C \mid DE$, then the combinations containing ABCDe, ABCd, ABcDE, aBCD̄E, are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if $B = C \mid DE$, or $BC = A$, then the combinations containing ABCDe, ABCd, ABc, aBcDe, aBed, are inconsistent and we eliminate them by making a figure 2 in those sections.

Make a letter S in the eliminated combinations.

From the eliminated combinations we can now obtain the following definitions:

- (1) CDE = BaCDE
- (2) CDe = ABCDe
- (3) CdE = ABCdE
- (4) Cde = ABCde
- (5) cDE = ABeDE
- (6) cDe = BcDe
- (7) cdE = BedE
- (8) cde = Bcde.

This set of propositions will be completely contradictory to the given propositions.

Make an ABCDE diagram:

AB	Ab	aB	ab	
1	1		1	CDE
		2	2	CDe
		3	3	CdE
		4	4	Cde
		5	5	cDE
		6		cDe
		7		cdE
		8		cde

Fig. 432.

Now, if CDE = Ba, then the combinations containing ACDE, abCDE, are inconsistent and we eliminate them by making a figure 1 in those sections.

Again, if CDe = AB, then the combinations containing acDe, AbCDe, are inconsistent and we eliminate them by making a figure 2 in those sections.

Again, if $CdE = AB$, then the combinations containing $aCdE$, $AbCdE$, are inconsistent, and we eliminate them by making a figure 3 in those sections.

Again, if $Cde = AB$, then the combinations containing $aCde$, $AbCde$, are inconsistent, and we eliminate them by making a figure 4 in those sections.

Again, if $cDE = AB$, then the combinations containing $acDE$, $AbcDE$, are inconsistent, and we eliminate them by making a figure 5 in those sections.

Again, if $cDe = B$, then the combinations containing $bcDe$ are inconsistent, and we eliminate them by making a figure 6 in those sections.

Again, if $cdE = B$, then the combinations containing $bedE$ are inconsistent, and we eliminate them by making a figure 7 in those sections.

Again, if $cde = B$, then the combinations containing $bede$ are inconsistent, and we eliminate them by making a figure 8 in those sections.

The result proves that we have found an octave of propositions completely contradictory to the given pair of propositions.

938. Let it be required to find a set of propositions contradictory to the following propositions:

$$(1) ab = Cd \mid cD$$

$$(2) Cd \mid cD = ab$$

$$(3) C = d$$

Make an ABCD diagram:

AB	Ab	aB	ab	
S 3	S 3	S 3	S 13	CD
S 2	S 2	S 2		Cd
S 2	S 2	S 2		eD
			S 1	ed

Fig. 433.

Now, if $ab = Cd \mid cD$, then the combinations $abCD$, $abcd$, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $Cd \mid cD = ab$, then the combinations containing ACd , $aBCd$, AcD , $aBcD$, are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if $C = d$, then the combinations containing CD are inconsistent, and we eliminate them by making a figure 3 in those sections.

Make a letter S in the eliminated sections.

From the eliminated combinations we can now get the following definitions:

- (1) $A = C \mid cD$
- (2) $B = C \mid cD$
- (3) $Cd = A \mid aB$
- (4) $cD = A \mid aB$

This set of propositions is completely contradictory to the given set of propositions.

Make an ABCD diagram:

AB	Ab	aB	ab	
				CD
				3 Cd
				4 cD
2 1	1	2		cd

Fig. 434.

Now, if $A = C \mid cD$, then the combinations containing Acd are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $B = C \mid cD$, then the combinations containing Bcd are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if $Cd = A \mid aB$, then the combination $abCd$ is

inconsistent, and we eliminate it by making a figure 3 in that section.

Again, if $cD = A | aB$, then the combination $abcD$ is inconsistent, and we eliminate it by making a figure 4 in that section.

The result now proves that we have found a set of propositions completely contradictory to the set of given propositions. The given propositions in this case were the propositions representing the tenth amendment to the Constitution of the United States.

939. Let it be required to find a set of propositions which shall be contradictory to the following proposition:

All the combinations are $ABCD | ABCd | ABed | AbCD$,
 $| AbcD | aBCD | aBeD | aBcd | abCd$.

Make an $ABCD$ diagram:

AB	Ab	aB	ab	
			S	CD
	S	S		Cd
S			S	cD
	S		S	cd

Fig. 435.

Now, if all the combinations are $ABCD | ABCd | ABed$
 $| AbCD | AbcD | aBCD | aBeD | aBcd | abCd$, then the combinations $ABeD$, $AbCd$, $Abcd$, $aBCd$, $abCD$, $abcD$, $abcd$, are inconsistent, and we eliminate them by making a letter S in those sections.

Make a letter S in the eliminated sections.

From the eliminated combinations we can get, among others, the following definitions:

- (1) $CD = abCD$
- (2) $AB = cDAB$
- (3) $Ab = dAb$

$$(4) \text{ aB} = \text{CdaB}$$

$$(5) \text{ abC} = \text{DabC}$$

This set of propositions is completely contradictory to the given proposition.

Make an ABCD diagram:

AB	Ab	aB	ab	
2 1	1 3	1 4		CD
2			5	Cd
	3	4		cD
2		4		cd

Fig. 436.

Now, if $CD = ab$, then the combinations containing ACD, ABCD, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $AB = cD$, then the combinations containing ABC, ABed, are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if $Ab = d$, then the combinations containing AbD are inconsistent, and we eliminate them by making a figure 3 in those sections.

Again, if $aB = Cd$, then the combinations containing aBe, aBCD, are inconsistent, and we eliminate them by making a figure 4 in those sections.

Again, if $abC = D$, then the combination abCd is inconsistent, and we eliminate it by making a figure 5 in that section.

The result proves that we have found a set of affirmative propositions completely contradictory to the given proposition.

940. Let it be required to find a set of propositions completely contradictory to the proposition,

No combinations are BA | Ca | D

Make an ABCD diagram:

AB	Ab	aB	ab	
	S			CD
S		S	S	Cd
	S	S	S	cD
S				cd

Fig. 437.

Now, if No combinations are BA | Ca | D, then the combinations containing ABd, AbD, aBCd, aBeD, abCd, abcD, are inconsistent, and we eliminate them by making a letter S in those sections.

Make a letter S in the eliminated sections.

From the eliminated combinations we can get the following definitions:

- (1) CD = AbCD
- (2) AB = dAB
- (3) Ab = DAB
- (4) cd = ABed

This set of propositions is completely contradictory to the given proposition.

Make an ABCD diagram:

AB	Ab	aB	ab	
1 2		1	1	CD
	3			Cd
2				cD
	4 3	4	4	cd

Fig. 438.

Now, if CD = Ab, then the combinations containing aCD,

ABCD, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $AB = d$, then the combinations containing ABD are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if $Ab = D$, then the combinations containing Abd are inconsistent, and we eliminate them by making a figure 3 in those sections.

Again, if $cd = AB$, then the combinations containing acd; Abed, are inconsistent, and we eliminate them by making a figure 4 in those sections.

The result proves that we have found a set of propositions completely contradictory to the given proposition.

941. Let it be required to find a set of propositions completely contradictory to the following proposition:

$$(1) \quad a = BCD$$

Make an ABCD-diagram:

AB	Ab	aB	ab	
			S 1	CD
		S 1	S 1	Cd
		S 1	S 1	cD
		S 1	S 1	cd

Fig. 439.

Now, if $a = BCD$, then all the combinations containing a, excepting aBCD, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Make a letter S in the eliminated sections.

From the eliminated combinations we can get the following definitions:

- (1) $CD = abCD$
- (2) $Cd = aCd$
- (3) $cD = acD$
- (4) $cd = acd$

This set of propositions is completely contradictory to the given proposition.

Make an ABCD diagram:

AB	Ab	aB	ab	
1	1	1		CD
2	2			Cd
3	3			cD
4	4			cd

Fig. 440.

Now, if $CD = ab$, then the combinations containing $\bar{A}CD$, $aBCD$, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $Cd = a$, then the combinations containing ACd are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if $cD = a$, then the combinations containing Acd are inconsistent, and we eliminate them by making a figure 3 in those sections.

Again, if $cd = a$, then the combinations containing Acd are inconsistent, and we eliminate them by making a figure 4 in those sections.

In this case, the set of propositions completely contradictory to the given proposition are inconsistent, because the letter A is eliminated.

942. Let it be required to find a set of propositions contradictory to the proposition,

$$A = Bc \mid bC$$

Make an ABC diagram:

AB	Ab	aB	ab	
S 1				C
	S 1			c

Fig. 441.

Now, if $A = Bc \mid bC$, then the combinations ABC, Abc, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Make a letter S in the eliminated sections.

From the eliminated combinations we can get the following definitions:

$$(1) \ C = ABC$$

$$(2) \ c = Abc$$

This pair of propositions is completely contradictory to the given proposition.

Make an ABC diagram:

AB	Ab	aB	ab	
	1	1	1	C
		2	2	c

Fig. 442.

Now, if $C = AB$, then the combinations containing aC, AbC, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $c = Ab$, then the combinations containing ac, ABC, are inconsistent, and we eliminate them by making a figure 2 in those sections.

The result proves that we have found a pair of propositions completely contradictory to the given proposition. The pair of propositions which we have found are contradictions because the letter a is eliminated.

943. Let the given propositions be,

$$Ac \mid aC = Ad \mid aD$$

$$Ad \mid aD = Ac \mid aC$$

Let it be required to find,

- (1) A set of propositions contradictory to the given propositions.

Make an ABCD diagram:

AB	Ab	aB	ab	
				CD
S 3	S 3	S 2	S 2	Cd
S 1	S 1	S 4	S 4	cD
				cd

Fig. 443.

Now, if $Ac = Ad$, then the combinations containing AcD are inconsistent, and we eliminate them by making a figure 1 in those sections.

The reader will observe that I pay no attention, in this connection, to the alternant aD . It would be impossible for A to equal a, hence, in a case of this kind, the alternant aD is superfluous.

Again, if $aC = aD$, then the combinations containing aCd are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if $Ad = Ac$, then the combinations containing ACd are inconsistent, and we eliminate them by making a figure 3 in those sections.

Again, if $aD = aC$, then the combinations containing acd are inconsistent, and we eliminate them by making a figure 4 in those sections.

The Reasoning Frame now shows the logical expression of the combination of the given propositions.

We can read the results thus:

$$(1) c = d, d = c$$

$$(2) C = D, D = C$$

Make a letter S in the eliminated combinations.

The contradictories are,

$$(1) C = d$$

$$(2) c = D$$

Make an ABCD diagram:

AB	Ab	aB	ab	
1	1	1	1	CD
				Ca
				cD
1	1	1	1	cd

Fig. 444.

Now, if $C = d$, then the combinations containing CD are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $c = D$, then the combinations containing cd are inconsistent, and we eliminate them by making a figure 2 in those sections.

The result proves that we have found a pair of propositions contradictory to the given propositions.

944. When we have worked an example out in the Reasoning Frame, we can always frame an affirmative proposition equivalent to the given propositions, by taking every combination for the subject, and all the uneliminated combinations in the alternative form for the predicate.

We can also get a single proposition equivalent to the given propositions, which shall have no combination for its subject, and all the eliminated combinations in the alternative form for the predicate.

We can also get a single proposition which shall be contradictory to the given proposition, by framing a proposition which shall have every combination for its subject and all the eliminated combinations in the alternative form for its predicate.

We can also get a single proposition contradictory to the given proposition or propositions which shall have no combination for its subject and the uneliminated combinations in the alternative form for its predicate.

945. Let us take the tenth amendment to the Constitution of the United States for an example:

It reads,

"The powers not delegated to the United States by the Constitution, nor prohibited by it to the states, are reserved to the states respectively, or to the people."

Let $a =$ powers not delegated to the United States,

$b =$ powers not prohibited to the states,

$C =$ powers reserved to the states,

$D =$ powers reserved to the people.

The propositions can be stated thus:

$$(1) ab = Cd \mid cD$$

$$(2) Cd \mid cD = ab$$

$$(3) \text{No } C = D$$

If any one should doubt the second proposition, i. e., $Cd \mid cD = ab$, it can be easily proved by using the Law of the Excluded Middle, for $Cd \mid cD$ must $= ab$, or $Cd \mid cD$ must $= AB \mid Ab \mid aB$.

Now if $Cd \mid cD = AB \mid Ab \mid aB$, then, since $ab = Cd \mid cD$, $ab =$ either $AB \mid Ab \mid aB$. But this is impossible by the Law of Contradiction, which says that a thing cannot both be and not be at the same time, hence it follows that $Cd \mid cD = ab$.

Make an ABCD diagram:

AB	Ab	aB	ab	
3	3	3	3 1	CD
2	2	2		Cd
2	2	2		cD
			1	cd

Fig. 445.

Now, if $ab = Cd \mid cD$, then the combinations $abCD$, $abcd$, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $Cd \mid cD = ab$, then the combinations containing ACd , $aBCd$, AcD , $aBcD$, are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if $No C = D$, then the combinations containing CD are inconsistent, and we eliminate them by making a figure 3 in those sections.

The Reasoning Frame now shows the logical expression of the combination of the given propositions.

We can now read in the Reasoning Frame the following propositions:

- (1) Every combination is $ABcd \mid Abcd \mid aBcd \mid abCd \mid abcD$, which can be translated,

By the Constitution all powers are either delegated to the United states, prohibited to the states, not reserved to the states and not reserved to the people, or,
 delegated to the United States, not prohibited to the states, not reserved to the states, and not reserved to the people, or,
 not delegated to the United States, prohibited to the states, not reserved to the states and not reserved to the people, or,
 not delegated to the United States, not prohibited to the states, reserved to the states, and not reserved to the people, or,
 not delegated to the United States, not prohibited to the states, not reserved to the states, but reserved to the people.

(2) No combination is, AC | AcD | aBC | aBcD | abCD
| abcd, which can be translated thus:

By the Constitution no power is delegated to the United States and reserved to the states, or,
delegated to the United States and reserved to the people, or,
prohibited to the states, and reserved to the states, or,
prohibited to the states and reserved to the people, or,
reserved to the states and reserved to the people, or,
not delegated to the United States, not reserved to the states,
not prohibited to the States and not reserved to the people.

(3) Every combination is AC | AcD | aBC | aBcD | abCD
| abcd.

This is contradictory to the given propositions.

It can be translated thus:

Every power is delegated to the United States and reserved to the States; or,
delegated to the United States and reserved to the people, or,
prohibited to the states and reserved to the states, or,
prohibited to the states and reserved to the people, or,
reserved to the states and reserved to the people, or,
not delegated to the United States, not prohibited to the states,
not reserved to the states, and not reserved to the people.

(4) No combination is ABcd | Abcd | aBcd | abCd | abcD.

This is contradictory to the given propositions.

It can be translated thus:

By the Constitution no power is delegated to the United States, not reserved to the States, and not reserved to the people, or,
not delegated to the United States, prohibited to the states, not reserved to the states, and not reserved to the people, or,
not delegated to the United States, not prohibited to the states, reserved to the states, and not reserved to the people, or,
not delegated to the United States, not prohibited to the states, not reserved to the states, and reserved to the people.

It will be seen that the tenth amendment to the Constitution does not recognize the doctrine of concurrent powers.

I think that this method of finding the complete contradictionies of given propositions is an important and useful discovery.

946. The reader will now have learned that there is a great difference between our system and the old logic. We make an exhaustive representation of all the possible propositions which can be made out of the terms used in the given propositions and then ascertain what propositions are inconsistent to any given proposition, and proceed to eliminate the combinations representing the inconsistent propositions, by making a figure against them in the diagram.

The first proposition will eliminate a certain number of combinations. These eliminations are complete and final, so far as they go.

The next proposition will eliminate more combinations and thus we go on until all the given propositions have had their say and the result shows the survivors.

947. Eulerian circles will answer for the simple cases used in syllogisms, but they are quite useless when we come to work with complex propositions involving five or six terms.

While experimenting with Eulerian circles, in solving logical problems, I discovered the method of squares. At that time I had no idea that any one else had ever thought of squares as a means of solving logical problems.

948. Every logician has recognized the fact that an affirmative proposition can be put into a negative form, but in our system, the negative terms are on a par with affirmative ones, and are as uniformly developed and used as are affirmative ones.

Affirmative terms, with us, have no special privileges. In working our examples we ask one question, What combinations are inconsistent with the given proposition?

949. The common logic talks of "the conclusion," as if there were but one conclusion.

Our system shows that there are many conclusions.

Our conclusions are so many various modes of expression. The same conclusion substantially, is expressed in a great many different forms.

950. The old system is limited to three terms.

It makes little difference to us how many terms there are. The more propositions we have on a given subject, the more able we are to get rid of all ambiguities and to make explicit in language everything that is implied in the thought.

951. Our rules, like the rules of practical arithmetic, do away with the tediousness and uncertainty of mental calculations made with great labor and by which different persons arrive at contradictory results without any one of them being able to show how the others have made a mistake.

952. Our system cuts off all debate and brings the parties at once to either admit or deny our fundamental principles.

953. I also frequently use the following method to get the complete contradictory of a very complex proposition.

First, make the proper diagram on common paper and eliminate the inconsistent combinations so as to get the visible expression of the given proposition.

Second, make a similar diagram on a piece of tracing paper. Place the diagram on the tracing paper, over the diagram on the common paper, and in the diagram on the tracing paper make a figure 1 in the combinations which are uneliminated in the diagram on the common paper. We now have in the diagram on the tracing paper, the visible expression of the complete contradictory of the given proposition.

By reading the uneliminated combinations in the contradictory of the given proposition, we can get all the propositions which are inconsistent with the given proposition.

CHAPTER XXXV.

FALLACIES.

954. A fallacy is a false or inconclusive reasoning. There are two kinds of fallacies. A fallacy is termed formal, when it is in the form of expression. When the proposition is not true, it is called a material fallacy. Logic, really, has nothing to do with material fallacies.

955. Examples of fallacies are:

(1) Money is wealth,

Corn is not money, therefore,

Corn is not wealth.

(2) Every tree is a vegetable,

Grass is not a tree, therefore,

Grass is not a vegetable.

(3) Horses are animals,

Sheep are not horses, therefore,

Sheep are not animals.

As these examples are similar we will simply work out the first one:

Let $A = \text{money}$,

$B = \text{wealth}$,

$C = \text{corn}$.

The propositions can be stated thus:

(1) $A = AB$

(2) $C = Ca$

Make an ABC diagram:

AB	Ab	aB	ab	
2	2 1			C
	1			c

Fig. 446.

Now, if $A = AB$, then the combinations containing Ab are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $C = Ca$, then the combinations containing CA are inconsistent, and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that the definition of C is:

$C = aB \mid ab$, which can be translated,

Corn is not money and it is wealth or not wealth.

956. The fallacy of equivocation is where the middle term is used in two different senses. An example is,

Repentance is a good thing.

Wicked men abound in repentance, therefore,

They abound in what is good.

In the first proposition repentence means genuine sorrow; in the second it means regret arising from pain or loss.

The premises should be stated as follows:

$$A = AB$$

$$C = CD$$

Make an ABCD diagram:

AB	Ab	aB	ab	
	1			CD
2	2 1	2	2	Cd
	1			cD
	1			cd

Fig. 447.

Now, if $A = AB$, then the combinations containing Ab are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $C = CD$, then the combinations containing Cd are inconsistent, and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that the definition of C is,

$$C = ABD \mid aBD \mid abD,$$

which can be translated,

Wicked men abound in regret, and they do or do not
abound in genuine sorrow, and they do or do not
abound in a good thing.

957. The fallacy of reasoning in a circle is where a person pretends to prove the truth of a proposition by asserting the truth of the conclusion. It is like saying A is B because it is B .

Whately gives this example:

“To allow every man an unbounded freedom of speech must always be, on the whole, advantageous to the state; for it is highly conducive to the interests of the community that each individual should enjoy liberty, perfectly unlimited, of expressing his sentiments.”

This is rant, not reasoning.

958. The fallacy of *Petitio Principii*, or begging the ques-

tion, is where a person reasons on a supposition which is not proved or granted.

959. The fallacy of self-contradiction is where arguments are advanced which contradict themselves.

An example is: "There are three points in this case," said the defendant's counsel; "in the first place we contend that the kettle was cracked when we borrowed it; secondly, that it was whole when we returned it; and, thirdly, that we never had it at all."

960. The fallacy of *Ignoratio Elenchi*, or irrelevant conclusion, is when a conclusion is substituted for the one which ought to have been proved. An example is:

The fine arts please the imagination and adorn and polish life.

But the fine arts are the parents of luxury, therefore,
the fine arts are a frivolous amusement.

961. The fallacy of the *Argumentum ad Hominem* is where a reference is made to something in the condition of the person who is addressed, to prove the truth of the argument. This argument is fair when it is applied solely to the principles of the person spoken to. Christ once used it with telling effect on the Pharisees. Luke's gospel, chap. 13, v. 5. The Pharisees pretended to be scandalized because Christ did works of mercy on the Sabbath. He said to them: "Which of you shall have an ass or an ox fallen into a pit, and will not straightway pull him out on the Sabbath day?"

962. The fallacy of Confusion of Ideas is where a person gets mixed up and perplexed in his reasoning.

A tricky man went into the shop of a rather simple-minded woman and asked for a penny loaf and a penny glass of gin. The articles being given, he drank the gin and addressed the woman as follows:

"On second thoughts, I will not take the bread; therefore, I just give it back in payment of the gin." The woman, somewhat perplexed, answered: "But you did not pay me for the bread." "Well," said the man, "I have not taken it." "But

“where is the payment for the gin?” “My good woman,” replied the man, “haven’t I told you already that I have given you back the penny loaf for it?”

This piece of sophistry so confused the ideas of the poor woman that she allowed the villain to depart.

A herring and a half for three half pence, how many for eleven pence? has perplexed many people at first sight.

963. The fallacy of *Suppressio Veri*, or the suppression of truth, is a common and dishonest way of reasoning. An example is, where a person was openly accused in an assembly of being concerned in appropriating the public money. Another rose to refute the calumny, and said that the accused was a most estimable individual. He was a good father and an exemplary husband, but not one word on the actual merits of the question, and by this sort of clap-trap appeal he prevented all inquiry as to the charge made.

The foregoing examples have been selected from Chambers’ “Information for the People.”

964. Let us take this example of a fallacy:

- (1) You are not what I am
- (2) I am a man, therefore,
You are not a man.

Let A = you

B = I

C = man

The premises can be stated thus:

- (1) A = Ab
- (2) B = BC

Make an ABC diagram:

AB	Ab	aB	ab	
1				C
1 2		2		c

Fig. 448.

Now, if $A = Ab$, then the combinations containing AB are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $B = BC$, then the combinations containing Bc are inconsistent, and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that the definition of A is,

$$A = bC \mid bc,$$

which can be translated thus:

You are a man or not a man.

965. Let us take this example of a fallacy:

- (1) Italy is a Catholic country and abounds in beggars,
France is also a Catholic country; therefore,
France abounds in beggars.

Let $A = \text{Italy}$

$B = \text{Catholic country}$

$C = \text{abounds in beggars}$

$D = \text{France}$

The premises can be stated as follows:

- (1) $A = ABC$
- (2) $D = DB$
- (3) No $A = D$

I assume that Italy and France are different countries.

Make an ABCD diagram:

AB	Ab	aB	ab	
3	3 1 2		2	CD
	1			Cd
3 1	3 1 2		2	cD
1	1			cd

Fig. 449.

Now, if $A = ABC$, then the combinations containing ABC , Ab , are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $D = DB$, then the combinations containing Db are inconsistent, and we eliminate them by making a figure 2 in those sections.

Again, if $No\ A = D$, then the combinations containing AD are inconsistent, and we eliminate them by making a figure 3 in those sections.

The Reasoning Frame now shows that the definition of D is,

$$D = aBC \mid abC,$$

which can be translated,

France is not Italy and it is a Catholic country, and it abounds or it does not abound in beggars.

These examples demonstrate that if we know how to state propositions correctly, our system will always enable us to detect fallacies.

966. Let us take this example:

Two and three are even and odd,
 Five are two and three, therefore,
 Five are even and odd.

These premises really mean,

- (1) Two and three taken separately are even and odd.
- (2) Five is two and three taken together.

Let A = two and three taken separately,
 B = even and odd,
 C = five,
 D = two and three taken together.

The premises may be stated thus:

- (1) A = AB
- (2) C = CD

Make an ABCD diagram:

AB	Ab	aB	ab	
	1			CD
2	2 1	2	2	Cd
	1			cD
	1			cd

Fig. 450.

Now, if A = AB, then the combinations containing Ab are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if C = CD, then the combinations containing Cd are inconsistent, and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows us that the definition of C is,

$$C = D (A \mid a) (B \mid b)$$

which can be translated,

Five is equal to two and three taken together

967. Let us take this example:

All the musical instruments of the Jewish temple made a noble concert.

The harp was a musical instrument of the Jewish temple, therefore,

The harp made a noble concert.

In the first premise the word "all" is collective, and not uni-

versal. It does not mean each and every. The subject of the premise is,

All the musical instruments of the Jewish temple.

Let A = all the musical instruments, etc.,

B = a noble concert,

C = harp,

D = a musical instrument, etc.

The premises can be stated thus:

$$(1) \text{A} = \text{AB}$$

$$(2) \text{C} = \text{CD}$$

The symbolic conclusion in this case will be similar to the one in the preceding case,

$$\text{C} = \text{D} (\text{A} | \text{a}) (\text{B} | \text{b})$$

and it can be translated thus:

The harp was a musical instrument of the Jewish temple, and made or did not make a noble concert.

968. Let us take this example:

All animals were in Noah's Ark, therefore,

No animals perished in the flood.

In this case the word "all" means every kind of, and refers to species. The word "no" refers to individual animals, hence, it is plain that the conclusion given is not warranted by the premise.

969. Let us take this example:

He that sends forth a book into the light, desires it to be read.

He that throws a book into the fire, sends it into the light, therefore,

He that throws a book into the fire desires it to be read.

(Watts' Logic, p. 322.)

In this case the word "light" in the major proposition means the public view of the world. In the minor proposition it signifies the brightness of the fire.

Let A = he that sends forth, etc.,

B = desires it to be read,

C = he that throws a book, etc.,

D = sends it into the light.

The propositions can be stated thus:

$$(1) A = AB$$

$$(2) C = CD$$

When this is worked out in the Reasoning Frame, we can then read,

He that throws a book into the fire sends it into the light, and desires or does not desire that it be read.

970. A similar example is,

He who thrusts a knife into another person should be punished.

A surgeon in operating does so, therefore,

He should be punished.

The major premise means,

He who thrusts a knife into another person maliciously.

The minor premise means,

A surgeon in operating does so without malice.

Hence, there are four terms, and the premises can be stated,

$$A = AB$$

$$C = CD$$

and then we can draw this conclusion, therefore,

He should be or should not be punished.

971. Dr. Bain in his work on Logic, makes some interesting remarks on fallacies.

He says, "A large class of fallacies consists in denying or suppressing the correlatives of an admitted fact. According to Relativity, the simplest affirmation has two sides; while complicated operations may involve unobvious correlates. Thus, the daily rotation of the starry sphere is either a real motion of the stars, the earth being at rest, or an apparent motion caused by the earth's rotation. Plato seems to have fallen into the confusion of supposing that both stars and earth moved concurrently, which would have the effect of making the stars, to appearance, stationary."

972. "Every mode of stating the doctrine of innate ideas,

commits or borders upon a Fallacy of Relativity, provided we accept the theory of Nominalism. A general notion is the affirmation of likeness among particular notions; it, therefore, subsists only in the particulars. It cannot precede them in the evolution of the mind; it cannot arise from a source apart, and then come into their embrace. A generality not embodied in particulars is a self-contradiction, unless on some form of Realism."

973. Kant's autonomy, or self-government of the will, is a fallacy of suppressed relative. No man is a law to himself; a law co-implicates a superior who gives the law and an inferior who obeys it; but the same person cannot both be ruler and subject in the same department."

974. "A fallacy of Relativity is pointed out, by Mr. Venn, in the doctrine of Fatalism; a doctrine implying that events, depending upon human agency, will yet be equally brought to pass, whether men try to oppose or try to forward them."

(Logic of Chance, p. 366.)

975. "Fallacies of Relativity often arise in the hyperboles of Rhetoric. In order to reconcile to their lot the more humble class of manual laborers, the rhetorician proclaims the dignity of all labor, without being conscious that if all labor is dignified, none is; dignity supposes inferior grades, a mountain height is abolished if all the surrounding plains are raised to the level of its highest peak.

So, in spurring men to industry and perseverance, examples of distinguished success are held up for universal imitation while, in fact, these cases owe their distinction to the general backwardness."

(Bain's Logic, pp. 621-22.)

976. The fallacy of the Irrelevant Question occurs when a person is decoyed into committing himself to a categorical answer—"Have you cast your horns?" If you answer, "I have," it is rejoined, "Then you have had horns;" if you answer, "I have not," it is rejoined, "Then you have them still?"

977. Another sophism is the fallacy of putting more questions than one as one.

An example is,

“Why did you strike your father?”

978. Prof. DeMorgan, in his work on Formal Logic, has an excellent chapter on Fallacies. He gives on page 242, the following examples of ambiguities:

Every dog runs on four legs,

Sirius (the dog star) is a dog, therefore,

Sirius runs on four legs.

Nothing is better than wisdom and virtue,

Dry bread is better than nothing, therefore,

Dry bread is better than wisdom and virtue.

A mouse eats cheese,

A mouse is one syllable, therefore,

One syllable eats cheese.

In these examples there are four terms. They can be stated thus:

$$A = AB$$

$$C = CD$$

and when worked out the conclusions will be,

Sirius runs or does not run on four legs.

Dry bread is better or not better than wisdom and virtue.

One syllable eats or does not eat cheese.

979. De Morgan gives this as the most difficult example:

To call you an animal is to speak truth,

To call you an ass is to call you an animal, therefore,

To call you an ass is to speak truth.

I think that these propositions mean,

You are an animal,

* An ass is an animal, therefore,

You are an ass.

Let A = you,

B = animal,

C = ass.

The propositions can be stated thus:

$$A = AB$$

$$C = CB$$

Make an ABC diagram:

AB	Ab	aB	ab	
1 2			2	C
1				c

Fig. 451.

Now, if $A = AB$, then the combinations containing Ab are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $C = CB$, then the combinations containing Cb are inconsistent, and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that the definition of A is:

$$A = BC \mid Be,$$

which can be translated,

You are or are not an ass.

980. DeMorgan says, on page 248, "It must be remembered that the word "all" in a proposition is not necessarily significative of a universal proposition; it may be a part of the description of the subject, thus, in

"All the peers are a House of Parliament," we do not use the words, "All the peers" in the same sense as when we say "All the peers derive their titles from the crown."

In the second case the subject of the proposition is "peer" and the term "all" is distributed, synonymous with each and every.

In the first case the subject is, "All the peers," and the term "all" is collective.

981. What you bought yesterday, you eat today. You bought raw meat yesterday, therefore, you eat raw meat today.

Let $A = \text{you}$,

$B = \text{bought}$,

$C = \text{what}$,

D = yesterday,

E = eat,

F = today,

G = raw meat.

The premises can be stated thus:

$$(1) \text{ CABD} = \text{ CAEF}$$

$$(2) \text{ A} = \text{ BDG}$$

Make an ABCDEFG diagram:

ABC	A _{BC}	A_BC	A _{bc}	a _{BC}	a _{Bc}	abC	abc	
		2	2					DEFG
2	2	2	2					DEFg
1		2	2					DEfG
2 1	2	2	2					DEfg
1		2	2					DeFG
2 1	2	2	2					DeFg
1		2	2					DefG
2 1	2	2	2					Defg
2	2	2	2					dEFG
2	2	2	2					dEFg
2	2	2	2					dEfG
2	2	2	2					dEfg
2	2	2	2					deFG
2	2	2	2					deFg
2	2	2	2					defG
2	2	2	2					defg

Fig. 452.

Now, if $CABD = CAEF$, then the combinations containing ABCDef, ABCDe, are inconsistent, and we eliminate them by making a figure 1 in those sections.

Again, if $A = BDG$, then all the combinations containing A excepting those containing ABDG, are inconsistent, and we eliminate them by making a figure 2 in those sections.

The Reasoning Frame now shows that the definition of A is:

$A = BCDEFG \mid BcDEFg \mid BcDEfG \mid BcDeFG \mid BcDefG$
which can be translated,

You eat or do not eat today or not today, raw meat.

982. Another fallacy may be called the fallacy of the Imperfect Dilemma.

DeMorgan gives this example:

A body must either be in the state A, or in the state B,
It cannot change in the state A, it cannot change in the
state B, therefore,

It cannot change at all.

Now, if the alternative A or B be necessary, the correct statement may be,

A body must be either in the state A, or in the state B,
or in the state of transition from one to the other.

983. Of this kind is the celebrated sophism of Diodorus Cronus, that motion is impossible, for all that a body does, it does either in the place in which it is, or in the place in which it is not and it cannot move in the place in which it is and certainly not in the place in which it is not."

This is an imperfect dilemma, because motion means transition from the place in which a body is, to the place in which it is not now but will be.

CHAPTER XXXVI.

UTILITY OF LOGIC.

984. The extravagant claims which were sometimes put forward by the advocates of the old logic, often ended in disappointment and brought the study of logic into disrepute. They represented logic as furnishing the only means for the discovery of all kinds of truth. Of course these pretensions were unfounded and the result was, that for a time the study of logic was generally abandoned.

985. I have already said that logic will not discover new facts, but our system will furnish a great many new meanings of old facts, and often this will be just as important to mankind as the discovery of a new fact.

986. Many good people think that in order to reason correctly it is only necessary to have common sense. Now, while common sense is an excellent quality and very necessary to a logician, it will not furnish him with those rules which are necessary to make a good logician.

987. It seems to me that the power to reason is the crowning glory of man and that it is of the utmost importance that he should learn to reason correctly. Every branch of learning has its necessary rules,—law, medicine, architecture, engineering, navigation, etc., etc., and a man might just as well expect to be a good lawyer, doctor, architect, engineer, navigator, etc., etc., without a knowledge of the technical rules of those subjects, as to be a good reasoner without learning the rules of logic, and just as the rules in those branches of science will tend to make a man a proficient and useful member of society in those professions, so will logic increase his usefulness in whatever profession he may engage.

988. The ability to reason correctly lies at the foundation of a knowledge of all the sciences, and the easier it is for a man to reason well, the easier will it be for him to master any branch

of learning. I believe that any ordinarily intelligent person can by our system become a first-class reasoner. If any one will master it thoroughly, he will be able to reason as correctly as the greatest logician who has ever lived.

989. This system will also afford a great deal of pleasure to those who learn it, by enabling them to combine different propositions and from the results of their combination get many new and hitherto unsuspected truths.

990. But its principal utility will lie in the fact that it will save a great deal of time and labor in bringing men who admit the premises to unanimous conclusions.

Where the principles are agreed on and the facts are admitted, it ought to put an end to all dispute. When it becomes generally known, I believe that it will be the means of bringing men to a substantial agreement on nearly all the disputed questions in law, theology, political economy, ethics and kindred sciences. Its usefulness to disputants and polemic writers is indisputable.

CHAPTER XXXVII.

PROBLEMS.

991. The reader is expected to find the categorical conclusions to the following problems:

- (1) All A is some B
- (2) All B is all C.
- 992. (1) No A is B
- (2) All B is some C.
- 993. (1) No A is B
- (2) No B is C.
- 994. (1) Some A is all B
- (2) All B is some C.
- 995. (1) All A is some not-B
- (2) Some not-B is all C.
- 996. (1) All A is some B
- (2) All not-C is some not-A.
- 997. (1) All not-A is some not-B
- (2) All not-B is some not-C.
- 998. (1) All A is some B
- (2) All C is some A.
- 999. (1) All A is some not-B
- (2) All C is some B.
- 1000. (1) All not-A is some B
- (2) All C is some B.
- 1001. (1) All not-A is some B
- (2) All not-B is some C.
- 1002. (1) All A is some not-B
- (2) All not-B is some C.
- 1003. (1) All not-A is some not-C
- (2) All not-B is some C.
- 1004. (1) All A is all B
- (2) All B is all A
- (3) All C is some D.
- 1005. (1) All AB is some C
- (2) All Ab is some D
- (3) All a is some BCD.
- 1006. (1) All A is some BCD
- (2) All a is some b
- (3) All c is some D.
- 1007. (1) All a is some BC
- (2) All C is some D

- (3) All D is some A.
 1008. (1) All a is some C
 (2) All b is some D
 (3) All D is some c.
 1009. (1) All a is some bcd
 (2) All A is some C
 (3) All Ab is some Cd.
 1010. (1) All AB is some CD
 (2) All Ab is some cd
 (3) All aB is some Cd
 (4) All ab is some cD
 1011. (1) All ABC is some D
 (2) All BCD is some e
 (3) All a is some bede
 (4) All Ab is some c.
 1012. (1) All A is some b
 (2) All a is some B
 (3) All D is some f
 (4) All dE is some F
 (5) All bC is some def.
 1013. (1) All ABCD is some ef
 (2) All ABCd is some ef
 (3) No A is b
 (4) No a is B
 (5) No D is ef
 (6) No b is EF
 (7) All a is all d
 (8) All Be is some D.
 1014. (1) If A is AB, then A = C
 (2) If C is CA, then C = B
 (3) If A is AC, then B is C.
 1015. (1) If A is Ab, then A is c
 (2) If c is ca, then c is B
 (3) If a is aB, then B is c
 (4) If a is aC, then a is b
 (5) If A is AB, then A is C
 (6) If C is Ca, then C is b.
 1016. (1) A or not-B is C
 (2) c is Ab.
 1017. (1) A or BC is D.
 1018. (1) C is a or b.
 1019. (1) ABC or aBc is D.
 1020. (1) A or B or C is D.
 (2) A or not-B or not-C is D
 (3) A is BC or D
 (4) AB or C is D.
 1021. A is B or C is D.

1022. C is B or A is not-D.
 1023. No A is Cd or c.
 1024. No A or B is C or D.
 1025. No A or B is CD or cd.
 1026. No AB or ab is C or D.
 1027. No A is B or C is D.
 1028. C is B or A is not-B.
 1029. A, B, or C is D.
 1030. A or B or C is D or E.
 1031. A is B or C is D.
 1032. B is C or A is not-D.
 1033. A or B is no C or D.
 1034. (1) A or B is B or C.
 (2) B or C is A or B.
 1035. (1) A or B is not-b or C.
 (2) b or C is A or B.
 1036. (1) A or B is A or C.
 (2) A or C is A or B.
 1037. (1) AB or CD is E
 (2) E is AB or CD.
 1038. (1) A is B or B is C
 (2) B is C or A is B.
 1039. (1) If A is B, E is F
 (2) If C is D, E is F
 (3) But A is B and C is D.
 1040. (1) A or B is C
 (2) C is A or B
 (3) A or B is D
 (4) D is A or B.
 1041. (1) A or not-C is B or not-D
 (2) B or not-D is A or not-C.
 1042. (1) A is not-D or B is D
 (2) B is D or A is not-D
 (3) C is not-D or A is B
 (4) A is B or C is not-D.
 1043. (1) AB is ABC or ABD or ABE
 (2) No A is BC or BD or BE.
 1044. Give the logical expression in the Reasoning Frame
 of the following propositions:

Some AB is all not-C

Some A is all not-B

Some A is not either B or C

All A is all B

Some A is all B

Some A is not any B

Not any A is B

Not some A is all B.

1045. Give the categorical premises which will produce the following conclusions:

ABc | Abc | aBc | abC.

1046. What categorical premises will produce the following conclusion?

ABcD | AbCd | aBCD | aBcd | abcD

1047. What categorical premises will produce the following conclusion:

Everything is ABCde | ABcde | AbCde | Abcde | aBCde
| aBcde | abCde | abcDE.

1048. The fact A is always to be found in company with the fact C and the fact D, but never in company with the fact B.

The facts C and D never occur together except in company with the fact A, what can we infer about the fact C?

1049. Wherever there is the circumstance A, there is also the circumstance B or the circumstance C. Wherever the circumstances A and C are both present, the circumstance B will also be present. The circumstance B never occurs without either the circumstance A or the circumstance C. What can you infer from the circumstance A, from circumstances where there is no A, from circumstances where there is no C?

1050. If the facts A and B are always found together, and if the facts D and E are never found together, and if the absence of A makes the presence of B, what relation is there between E and B?

1051. If A is B, A is CDe or CdE,
If A is bE, A is neither D nor C,
Ad is c
Ac is d,

what can we learn about B and E?

1052. (1) A is AB
(2) bA is bAC
(3) CA is CAD
(4) No EA is D or a

What can we learn about D and a?

1053. Four hunters went on a hunting trip. They were in camp seven days. A different party went hunting each day. B and D always went out together or stayed in together. Name the different parties that went out on seven different days.

APPENDIX.

HISTORICAL NOTES.

1054. Some of my readers may not have access to a library containing the principal works on logic, and for their benefit, I have compiled the following brief historical notes on logic and logicians, and a few extracts from works on logic.

For the first scientific treatment of logic, we are to look to the Greeks. Zeno of Elea is called the father of logic and dialectics, but it was then treated with particular reference to the art of disputation, and soon degenerated into sophistry. The Sophists and the Megarean school (founded by Euclid of Megara), greatly developed this art. The latter, therefore, became known under the name of the heuristic or dialectic school, and is famous for the invention of several sophisms."

(*Encyclopedia Americana*, vol. 8, p. 49.)

1055. "The first attempt to represent the forms of thinking, *in abstracto*, on a wide scale and in a purely scientific manner, was made by Aristotle. His logical writings were called, by later ages, *organon*, and for almost two thousand years after him, maintained authority in the schools of the philosophers. His investigations were directed, at the same time, to the criterion of truth, in which path Epicurus, Zeno, the founder of the Stoic school, Chrysippus, and others, followed him."

(Ib.)

1056. "With Epicurus, words were the signs of things, and not, as with the Stoics, of the ideas of things. One sought for the meanings of words, the other for a knowledge of things."

(*Studies in Logic*, p. 1.)

1057. "Perhaps the most interesting of these early thinkers, so far as the history of logic is concerned, is Antisthenes, whose extreme nominalism presents the most curious analogies to some recent logical work. According to Antisthenes * * * there is, therefore, no distinction of subject and predicate possible, even identical propositions, the only possible forms under this theory, are mere repetitions of the complex name. Predication is either impossible or reduces itself to naming in the predicate what is named in the subject. It is the simple result of so consistent a nominalism that all truth is arbitrary or relative; there is no possibility of contradiction, not even of one's self."

(*Encyclopedia Britannica*, Vol. 14, p. 786.)

1058. "In essence, the Stoic doctrine is identical with that of Antisthenes above noted, and it is interesting to observe that, under the purely nominalistic theory, logic became almost identical with the doctrine of expression or rhetoric."

(Ib. p. 791.)

1059. "Aristotle, one of the most celebrated philosophers of Greece, and the founder of the Peripatetic sect, was born at Stagyra, a town of Thrace, B. C. 384, being the son of Nicomachus, physician to Amyntas, King of Macedon. His parents dying during his childhood, he was brought up by Proxenus of Atarna in Mysia, and at the age of seventeen became the disciple of Plato, who used to call him 'the mind of his school.'"

(Gorton's Biographical Dictionary, v. I.)

1060. "Proclus (A. D. 409) endeavored to change the entire framework of human reason; but his logical views are so intimately blended with his theology, that we can scarcely separate them for a special notice. He cultivated the Greek logic, but founded upon it the Eastern ideas of illumination or intuition; and this led to almost impenetrable darkness and mysticism. The human mind, according to Proclus, may be viewed under two great categories,—identity and diversity."

(Blakey's Hist. Sketch of Logic, p. 102.)

1061. The analytical process, which formed such a conspicuous ingredient in the Socratic logic, was nothing more or less than an exhibition of that inward movement, which every man of sane mind, no matter what portion of acquired knowledge he may possess, carries on almost every moment of his life. Our minds are perpetually dividing the aggregate representations of things presented to its contemplation, whether of a physical or mental stamp, and resolving them, as it were, into their original or primary elements; and after this is effected, we sum them all up again, contemplate the representations as entire and perfect wholes or compound conceptions, and fix them as such in the mind. This mental process is so subtle and rapid, that we seldom can arrest the trains of thought which constitute it, a sufficient length of time to bring the faculty of attention to bear upon and observe them.

(Ib. p. 19.)

1062. "Sextus Empiricus (supposed to have flourished in the reign of the Emperor Commodus, A. D. 161). He taught: All categories such as genus and species are useless, one-sided, imperfect and often completely false. If we consider them as purely mental conceptions or controversies of the mind, how can we determine their relation to external things? For anything we know to the contrary, the mental instrument may have no real or true relation, whatever, to the thing on which it operates."

(Ib. p. 106.)

1063. "The schoolmen is a name given to the leaders of thought in the scholastic period. The most eminent were: Johannes Scotus Erigena, (died circ. 886.) Anselm, Archbishop of Canterbury, (1033-1109), William of Champeaux, (died 1121), Peter Lombard, (died 1164), Alexander of Halles, (died 1245), St. Bonaventure, (died 1274), Albertus Magnus (1193-1280), St. Thomas Aquinas (circ. 1225-74), Duns Scotus (died 1308), Buridan (died after 1350), and Johannes Gerson, who endeavored to combine mysticism with scholasticism (1363-1429)."

(Encyclopedic Dictionary.)

1064. "Another striking feature of the schoolmen is their incessant and pertinacious disputes on the nature of particular and universal ideas. This is one of the most conspicuous incidents in their history, and has served alike to hand down their fame to posterity and to make them, in the eyes of many, objects of commiseration and contempt."

For the sake of those who may not know the general merits of the question, we shall make a few explanatory observations upon it."

(Blakey's Hist. Sketch of Logic, p. 128.)

1065. "The point of dispute is simply this,—the Nominalists affirm that there are two classes of truths; one class relating to individual or single objects and their particular qualities or properties; the other class to general collections or assortments of things, which we designate by a general term or terms. A man is a particular idea; a multitude of men, a general idea. The Nominalists affirm that the difference between those two kinds of ideas is only a verbal one, i. e., when men talk or reason about these general ideas or attributes of things, the general term is the only thing with which the mind is conversant."

(Ib. p. 128.)

1066. "Now, the Realists denied this doctrine *in toto*. They maintained that though these general terms are used in our descriptions of the similar properties or qualities of things, yet, there is a general idea always present in the mind, when it thus characterizes the common attributes which belong to a particular genus or class. This general term is not a mere verbal instrument or symbol, but stands for a real permanent intellectual conception, which is always present to the mind and to which the name of general idea is uniformly given."

(Ib. p. 128.)

1067. "Some reasoners attempted to steer a middle course; they were called Conceptualists. They agreed with the Nominalists in denouncing general ideas or conceptions, as the Realists considered them to be; but they still thought the mind had the power of creating those general ideas, which they pre-

ferred to call conceptions. They said there were no essences or universal ideas to agree with general terms, and that the mind could reason about classes of individuals without the mediation of language."

(Ib. p. 129.)

1068. "It may be observed in passing, that the schoolmen must not be considered as the originators of this controversy about particular and universal ideas. We can trace it in the oldest records we have of logical philosophy. Plato, Aristotle, the Stoics and many other philosophers and sects, entered deeply into the entire question. They were all, however, unable to solve it, and it descended down to the schoolmen of the Middle Ages, with all its puzzling freshness and inherent mystery."

(Ib. p. 129.)

1069. "Logic or dialectics, enjoyed great esteem in later times, particularly in the Middle Ages, so that it was considered almost as the spring of all science, and was taught as a liberal art from the eighth century. The triumph of logic was the scholastic philosophy, which was but a new form of the ancient sophistry, and theology, particularly, became filled with verbal subtleties.

Raymond Lully strove to give logic another form. The scholastics were attacked by Campanella, Gassendi, Peter Ramus (Pierre de la Ramee), Bacon, and others, with well founded objections."

(Encyclopedia Americana, vol. 8, p. 49.)

1070. Roscellinus, A. D. 1089. "This scholastic was canon of Compeigne. He is commonly considered as the first writer who distinctly broached the Nominalist theory. He maintained that all general terms or names used in formal propositions, are but simple mental abstractions which the mind forms by comparing a certain number of individuals with each other. In fact, he went the full length of maintaining that universals were nothing but names.

This position appeared novel and startling to his age and hence it was that he drew upon himself ecclesiastical censure and rebuke.

Roscellinus was obliged to retract his opinions at the council of Soissons, held in the year 1092. He was afterwards banished both from England and France. The theological bearings of the logical questions were the real cause of his defeat and punishment. He taught, "Tres personas esse tres realitates differentes," a proposition, says his antagonist St. Anselm, that ought to warn every one how cautiously they should handle questions of Holy Writ."

(Blakey's Hist. Sketch of Logic, p. 134.)

1071. Pierre Abelard, A. D. 1142. "The name of Abelard is intimately connected with the early history of scholastic logic. He was a zealous Nominalist and jealously contended for the validity of his theory through every phase of his eventful life."

John of Salisbury says: "that Abelard and his disciples looked upon the proposition that we can affirm one thing from another thing, as a great absurdity, though this absurdity was backed by the authority of Aristotle."

(Ib. p. 140.)

1072. Raymond Lully, A. D. 1309.

"This was a zealous but eccentric logician. His life in connection with logical and philosophical studies is full of romantic interest. His *Ars Magna* is the exposition of a plan to enable the mind to work out all kinds of propositions through the means of a mechanical table of ideas, disposed in such a manner that their different correlations would furnish satisfactory answers to every imaginable sort of questions. A great deal of ingenuity is displayed in this logical scheme; and some degree of interest was at first excited in different schools of learning, as to its practical and successful application. But its barrenness and formality soon became apparent, and many of the scholastic doctors pronounced it as useless, and as little better than a severe satire upon the entire system of dialectic mechanism. During the life of Lully, and for nearly two centuries after his death, his opinions on logical science were pretty generally adopted in seminaries of learning, both in Majorca and in a part of Spain. Even in the colleges of Parma, Montpelier, Paris and Rome, he was cordially esteemed as a logician whose general views were both enlightened and highly favorable to sound religion and morality. His theory of reasoning was nearly, in all cases, however, adopted with some reservations; and he was admired more for his ingenuity than for soundness and comprehensiveness of judgment. The doctors of the Sorbonne protested against the system of Lully, although it was taught with great eclat at Toulouse by Raymond de Sebonde. Politian praises his method; and Leibnitz himself, thought his logical works a monument of genius and industry. He has been alike the object of ardent admiration and severe censure. Whilst it has been declared that the simple touch of his handkerchief frequently cured hundreds of the sick, yet the Church, at one time, pronounced himself and all his disciples as heretics, and Gregory IX placed his writings, by a formal bull, in the Index Expurgatorius. There seems to have been so much vitality in

his system as to maintain its remembrance for a considerable time after the death of its founder."

(Ib. p. 154.)

1073. William Occam, A. D., 1320. "Occam was a native of the county of Kent, studied at Merton College, Oxford, under the celebrated Duns Scotus, and was called the Invincible Doctor. The Realistic doctrines met with a bold and formidable opponent in Occam. He adopted a certain form of the Nominalists' theory. He maintained that general ideas could not have an existence independent of external things, and of the Deity * * * "Every substance," says he, "is numerically one and singular; it is itself, and no other. It is not the same with a universal. If the universal were a thing existing in a number of individual or particular things, it would then possess a distinct and independent existence; for everything which is superior to another thing, must, according to the established laws of God, be independent of that thing,—a consequence which leads to a gross absurdity in reference to universal notions." * * * The commentators and critics of Occam have been by no means agreed as to the precise nature of his own opinions. He is charged with arguing in the most decided manner against the Realists, stating the case of the Nominalists, and then leaving the question without offering his own opinions upon it. What these are, really seem to be that he could not go the whole length with the Nominalists' theory and that he was substantially what is denominated a Conceptualist."

(Ib. p. 156.)

1074. Peter Ramus, A. D. 1515. "The leading notion which seemed to have occupied the mind of Ramus relative to logic was, that all its formal rules should be pure transcripts of the laws of thought as these are displayed in the act of reasoning. Nothing should be admitted into any system that will not bear this test. He defined logic to be the art of discoursing correctly or justly; and the examples which he gives are chiefly taken from the ancient orators and poets."

(Ib. p. 172.)

1075. Thomas Hobbes, b. 1588, d. 1679. "The universality of one name to many things, has been the cause that men think the things are themselves universal, and so seriously contend that besides Peter and John and all the rest of the men that are, have been, or shall be, in the world, there is yet something else that we call man, viz.: man, in general, deceiving themselves by taking the universal or general appellation for the thing it signifieth." "Logic is," he says, "the art of computation." "Logicians add together two names to make an affirmation, and two affirmations to make a syllogism, and many

syllogisms to make a demonstration; and from the sum or conclusion of a syllogism they subtract one proposition to find another." "Reason is nothing but reckoning, i. e., adding and subtracting of the consequences of general names agreed upon, for the marking and signifying of our thoughts."

In the author's *Logica* we find the same doctrine maintained. "An universal," says he, "is not a name of many taken collectively, but of each thing taken separately. Man is not the name of the human family in general, but of each single member of it, as Peter, John, and the rest separately. Therefore, this universal name is not the name of anything existing in nature, or of any idea or fantasm formed in the mind, but remains so by some word or name. It thus happens that when an animal or a stone or a ghost, or anything else, is called universal, we are not to understand by this term that any man or stone, or anything else, was or is or can be an universal; but only that these terms, animal, stone, and the like, are universal names, i. e., names common to many things; and the ideas or conceptions corresponding to them in the intellect, are the images or fantasms of single animals or other things, and, consequently we do not need, in order to comprehend what is meant by an universal, any other faculty than that of imagination, by which we remember that such words have excited the ideas in our mind, sometimes of one particular thing, sometimes of another."

If speech be peculiar to man, as for aught I know it is, then is understanding peculiar to him also; understanding being nothing else but conception caused by speech. True and false are attributes of speech, not of things; where speech is not there is neither truth nor falsehood, though there may be error. Hence, as truth consists in the right ordering of names in our affirmations a man that seeks precise truth hath need to remember what every word he uses stands for and place it accordingly."

(Ib. p. 227.)

1076. Jacques Benigne Bossuet, born in 1627, died 1704. "He defines truth to be that which exists, and falsehood that which has no existence. Truth being eternal, it must of necessity rest upon Deity. All necessary truths and principles existed prior to the human understanding; and consequently we can only be said to find truths, not to create them."

(Ib. p. 268.)

1077. John Locke, born 1632, died 1704. "God has not been so sparing to men to make them barely two-legged creatures, and left it to Aristotle to make them rational, i. e., those few of them that he can get to examine the grounds of syllogisms, as to see, that in about three score ways that three

propositions may be laid together, there are but fourteen wherein one may be sure that the conclusion is right. God has been more bountiful to mankind. He has given them a mind that can reason without being instructed in the methods of syllogism."

(Ib. p. 276.)

1078. Christian Wolff, born 1679, died 1754. "It was a favorite opinion of Wolff's that all our reasonings could be greatly facilitated by having recourse to a uniform system of signs. He conceived that hieroglyphical emblems or figures might be so applied as to represent fully and forcibly all general notions and propositions."

(Ib. p. 288.)

1079. Gottf. Plouquet, born 1716, died 1790. "In his *Methodus Calculandi in Logicis*, and other works, labored hard to introduce new elements into the science of logic. His great aim was to reduce all human knowledge to one or two simple principles or rules, and to establish upon these a logical method which would mechanically, as it were, convey knowledge on every branch of science with infallible certainty and great expedition. Reasoning was to be reduced to its simple elements, and by means of algebraical signs rendered a matter of pure calculation.

Logic was only, according to Plouquet, the art of deducing by an immutable rule, the known from the unknown, and this is amply sufficient for the explanation of every department of human inquiry. He reduces all judgments on facts or experience to identical propositions, by the aid of the principle of sufficient reason."

(Ib. p. 293.)

1080. Denys Diderot, born 1713, died 1784. "In many parts of his philosophical writings Diderot reduces reasoning to a mere species of sensation. He says, 'Every idea must necessarily, when brought to its state of ultimate decomposition, resolve itself into a sensible representation or picture; and, since everything in our understanding has been introduced there by the channel of sensation, whatever proceeds out of the understanding is either chimerical or must be able in returning by the same road, to reattach itself to its sensible archetype. Hence, an important rule in philosophy, that every expression which cannot find an external and a sensible object to which it can thus establish its affinity, is destitute of signification.' Helvetius affirms, likewise, that all truths may be reduced to simple facts, or identical propositions, A = B. The reasoning process is nothing more, he says, than the development of this simple law of our intellectual existence."

(Ib. p. 316.)

1081. Etienne Bonnot de Condillac, born 1715, died 1780. "In Condillac's Logic he conceives that all reasoning may be ultimately resolved into the same form and certainty as mathematical evidence. The mode of accomplishing this would be to effect such improvements in language as to make it represent certain fixed and determined ideas. On the character of the logic of Condillac, the author of his life and writings in the last edition of the Encyclopedia Britannica, gives us the following opinion: he considers the justness of our reasonings as depending on the degree of perfection of the language we possess. The superior certainty of mathematical as compared with other knowledge, is ascribed by him to the superior certainty of mathematical language."

(Ib. p. 318.)

1082. George Campbell, born 1719, died 1796. "Dr. Campbell says the method of proving by syllogism appears, even on a superficial review, both unnatural and prolix. The rules laid down for distinguishing the conclusive from the inconclusive forms of argument, the true syllogism from the various kinds of sophism, are at once cumbersome to the memory and unnecessary in practice. No person, one may venture to pronounce, will ever be made a reasoner who stands in need of them. In a word, the whole bears the manifest indications of an artificial and ostentatious parade of learning, calculated for giving the appearance of great profundity to what, in fact, is very shallow. Such, I acknowledge, have been for a long time, my sentiments on the subject. On a mere inspection I cannot say I have found reason to alter them, though I think I have seen a little further into the nature of this disputative science, and, consequently, into the grounds of its futility."

(Ib. p. 348.)

1083. Lord Kames (Henry Homes) b. 1696, d. 1782.)

"Lord Kames was another Scottish writer who spoke lightly of the school logic. Speaking of reasoning in general, his lordship says, that all real knowledge of mankind may be divided into two parts, the first consisting of self-evident propositions; the second, of those which are deduced by just reasoning from self-evident propositions. The line which divides these two parts ought to be marked as distinctly as possible, and the principles that are self evident, reduced as far as can be done, to general axioms."

(Ib. p. 351.)

1084. L. H. Wagner, 1806.

"In his Logic, views the science of reasoning in a different

light from his contemporaries. His aim is to give a purely mathematical form to all logical rules, much after the same fashion as Lully and Bruno."

(Ib. p. 391.)

1085. George Wilhelm Frederick Hegel, 1816.

"In his *Wissenschaft der Logik*, denies that logic is merely expressive of the forms of thought; it constitutes its very essence and reality. Logic displays three different states or conditions. We simply consider and look at a thing. We then separate that thing from others, for nothing can exist in absolute unity; it must have two aspects and then out of these arises a certain relation which alone constitutes truth, reality being the absolute."

(Ib. p. 391.)

1086. "Ventura (1828) enters profoundly into the science of method. All logical methods proceed on the principle of analysis. The mind looks at an entire system or a large assemblage of general principles, and then seems to set about the work of analysis or separation with a view of realizing one general idea, which is known only to itself and which is often obtained by a mental process, which entirely eludes the most searching efforts of consciousness. Everything must be taken to pieces; every corner and crevice of the system must be examined before the several parts can be put together and adjusted agreeably to the scientific idea which we have in our own minds, and which we set out in our inquiries to establish and realize. These are the leading steps of the mental process in every philosophical method."

(Ib. p. 409.)

1087. "Sir William Hamilton succeeded Dr. Ritchie in 1836 in the Edinburg University. About four years after this, it is said that the Professor introduced what is termed his new analytic method of teaching formal logic. This method proceeds on a thoroughgoing quantification of the predicate. By the adoption of this principle we are told that past evils are corrected, past omissions supplied and logic receives its highest development in the perfection and simplicity of its form."

"The entire doctrine of the conversion of syllogisms is, (on the principle of this new analytic method of Sir William) pronounced to be useless and false. This inconsistent and cumbersome doctrine resulted, as we have said, from a false analysis by logicians of the elements with which they had to deal. The whole doctrine is founded upon the relation of quantity between the subject and predicate in a proposition; but if a principle element of that relation be left out, the doctrine will of course be defective. Logicians stand chargeable with this neglect. They commenced to recompose their system before,

by thorough decomposition, they had obtained all the elements requisite for that system."

(Ib. p. 445.)

1088. When Dr. Thomas Brown's lectures on the Philosophy of the Human Mind (1822) made their appearance, a new direction was given to mental science. "What is it which the syllogistic art would confer on us in addition? To shorten the process of arriving at truth, it forces us to use, in every case, three propositions instead of the two which nature directs us to use. Instead of allowing us to say: Man is fallible, he may therefore err, even when he thinks himself most secure from error,—which is the spontaneous order of analysis in reasoning,—the syllogistic art compels us to take a longer journey to the same conclusion by the use of what it calls a major proposition,—a proposition which never rises spontaneously, for the best of all reasons, that it cannot rise without knowledge of the very truth which is by supposition unknown. To proceed in the regular form of a syllogism, we must say, All things that are fallible may err, even when they think themselves most secure from error. But man is a fallible being, he may therefore err, even when he thinks himself most secure from error.

In our spontaneous reasonings, in which we arrive at precisely the same conclusions, and with a feeling of evidence precisely the same, there are, as I have said, no major propositions, but simply what in this futile art are termed technically the minor and the conclusion. The invention and formal statement of a major proposition then, in every case, serve only to retard the progress of discovery, not to quicken it or render it in the slightest degree more sure.

Again, he observes, the syllogism, therefore, which proceeds from the axiom to the demonstration of particulars, reverses completely the order of reasoning and begins with the conclusion, in order to teach us how we may arrive at it."

(Ib. p. 452.)

1089. John Stuart Mill, b. 1806, d. 1873.

"It must be granted," says Mr. Mill, "that in every syllogism considered as an argument to prove the conclusion, there is a *petitio principii*. When we say,—All men are mortal; Socrates is a man, therefore, Socrates is mortal; it is unanswerably urged by the adversaries of the syllogistic theory that the proposition, Socrates is mortal, is pre-supposed in the more general assumption, All men are mortal; that we cannot be assured of the mortality of all men, unless we were previously certain of the mortality of every individual man; that if it be still doubted whether Socrates or any other individual you choose to name be mortal or not, the same degree

of uncertainty must hang over the assertion, All men are mortal; that the general principle, instead of being given as evidence of the particular case, cannot itself be taken for true, without exception, until every shadow of doubt which could affect any case comprised within it is dispelled by evidence *aliunde*; and then what remains for the syllogism to prove?

That, in short, no reasonings from generals to particulars can, as such, prove anything, since from a general principle we cannot infer any particulars but those which the principle itself assumes as foreknown. This doctrine is irrefragable."

(Ib. p. 467.)

1090. George Boole, d. 1864.

"The time must come when the inevitable result of the admirable investigations of the late Dr. Boole must be recognized at their true value, and the plain and palpable form in which the machine presents those results will, I hope, hasten the time. Undoubtedly Boole's life marks an era in the science of human reason. It may seem strange that it had remained for him first to set forth in its full extent, the problem of logic, but I am not aware that anyone before him had treated logic as a symbolic method for evolving from any premises the description of any class whatsoever, as defined by these premises. In spite of several serious errors into which he fell, it will probably be allowed that Boole discovered the true and general form of logic, and put the science substantially into the form which it must hold for evermore. He thus effected a reform with which there is hardly anything comparable in the history of logic between his time and the remote age of Aristotle. Nevertheless, Boole's quasi-mathematical system could hardly be regarded as a final and unexceptionable solution of the problem. Not only did it require the manipulation of mathematical symbols in a very intricate and perplexing manner, but the results when obtained were devoid of demonstrative force because they turned upon the employment of unintelligible symbols, acquiring meaning only by analogy."

(Jevon's Principles of Science, p. 113.)

1091. Archbishop Richard Whately, b. 1789, d. 1863.

"In his work on Logic, pp. 82-86, makes some excellent remarks on probabilities. He says, 'But though when one premise is certain, and the other only probable, it is evident that the conclusion will be exactly as probable as the doubtful premise, and there is some liability to mistake in cases where each premise is merely probable. For, though most every one would perceive that in this case, the probability of the conclusion must be less than that of either premise, the precise degree in which its probability is diminished is not always so readily

apprehended. And yet this is a matter of exact and easy arithmetical calculation. I mean, that given the probability of each premise, we can readily calculate, and with perfect exactness, the probability of the conclusion. As for the probability of the premises themselves, that are put before us, that of course must depend on our knowledge of the subject-matter to which they relate. But supposing it agreed what the amount of probability is in each premise, then, we have only to state that probability in the form of a fraction, and to multiply the two fractions together, the product of which will give the degree of probability of the conclusion. Let the probability, for instance, of each premise be supposed the same, and let it in each be 2-3, (that is, let each premise be supposed to have two to one in its favor, that is, to be twice as likely to be true as to be false), then the probability of the conclusion will be two-thirds of two-thirds, 4-9;—rather less than one-half. For, since twice two is four, and thrice three nine, the fraction expressing the probability of the conclusion will be four-ninths. When you have two (or more) distinct arguments each separately establishing as probable the same conclusion, the mode of proceeding to compute the total probability is the reverse of that mentioned just above, for there, in the case of two probable premises, we consider what is the probability of their being both true, which is requisite in order that the conclusion may be established by them.

But in the case of a conclusion twice (or oftener) proved probable by separate arguments, if these indications of truth do not all of them fail, the conclusion is established. You consider, therefore, what is the probability of both of these indications of truth being combined in favor of any conclusion that is not true. Hence, the mode of computation is to state (as a fraction) the chances against the conclusion as proved by each argument, and to multiply these fractions together to ascertain the chances against the conclusion as resting on both the arguments combined, and this fraction being subtracted from unity, the remainder will be the probability for the conclusion. For instance, let the probability of a conclusion as established by a certain argument, be 4-9: (suppose that this man is the perpetrator of a certain murder, from stains of blood being found on his clothes), and again, of the same conclusion as established by another argument, 2-5: (suppose from the testimony of some witness of somewhat doubtful character), then, the chances against the conclusion in each case respectively, will be 5-9 and 3-5, which multiplied together give 15-45ths or 1-3 against the conclusion. The probability, therefore, for the conclusion as depending on these two argu-

ments jointly, (i. e., that he is guilty of the murder) will be 2-3, or two to one.

As for the degree of probability of each premise, that, as we have said, must depend on the subject matter before us, and it would be manifestly impossible to lay down any fixed rules for judging this. But it would be absurd to complain of the want of rules determining a point for which it is plain no precise rules can be given, or to disparage for that reason, such rules as can be given for the determining of another point.

Mathematical science will enable us, given one side of a triangle and the adjacent angles, to ascertain the other sides, and this is acknowledged to be something worth learning, although mathematics will not enable us to answer the question which is sometimes proposed in jest, of "How long is a rope?"

1092. In Brewster's Encyclopedia there is an article on Logic, from which I make the following quotations:

"The name of Aristotle is inseparably connected with the history of Dialectics. He claims the sole honor of the invention of syllogism; and his claims have been generally recognized; for, though the syllogistic mode of reasoning had been long known and much practiced before his time, yet, he had the merit of reducing to a system, those principles which, till then, had lain as a chaos of undigested materials. The utility of his labors, indeed, has not only been disputed in modern times, but they have been denounced as the chief cause of retardation in the progressive march of the human intellect; yet no one ever doubted the ingenuity which planned, and the industry which perfected a system which exerted a predominant influence over the human mind for many ages."

(B. E. XIII, p. 142.)

1093. "But the system, by whomsoever raised, is a stupendous monument to misapplied ingenuity. It owed its attraction to its high pretensions, for there is something very imposing in the idea of possessing a rule by which we may measure the pretensions of every doctrine and subject to the dominion of our own mind, every branch of human knowledge. We need not be surprised then, that Aristotle, the real or the supposed inventor of this art, should be regarded with veneration and considered as a benefactor to the human race.

It was long before mankind could be induced to doubt the infallibility of their guide, till at last his influence was shaken by experience of the inefficacy of his method for the discovery of truth, and by the splendid labors of a few, who dared to abandon the intellectual weapons of dialecticians and trust their fame to a humble investigation of the laws of nature."

(Ib. p. 142.)

1094. "We once witnessed an instance of this confusion of ideas in a court of justice. The judge thus questioned a witness—Did you hear A say that B had given him a blow which would be his death? The witness answered, No. Did you hear A say that B had not given him a blow? To this the witness also answered, No. The judge not perceiving that his evidence implied no contradiction, threatened to commit him to prison for perjury and equivocation."

(Ib. p. 145.)

1095. "Such has been the revolution of philosophical opinion, that syllogism, which for so many centuries had been considered as the grand bulwark of reasoning, is now almost universally exploded; and any man who should digest his arguments into a syllogistic form, or who should say a word in favor of the practice, would excite ridicule rather than produce conviction.

The authority of Bacon, of Locke or Reid, and of Stewart, (not to mention hundreds of their followers, who, under their protecting shield, have furiously attacked a doctrine which they never attempted to comprehend), would of itself, have been sufficient to shake the credit of a system to which the philosophical world had so long bowed with submission."

(Ib. p. 147.)

1096. Victor Cousin, b. 1792, d. 1867.

"Cousin wrote a very interesting work called "History of Mental Philosophy." From it I take the following selections:

"The inquiry that we are about to make in order to be methodically directed, should be divided into three points. First, it is necessary to state and enumerate in their integrity, the elements or essential ideas of reason. We must have them all and be sure, at the same time, that we suppose none and that we omit none; for, if we imagine a single one, a hypothetical element would lead us to hypothetical relations and thence to a hypothetical system. The first law of a wise method is then, a complete enumeration.

The second is an examination so profound, of all these elements, that it may result in their reduction and that we may finish by having in hand the determinate number of elements, simple and indecomposable, which form the boundary of analysis.

The third law of method is the examination of the different relations of these elements among themselves. I say the different relations, for these elements may sustain a great number of different relations. None must be supposed, nor must be neglected. It is when we shall have all these elements, when we shall have reduced them, when we shall have seized

upon all their relations, that we shall be in possession of the foundations of reason and of its history."

(V. C. Ment. Philos. v. I, 66-67.)

1097. "I speak to you of unity, you cannot avoid thinking of variety. When I speak to you of the infinite you cannot avoid conceiving the finite. We must not say, as is said by two great rival schools, that the human mind begins by unity and the infinite; or by the finite, the contingent and the multiple; for, if it begins by unity, I defy it ever to arrive at multiplicity; or, if it starts at multiplicity alone, I defy it equally to arrive ever at unity; if it starts from phenomena alone, it would not arrive at substance; if it starts from the idea of imperfection it would not arrive at perfection; and reciprocally.

The two fundamental ideas to which reason is reduced, are then, two contemporaneous ideas. The one supposes the other in the order of the acquisition of our knowledge. As then, we do not begin only by the senses and experience, and as we do not any more begin by abstract thought and by intelligence alone, so the human mind begins neither by unity nor by multiplicity; it begins and cannot avoid beginning by both; the one is the opposite of the other, a contrary implying its contrary; the one exists only on condition that the other exists at the same time. Such is the order of the acquisition of our knowledge. The order of nature is different."

(Ib. p. 70.)

1098. "Call to mind the conclusions of the last lecture. Reason, in whatever way it may occupy itself, can conceive nothing except under the condition of two ideas which preside over the exercise of its activity: idea of the unit and of the multiple, of the finite and of the infinite, of being and of appearing, of substance and of phenomenon, of the absolute cause and of secondary causes, of the absolute and of the relative, of the necessary and of the contingent, of immensity and of space, of eternity and of time, etc.

Analysis in bringing together all these propositions, in bringing together, for example, all their first terms, identifies them; it identifies equally, all the second terms; so that of all of these propositions, compared and combined, it forms a single proposition, a single formula, which is the formula itself, of thought and which you can express, according to the case, by unit and by the multiple, the absolute being and the relative being, unity and variety, etc.

Finally, the two terms of this formula, so comprehensive, do not constitute a dualism in which the first term is on one side, the second on the other, without any other relation than that of being perceived at the same time by reason."

(Ib. p. 73.)

1099. "Reason conceives a mathematical truth; can it change this conception as my will changed just now my resolution? Can it conceive that two and two do not make four? Try and you will not succeed; and not only in mathematics but in all the other spheres of reason, the same phenomenon takes place. In morals, try to conceive that the just is not obligatory; in art try to conceive that such or such a form is not beautiful; you will try it in vain, reason will always impose upon you the same conception. Reason does not modify itself according to our taste; you do not think as you wish."

(Ib. pp. 75-76.)

1100. "The necessary condition of intelligence is consciousness, that is difference."

(Ib. p. 77.)

1101. "At the bottom of every negation lies an affirmation."

(Ib. p. 81.)

1102. From Herbert Spencer's great work, "Principles of Psychology," vol. 2, I make the following quotations:

"The syllogism then, if taken to represent the form of the inferential act, has the fundamental fault that it fails to cover the whole of the ground it professes to cover. It falls short at both ends. There are simple deliverances of reason and complex deliverances of reason, both of them having the highest degree of certainty, which are entirely extra-syllogistic, cannot however, violently dislocated, be brought within the syllogistic form. Consequently, if it be admitted that a true expression of the ratiocinative act must be one applicable to all ratiocinative acts, it must be concluded that the ratiocinative act is not truly expressed by the syllogism."

(Ib. pp. 96-97.)

1103. "Ability to perceive equality implies a correlative ability to perceive inequality; neither can exist without the other. But, though inseparable in origin, the cognitions of equality and inequality, whether between things or relations, differ in this, that while the one is definite, the other is indefinite. There is but one equality, but there are numberless degrees of inequality. To assert an inequality involves the affirmation of no fact, but merely the denial of a fact; and, therefore, as positing nothing specific, the cognition of inequality can never be a premise to any conclusion."

(Ib. p. 26.)

1104. "Obviously, then, the process of thought formulated by the syllogism, is in various ways irreconcilable with the process of reasoning as normally conducted, irreconcilable as presenting the class, while yet there is nothing to account for its presentation; irreconcilable as predicated of that class a special attribute while yet there is nothing to account for

its being thought of in connection with that attribute; irreconcilable as embodying in the minor premise, an assertory judgment (this is a man), while the previous reference to the class men, implies that that judgment had been tacitly formed beforehand; irreconcilable as separating the minor premise and the conclusion, which ever present themselves to the mind in relation."

(Ib. p. 99.)

1105. "The proposition, 'I have a pain,' may be called in contrast with most propositions a simple one; though even it involves the unexpressed propositions that I have a body, and this body has a part in which this pain is localized, and that I have before had pains with which I class this as like in general nature. Strictly speaking, no such thing exists as an absolutely simple proposition, implying nothing beyond one subject and one predicate known in relation. Nevertheless, though the simplest proposition connotes sundry other propositions, there is a broad line to be drawn between it and the great mass of propositions, which severally make multitudes of predictions beyond that which they appear to make."

(Ib. pp. 395-6.)

1106. "Clearly, then, that we can compare conclusions with scientific rigour, we must not only resolve arguments into their constituent propositions, but must resolve each complex proposition into the simple propositions composing it. And only when each of these simple propositions has been separately tested, can the complex proposition, made up of them, be regarded as having approximately, a validity equal with that of a simple proposition which has been tested."

(Ib. p. 399.)

1107. "Still, there rises the question, how are we to choose between opposing conclusions, each of which claims to be legitimately drawn from premises alleged to be beyond doubt? Arguments of all kinds, including those of metaphysicians, which we have here to value, proceed upon the tacit assumption that each datum and each successive step has that indubitable warrant, the nature of which we have been examining."

(Ib. pp. 428-29.)

1108. "Two reasons may be distinguished for insisting on this testing process. One is, that in proportion as propositions are compound, direct comparisons of them must be hazardous; because their component propositions, each of which is an inlet to possible error, cannot be severally tested and verified. The other is, that only when compound propositions are resolved into their constituents, can it be seen what are the

relative numbers of assumptions in the two, and what are the relative possibilities of error hence resulting."

(Ib. p. 429.)

1109. "The general notions of agreement and disagreement, apply equally to two lines compared in their lengths and to two accounts of an event; and hence, in the absence of experiences that yield this general notion, accuracy of thought and precision of statement are not possible."

(Ib. p. 530.)

1110. "To say—'This is an animal,' or 'This is a circle,' or 'This is the color red,' necessarily implies that animals, circles and colors have been previously presented to consciousness. And the assertion that this is an animal, a circle, or a color, is a grouping of the new object perceived, with the similar objects remembered. In like manner, the inferences—'That berry is poisonous,' 'This solution will crystallize,' are impossible, even as conceptions, unless a knowledge of the relations between poison and death, between solution and crystallization, have been previously put into the mind, either immediately by experience or mediately by description."

(Ib. pp. 114-115.)

1111. "Thus, the belief in an unchanging order, the belief in law, now spreading among the more cultivated throughout the civilized world, is a belief of which the primitive man is absolutely incapable. Not simply does he lack the experiences that give materials for the conception, but he lacks the power of framing the conception; he is unable to think of a single law much less of law in general. The needful representativeness of thought is to be acquired only by the inheritance of accumulated increments of faculty successively organized, and it is even now possessed in a high degree only by a very small minority."

(Ib. p. 529.)

1112. From a fine article on Logic in the *Encyclopaedia Britannica*, vol. 14, written by Prof. Adamson, I make the following extracts:

"Even more radical is the divergence of modern logic from the Aristotelian ideal and method. The thinker who claimed for logic a special preeminence among sciences, because "since Aristotle it has not had to retrace a single step, * * * and to the present day has not been able to make one step in advance," has himself, in his general modification of all philosophy, placed logic on so new a basis that the only point of connection retained by it in his system, with the Aristotelian, may not be unfairly described as the community of subject."

1113. "If, adopting a simpler method, one were to inspect a fair proportion of the more extensive recent works on Logic,

the conclusion drawn would probably be the same,—that, while the matters treated show a slight similarity, no more than would naturally result from the fact that thought is the subject analyzed, the diversity in mode of treatment is so great that it would be impossible to select by comparison and criticism a certain body of theorems and methods, and assign to them the title of logic. That such works as those of Tredelenburg, Ueberweg, Ulrici, Lotze, Sigwart, Wundt, Bergmann, Schuppe, DeMorgan, Boole, Jevons, and these are but a selection from the most recent, treat of notions, judgments and methods of reasoning, gives to them, indeed, a certain common character, but what other features do they possess in common? In tone, in method, in aim, in fundamental principles, in extent of field, they diverge so widely as to appear, not so many different expositions of the same science, but of so many different sciences. In short, looking to the chaotic state of logical text books at the present time, one would be inclined to say that there does not exist anywhere a recognized currently received body of speculations to which the title of logic can be unambiguously assigned, and that we must therefore resign the hope of attaining by any empirical consideration of the received doctrine, a precise determination of the nature and limits of logical theory.”

(Enc. Br. p. 783.)

1114. “Each perception is itself and is only itself; no judgment is possible save that of identity. In other words, if there be judgment at all, it can consist only in the assertion that the unanalyzed perception is identical with that into which it is analyzed, and as each perception and each analytic portion of a perception may be signified by an arbitrary sign (name or other hieroglyphic,) judgment is essentially an affair of naming, a declaration that different names are identical or belong to the same perception.

Reasoning is simply the transition from identity to another—a more developed result of analysis. Scientific or real knowledge, is an accurately framed system of signs, i. e., a collection of signs which expresses precisely the results of the analysis of complex perceptions. Logic, under this doctrine of knowledge, is merely a statement of the various modes in which analysis is carried out; of the ways in which names are applied, and of the forms in which names are combined. Such is the theory of logic presented by Condillac.”

(Ib. p. 794.)

1115. “One development from the Psychology of Locke has thus appeared as an extreme formalism, which, if carried out consistently, must needs assume the aspect of a numerical or mechanical system of computation. It is remarkable that

a very similar result was reached by Leibnitz, a thinker who proceeded from a quite opposed psychological conception.

The characteristics of *Scientia Generalis* are at once deducible from the two general principles, which in Leibnitz's view, dominate all our thinking,—the law of sufficient reason and the law of non-contradiction. It must contain a complete account of the modes in which, from data, conclusions are drawn, and in which, from given facts, data are inferred, and since the only logical relations are those of identity and non-contradiction, the forms of inference from or to data, must be the general modes of combinations of simple elementary facts which are possible under the law of non-contradiction.

The statement of the data of any logical problem, and the description of the processes involved in combining them or arriving at them, are much assisted by, if not dependent on, the employment of a general characteristic or symbolic art.

The fundamental divisions of *Scientia Generalis*, so far at least as its groundwork are concerned, (for Leibnitz sometimes includes under one head all possible applications of the theory,) are, (1) the synthetical or combinatorical art, the theory of the processes by which from given facts complex results may be obtained (of these processes which make up general *mathesis*, syllogistic and mathematical demonstrations are special varieties); (2) the analytic or regressive art, which starting from a complex fact, endeavors to attain knowledge of the data from whose combination it arose.

Of the first art, the logical calculus in particular, a somewhat clearer and fuller outline is given. The logical calculus implicated,

- (1) the statement of data in their simplest form,
- (2) the assignment of the general laws under which combination of these data is possible,
- (3) the complete exposition of the forms of combination,
- (4) the employment of a definite set of symbols, both of data and modes of combination, subject to symbolic laws arising from the laws under which combination is possible."

(Ib. p. 794.)

1116. "In the *Fundamenta Calculi Ratiocinatoris* and the *Non-inclegans Speciman Demonstrandi*, something is effected toward filling up the first, second and fourth of these rubrics, but in no case is the treatment exhaustive. The simple data, called characters or formulæ, are symbolized by letters, relations of data by a somewhat complicated and varying system of algebraic signs; for the calculus, or set of operations exercised upon relations given, so as to produce a new formulæ, no comprehensive system of symbols is adopted.

Formulæ, relations and operations take the place of notions, judgments and syllogism. The general laws of combination of data are stated without much precision. Leibnitz recognizes the law of substitution, notes also what have been called the laws of reduplication and commutativeness, but, in actual realization of his method, employs indifferently, the relation of containing and contained, of the relation of identical substitution (*æquippollence*.) No attempt is made to develop a complete scheme of possible modes of combination.

At the root of Leibnitz's universal calculus, as of Condillac's method of analysis, and generally of nominalist logic, there lies a peculiar acceptance of the abstract law of identity. That a thing is what it is,—that knowledge of a thing is a single, indivisible, mechanical fact, susceptible only of explication or of expanded statement,—that is the principle dominating logical theories which in other respects may differ widely. Insistence upon the aspect of knowledge or of the object known is the ground for assigning to thought a function purely analytic, which is the very keynote of nominalism.”

(Ib. p. 794.)

1117. “Under all circumstances, difference is as important an element as identity, in the judgment, and to concentrate attention upon the identity, is to take a one-sided and imperfect view.

So soon, however, as the real nature of thought has been thrown out of account as not concerned in the processes of logic, so soon as the law of non-contradiction, in its manifold statement, has been formulated as the one principle of logical or formal thinking, there appears the possibility of evolving an exact system of the conditions of non-contradictoriness.”

(Ib. p. 800.)

1118. “The ultimate units of knowledge, whatsoever we call them, whether notions or ideas of classes of names, have at least one characteristic,—they are what they are, and therefore, exclude from themselves whatever is contradictory of their nature. They are combined positions and negations, that which is posited or negated being left undetermined,—referred, in fact, to matter as opposed to form. With respect to any article of thought, therefore, the only logical requirement is that it shall possess the characteristic of not being self-contradictory, and the only logical question is, what exactly is posited and negated thereby.”

(Ib. p. 800.)

1119. “Hobbes' doctrine of thought as dealing with names and as essentially addition and subtraction of nameable features, Boole's doctrine of thought, as the determination of a class, Jevon's view of thought, as simple apprehension of

qualities,—any of these will serve as starting point, for in all of them the fruitful element is the same.

The further step that the generalization of the system of thought must take a symbolic form, presents itself as an immediate and natural consequence."

(Ib. p. 800.)

1120. "The first question which suggests itself in connection with Boole's symbolic logic, is the necessity or advisability of retaining the reference to classes, or the description of thought as classification.

Do the symbolic laws really depend to any great extent on the logical peculiarities of class arrangement? Mr. Venn, who emphasizes this feature in Boole's scheme, has, however, done good service in leading up to a different explanation. The general reference to objects, which is also noted as implied in all Boole's formulas, has nothing to do with the possible difference of conceptualist or materialist doctrines of the proposition, and, in fact, as all distinctions of thing and quality, resemblance and difference, higher and lower, subject and predicate vanish or are absorbed in the more general principle underlying the symbolic method, phrases such as classification, extension, intention and the like, should be banished as not pertinent. Nay, the usual distinctions of quantity and even of quality, either disappear or acquire a new significance, when they are brought under the scope of the new principle. What symbolic logic works upon by preference, is a system of dichotomy, of x and not x , y and not y and so forth. In other words, quantitative differences require to find expression through some combination of the positions and negations of the elements making up the objects dealt with, while the usual quantitative distinctions are merged in the position or negation of various combinations."

(Ib. p. 801.)

1121. "There appears very clearly in Grassman's treatment the essence of the principle on which symbolic logic proceeds. Thought is viewed as simply the process of positing and negating definite contents or units, and the operations of logic become methods for rendering explicit that which is in each case posited or negated. To apply symbolic methods, we require units as definite as those of quantitative science, and the only laws we can employ are those which spring from the nature of units as definite."

(Ib. p. 801.)

1122. Augustus DeMorgan, b. 1806, d. 1871. From DeMorgan's work on "Formal Logic," I make the following extracts:

"Whether the premises be true or false, is not a question of logic, but of morals, philosophy, history, or any other knowl-

edge to which their subject-matter belongs: the question of logic is, Does the conclusion certainly follow if the premises be true?"

(Formal Logic pp. 2, 4.)

1123. "Every X is Y, affirms, Some X's are Y's, and denies No X is Y, Some X's are not Y's.

No X is Y affirms Some X's are not Y's and denies Every X is Y, Some X's are Y's.

Some X's are Y's does not contradict Every X is Y, Some X's are not Y's, but denies no X is Y.

Some X's are not Y's does not contradict No X is Y, Some X's are Y's, but denies Every X is Y."

(Ib. pp. 4, 5.)

1124. "Thus, the honest witness who said, 'I always thought him a respectable man—he kept his gig,' would probably not have admitted in direct terms, 'Every man who keeps a gig must be respectable.'"

(Ib. p. 20.)

1125. "Thus, the idea of a horse is the horse in the mind: and we know no other horse. We admit that there is an external object, a horse, which may give a horse in the mind to twenty different persons: but no one of these twenty knows the object; each one only knows his idea."

(Ib. p. 29.)

1126. "Connected with ideas are the names we give them; the spoken or written sounds by which we think of them, and communicate with others about them. To have an idea and to make it the subject of thought as an idea, are two perfectly distinct things: the idea of an idea is not the idea itself. I doubt whether we could have made thought itself the subject of thought without language. As it is, we give names to our ideas, meaning by a name not merely a single word, but any collection of words which conveys to one mind the idea in another. Thus, a-man-in-a-black-coat-riding-along-the-high-road-on-a-bay-horse, is as much the name of an idea as man, black, or horse. We can coin words at pleasure; and were it worth while, might invent a single word to stand for the preceding phrase."

(Ib. 34.)

1127. "Every name has a reference to every idea, either affirmative or negative. The term horse applies to every thing, either positively or negatively. This (no matter what I am speaking of) either is or is not a horse. If there be any doubt about it, either the idea is not precise, or the term horse is ill understood. A name ought to be like a boundary, which clearly and undeniably either shuts in or shuts out every idea

that can be suggested. It is the imperfection of our minds, our language, and our knowledge of external things, that this clear and undeniable inclusion or exclusion is seldom attainable, except as to ideas which are well within the boundary: at and near the boundary itself all is vague.

There are decided greens and decided blues, but between the two colors there are shades of which it must be unsettled by universal agreement to which of the two colors they belong. To the eye, green passes into blue by imperceptible gradations: our senses will suggest no place on which all agree, at which one is to end and the other to begin."

(Ib. p. 35.)

1128. "When a name is clearly understood, by which we mean when of every object of thought we can distinctly say, this name does or does not contain that object—we have said that the name applies to everything, in one way or the other. The word man has an application both to Alexander and Bucephalus: the first was a man, the second was not.

In the formation of language, a great many names are, as to their original signification, of a purely negative character: thus, parallels are only lines which do not meet. Aliens are men who are not Britons (that is, in our country)."

(Ib. p. 37.)

1129. "Let us take a pair of contrary names, as man and not-man. It is plain that between them they represent everything imaginable or real in the universe. But the contraries of common language usually embrace, not the whole universe, but some one general idea. Thus, of men, Briton and alien are contraries: every man must be one of the two, no man can be both. Not-Briton and alien are identical terms, and so are not-alien and Briton. The same may be said of integer and fraction among numbers, peer and commoner among subjects of the realm, male and female among animals, and so on. In order to express this, let us say that the whole idea under consideration is the universe, (meaning merely the whole of which we are considering parts), and let names which have nothing in common, but which between them contain the whole idea under consideration, be called contraries in, or with respect to, that universe. Thus, the universe being mankind, Briton and alien are contraries, as are soldier and civilian, male and female, etc.; the universe being animal, man and brute are contraries, etc."

(Ib. pp. 37, 38.)

1130. "One inflexion, or one additional word, may serve to signify a contrary of any kind: thus, not man is effective to denote all that is other than man."

(Ib. p. 39.)

1131. "Every negative proposition is affirmative and every affirmative is negative. Whatever completely does one of the two, include or exclude, also does the other. If I say that 'No A is B,' then b being the name of everything not B in the universe of the proposition, I say that 'Every A is b'; and if I say that 'Every A is B,' I say that 'No A is b.' Whether a language will happen to possess the name B or b, or both, depends on circumstances of which logical preference is never one, except in treatises of science. The English may possess a term for B, the French only for b: so that the same idea must be presented in an affirmative form to an Englishman, as in 'Every A is B,' and in a negative one to a Frenchman, as 'No A is b.'

From all this it follows that it is an accident of language whether a proposition is universal or particular, positive or negative.

We having the names A and B, may be able to say, 'Every A is B': another language which only names the contrary of B must say, 'No A is b.' A third language in which A's have not a separate name, but are only individuals of the class C, must say, 'Some C's are B's; while a fourth, which is in the further predicament of naming only b, must have it, 'Some C's are not b's.'

(Ib. p. 40.)

1132. "In all assertions, however, it is to be noted, once for all, that formal logic, the object of this treatise, deals with names and not with either ideas or things to which these names belong."

(Ib. p. 42.)

1133. "Thus A and B divested of all specific meaning, are really names as names, independently of things: or at least may be so considered. For the truth of the proposition, under all meanings, gives us a right to suppose, if we like, that names are the meanings, that is to say, that we may put it thus: 'When the name A is, the name B is: but the name B is not; therefore the name A is not.'

It is not, therefore, the object of logic to determine whether conclusions be true or false; but whether what are asserted to be conclusions are conclusions. By a conclusion is meant that which is and must be shut in with certain other preceding things put in first. It is that which must have been put into a sentence, because certain other things were put in. To infer a conclusion is to bring in, as it were, the direct statement of that which has been virtually stated already—has been shut in.

When we say 'A is B, B is C,' we conclude 'A is C,' it would be more correct to say, 'A is B, B is C,' we have concluded 'A is C.'

(Ib. p. 43.)

1134. "Inference does not give us more than there was before: but it may make us see more than we saw before: ideally speaking, then, it does give us (in the mind) more than there was before. But the homely truth that no more can come out than was in, though accepted as to all material objects, even by metaphysicians—who are generally well pleased to find the key of a box which contains what they want, though sure that it will put in no more than was there already, has been applied to logic, and even to mathematics, in depreciation of their rank as branches of knowledge. Those who have made this strangest of human errors must have assumed an ideal omniscience and looked at human imperfection objectively. Omniscience need neither compare ideas nor draw inferences: the conclusion which we deduce from premises is always present with them; truths are concomitants, not consequences. When we say that one assertion follows from another, we speak purely ideally, and describe an imperfection of our own minds: it is not that the consequence follows from the premises, but that our perception of the consequence follows our perception of the premises: the consequence, objectively speaking is in and with, and of, the premises. We speak wrongly if we speak ideally, when we say that 'A is C,' is in 'A is B and B is C': in fact, it is only by giving an objective view to the argument that we can even assert that it will be seen. To uncultivated minds, this simple conclusion is never concomitant with the premises, and only with some difficulty a consequence. From the certainty that a consequence may be made to come out, which is an allegorical use of the word out, we assume a right to declare by the same sort of allegory that it was in. The premises, therefore, contain the conclusion: and hence, some have spoken, as if in studying how to draw the conclusion, we were studying to know what we knew before. All the propositions of pure geometry, which multiply so fast that it is only a small and isolated class, even among mathematicians, who know all that has been done in that science, are certainly contained in, that is necessarily deducible from, a very few simple notions. But to be known from these premises is very different from being known with them."

(Ib. pp. 44, 45.)

1135. "The study of logic, then, considered relatively to human knowledge, stands in as low a place as that of the humble rules of arithmetic, with reference to the vast extent of mathematics and their physical applications. Neither is the less important for its lowness: but it is not everyone who can see that. Writers on the subject frequently take a scope which entitles them to claim for logic one of the highest places:

they do not confine themselves to the connection of premises and conclusion, but enter upon the *periculum et commodum* of the formation of the premises themselves. In the hands of Mr. Mill, for example, (and to some extent in those of Dr. Whately), logic is the science of distinguishing truth from falsehood, so as both to judge the premises and draw the conclusion, to compare name with name, not only as to identity or difference, but in all the varied associations of thought which arise out of this comparison."

(Ib. p. 46.)

1136. "But *is* in the sense 'is equal to,' does satisfy all the conditions. This sense of *is*, namely, agreement in magnitude, is the copula of the mathematician's syllogism when he is reasoning on quantity only."

(Ib. p. 52.)

1137. "There are common uses of the word which are not admitted in logic: and, among them, one of the most common, connection of an object with its quality and of an idea with one of its constituent or associated ideas.

As when we say, 'The rose is red,' Prudence is desirable,' here the logical conditions are not satisfied. For example, 'Red is the rose,' though a poetical inversion of the first assertion, is not logically true. It is usual to consider such propositions in logic as elliptical; thus, 'The rose is red,' is considered as 'The rose is a red object, or an object of red color,' in which the *is* now takes one of the senses which allows of conversion."

(Ib. p. 52.)

1138. "The *is* of agreement in particulars may always be reduced to the *is* of identity by alteration of the predicate; thus, 'Every A is B in color,' is 'Every A is a thing having the color of one of the B's.'

(Ib. p. 53.)

1139. "Thus, when we say, 'All animals require air,' or that the name *requiring air* belongs to everything to which the name animal belongs, we should understand that we are speaking of things on this earth: the planets, etc., of which we know nothing, not being included."

(Ib. p. 55.)

1140. "Contrary names, with reference to any one universe, are those which cannot both apply at once, but one or the other of which always applies. Thus the universe being *man*, *Briton*, and *alien* are contraries; the universe being property, real and personal are contraries. Names which are contraries in one universe are not necessarily so in a larger one. Thus, in geometry, when the universe is one plane, pairs of straight lines are either parallels or intersectors, and never both: parallels and

intersectors are then contraries. But when the student comes to solid geometry, in which *all space* is the universe, there are lines which are neither parallels nor intersectors; and these words are then not contraries. But names which are contraries in the larger and containing universe are necessarily contraries in the smaller and contained, unless the smaller universe absolutely exclude one name, and then the other name is *the universe*."

(Ib. p. 55.)

1141. "It has been proposed to consider the universal propositions as definite with respect to quantity; but this is not correct. The phrase 'All X's are Y's,' does not tell us how many X's there are, but that, be the unknown number of X's in existence what it may, the unknown number mentioned in the proposition is the same. That which is definite is the ratio of the number of X's of the proposition to the X's of the universe. So understood, however, the 'definite quantity,' as an abbreviation, may be said to belong to universals. And the indefiniteness of the particular proposition is only hypothetical. It is in our power to suppose the sum to be one-half of the whole, or two-thirds, or any other fraction.

The quantity of the subject is expressed; that of the predicate, though not expressed, is necessarily implied by the meaning of the language. The predicate of an affirmative is particular; the predicate of a negative is universal. If I say 'X's are Y's,' even though I speak of all the X's, I only really speak of so many Y's as are compared with X's and found to agree; and these need not be all the Y's. 'Every horse is an animal' declares that so many horses as there are to speak of, so many animals are spoken of: and leaves it wholly unsettled whether there be or be not more animals left. But if I should say 'X's are not Y's,' though it should be only one X, as in 'this X is not a Y,' yet I speak of every Y which exists. The assertion is, 'this X is not any one whatsoever of all the Y's in existence.'

(Ib. p. 57.)

1142. "'Some' usually means a rather small fraction of the whole; a larger fraction would be expressed by 'a good many'; and somewhat more than half by 'most;' while a still larger proportion would be 'a great majority,' or 'nearly all.' A perfectly *definite particular*, as to quantity, would express how many X's are in existence, how many Y's, and how many of the X's are or are not Y's: as in '70 out of the 100 X's are among the 200 Y's.'

(Ib. p. 58.)

1143. "Again the word negative had better be viewed as not

so much presenting exclusion for its first idea, as inclusion in the contrary."

(Ib. p. 68.)

1144. "The other view which I here propose is really a different mode of looking at that just given. By the time we have made every name carry its contrary, as a matter of course, we become prepared to take the following view of the nature of a proposition.

A name by itself is a sound or a symbol: its relation to things (be they objects or ideas) is twofold. There may be *in rerum natura*, that to which the name applies, or there may not. I do not here speak of how many things there may be to which a name applies: it is not essential to know whether they be more or fewer, either absolutely or relatively. The introduction of contraries may be made the expulsion of quantity. With reference to application then, let a name be called possible or impossible, according as the thing to which it applies can be found or not. A name may be compounded of others; the compound name being that of everything to which all the components apply. Thus, wild animal is the name of all things to which both the names of wild and animal apply. To call this compound name impossible, is to say that there is not such a thing as a wild animal: to call it possible is to say that there is such a thing."

(Ib. pp. 105, 106.)

1145. "The affirmative proposition requires the existence of both terms."

(Ib. p. 111.)

1146. "Thus, P, Q, R, being certain names, if we wish to give a name to everything which is all three, we may join them thus, PQR."

(Ib. p. 115.)

1147. "The other exclusion may involve, on the same terms, an error of the same kind; or may equally be the expression of arbitrary will: but there is what is more reasonably matter of opinion about it.

Aristotle will have no contrary terms: not-man, he says, is not the name of anything. He afterwards calls it an indefinite or aorist name, because, as he asserts, it is both the name of existing and non-existing things. If he had here made the distinction between ideal and objective, he would have seen that man and not-man equally belong to both (objectively) existing and non-existing things: man, for example, belongs as a name to Achilles and the seven champions of Christendom, whether they ever existed in objective reality or not: and not-man belongs, in either case, to their horses. I think, however,

that the exclusion was probably dictated by the want of a definite notion of the extent of the field of argument, which I have called the *universe* of the proposition. Adopt such a definite notion, and, as sufficiently shown, there is no more reason to attach the mere idea of negation to the contrary, than to the direct term."

(Ib. p. 128.)

1148. "But it can hardly be affirmed that any one admitting not-man as a name, should thereupon refuse to recognize the identity of 'horse is not man with horse is not-man.' The middle term is to be distributed in one or the other of the premises. By distributed is here meant universally spoken of. I do not use this term in the present work, because I do not see why, in any deducible meaning of the word distributed, it can be applied to universal as distinguished from particular. In using a name, it seems to me that we always distribute: that is, scatter as it were, the general name over the instances to which it is to apply. When I say, some horses are animals, I distribute certain horses among the animals; and when all, all."

(Ib. p. 137.)

1149. "When contraries are introduced, the distinction between positive and negative is made to appear what it really is, one of language, or rather one of choice of names. But the distinction of form is not abolished, but is exactly what it was before."

(Ib. p. 139.)

1150. "The distinction may be easily illustrated by example. 'All the planets but one,' is a particular proposition; it is 'some planets,' there is no one planet of right included in it, but 'all the planets except Neptune' is a universal proposition: 'a-planet-not-Neptune' is a name of Mercury, of Venus, etc.; and of every planet it can be stated whether it be in the name or not. That which is true inferentially of 'all the planets but one' left particular, is true of all the planets but Neptune; but that which is true of the latter, is not necessarily true of the former."

(Ib. p. 143.)

1151. "Thus, *man* and *rational animal* are not identical names, *qua* names, for they neither spell nor sound alike: the identity understood is that of meaning; where one applies, there shall the other apply also."

(Ib. pp. 146-147.)

1152. "All the theory of names, their application or non-application, may be applied to propositions, their truth or falsehood. To say that a proposition is true in a certain case, is to say that a certain name applies to a certain case; to say that it

is false, is to say that a certain name does not apply, but that its contrary does. That contrary is what logicians usually call contradictory, and the name is not simply true or false, but the adjective attached to the proposition. The conditions under which we are to speak, limit us to a number of cases which constitute what we may now call, not the universe of the names in the propositions, but the universe of the truth or falsehood of the propositions."

(Ib. p. 147.)

1153. "I am compelled to use the words contrary and contradictory as synonymous: at which compulsion I am well pleased, never having seen any good reason why, in the science which considers the relations of *dicta*, the *contraria* should be anything but the *contra dicta*."

(Ib. p. 148.)

1154. "And just as in a universe of names, every name introduced is supposed to belong or not to belong, to every instance in that universe: so in a universe of propositions, I suppose every proposition, or its contrary, to apply (whether it be or be not known which applies) in every instance."

(Ib. p. 149.)

1155. "The question of a premise being right or wrong, in fact or principle, unless, indeed, it contradicts itself, does not belong to logic: nor could it so belong unless logic were made, in the widest sense, that attempt at the attainment of the *nitio veri*, which some have defined it to be. All that relates to the collection of true premises with respect to the vegetable world, belongs to botany; with respect to the heavenly bodies, to astronomy; with respect to the relation of man to his creator, to theology."

(Ib. p. 239.)

1156. "As regards knowledge, there must likewise be a transition or change; and the act of knowing includes always two things. When we consider our mental states as knowledge, the same law holds. We know heat by a transition from cold; light, by passing out of the dark; up, by contrast to down. There is no such thing as an absolute knowledge of any one property; we could not know "motion," if we were debarred from knowing "rest." No one could understand the meaning of a straight line without being shown a line not straight, a bent or crooked line."

(Bain's Deductive and Inductive Logic, p. 3.)

1157. "Our knowledge of a fact is the discrimination of it from differing facts, and the agreement or identification of it with agreeing facts. The only other element in knowledge is

the retentive power of the mind, or memory, which is implied in these two powers."

(Ib. p. 4.)

1158. "The Contradictory Opposition of terms is when they differ only in respectively wanting and having the particle "not," or its equivalent. One or other of such terms is applicable to every object."

(Krause's Vocabulary of Philosophy, p. 513.)

1159. 'Elimination (*elimino*, to throw out), in Mathematics, is the process of causing a function to disappear from an equation, the solution of which would be embarrassed by its presence there. In other writings the correct signification is, 'the extrusion of that which is superfluous or irrelevant.'

(Ib. p. 155.)

1160. "As all nature is bound together by certain common qualities and relations, human knowledge will be found to consist chiefly, if not wholly, in comparing resemblances, or contrasting differences: and this is done with little trouble, for we perceive intuitively the agreement or disagreement of the objects presented to us."

(Brewster's Encyclopedia Art. Logic, p. 125.)

1161. "It may confidently be asserted that there is no department of human speculation and inquiry in which so many contradictory opinions are entertained as in the science or art of logic. For the last five-and-twenty centuries, system has followed system in rapid succession; and one generation of logicians after another have been chiefly occupied in refuting or modifying the principles and correcting the misstatements of their predecessors. No sooner has a particular logical system obtained a footing in some locality in the republic of letters and become incorporated with the general routine of philosophical education, than some aspiring and ambitious speculator has called in question its fundamental principles, or subjected its practical rules to supervision and amendment.

From Zeno to modern times, every theoretical logician has flattered himself in his day, that he had placed logic on a firm basis, not to be disturbed as long as the world lasted.

He has flattered himself with the idea that it was his fortunate lot to chase from the science every vestige of doubt, to reconcile every real and apparent contradiction and to make to all future generations the path of knowledge and science indisputably plain and of ready and agreeable access.

And the same spirit animates the philosophical logician of the present hour in every direction where his science is known and cultivated. Every speculator has a system of his own with which strangers do not intermeddle. He is the sole

champion of his own theory and the herald of his own fame. He, too, labors under the cheering anticipation that he is putting the finishing stroke to the science, and silencing forever, throughout the philosophical world, the voice of doubt and contention."

(*Blakey's Hist. Sketch of Logic*, p. 16.)

1162. "Though he may have all the learning of the East, and all the talent of Christendom centered in his own person, yet he knows full well that apart from his own professional chair or private study, he will not find a single cultivator of the same science entirely agreeing with him, either on the fundamental principles of logical philosophy, or on the best modes of applying them.

But this does not discourage him or ruffle the equable current of his self-complacency. He has the advantage over those who have gone before him, hoping unto death the same thing as himself; inasmuch as he reasons that if there ever is to be a time when the principles of his science are to be known and unalterably fixed, he may be the fortunate instrument in this grand and noble achievement. While there is life there is hope; and this consideration is sufficient to sustain him in his labors amidst the mass of disappointment that lies behind him."

(Ib. p 16.)

1163. "The speculative aspects under which logic has appeared in different ages and countries have not been more checkered and varied than its external fortunes. It has at one time revelled in unbounded authority and power, and, yet, at another, been doomed to the bitter humiliation of abject servitude and dependence. It has been the petted child of courts and monarchs, and yet been rivalled by the beggar in the street. It was once the art of arts, the science of sciences, and the proudest emblem in the escutcheon of the philosopher. The warrior ventured not to battle with it, nor could the lawyer on the bench, or the theologian in the pulpit acquit himself with grace, unless versed in its canons and rules. Notwithstanding however, all this power and grandeur, we have witnessed the science scouted from many influential universities, and, where admitted, it was only on the condition of becoming a humble menial and a willing slave."

(Ib. p. 17.)

1164. "In spite, however, of all such reverses logic has within it a vigorous principle of vitality. Like the Phœnix, it is continually rising from its own ashes. It never allows mankind to wander far nor long without pressing its claims and obtruding its counsels and admonitions upon them. It must, therefore, have a permanent hold on our sympathies,

some fixed root in our nature, or it would have been obliterated long ago from the book of knowledge. Astrology and alchemy never tantalized human reason so severely, for, what can present a greater anomaly to the understanding than that logic calling itself a science; having chairs in universities set apart for its special cultivation; witnessing its professors taking the first rank among the acute and profound of our race, and pointing with exulting pride to more than a thousand distinct treatises on the subject which have emanated from their pens within the last three hundred years; that logic, we say, should under these circumstances not be able to furnish two logicians of any country who can agree in any one common principle of this science, nor be able to state to what particular or general uses it can be applied, must present to the candid mind one of the most striking phenomena in the entire range of human thought. Can any subject in the whole circle of the sciences present such a lack of unanimity or a more cheerless and desponding aspect? The use of the word logic is almost the only thing which disputants have in common. If we venture a step beyond this and ask for a definition of what is implied in it, we are instantly stunned with a thousand discordant voices from all parts of the world."

(Ib. p. 18.)

1165. This work is not a book on psychology or metaphysics, and, although certain writers on logic have had considerable to say on those subjects, I have avoided them as far as possible. And yet the use of the Reasoning Frame has led me to adopt the views of Mr. Prince in his work on "The Nature of Mind," and several of Mr. Haig's views which are contained in his work on "Symbolism."

I make the following extracts from those works:

1166. "When men have once acquiesced in untrue opinions," remarks Hobbes, "and registered them as authenticated records in their minds, it is no less impossible to speak intelligibly to such persons than to write legibly on a piece of paper already scribbled over."

(Prince's *Nature of Mind*, p. 3.)

1167. "According to the Theory of Aspects, consciousness and nerve motions (vibrations,) are only different aspects of one and the same underlying substance which is unknown. This view has perhaps been as clearly expressed by Prof. Bain, as by anyone else, when he says, 'The one substance, with two sets of properties, two sides, (the physical and the mental,) a double-faced unity, would seem to comply with all the exigencies of the case.'"

(Ib. p. 14.)

1168. "The same notion has thus been described by Lewes: 'There may be every ground for concluding that a logical process has its correlative physical process and that the two processes are merely two aspects of one event.'"

(Ib. p. 14.)

1169. And again: "The two processes are equivalent and the difference arises from the difference in the mode of apprehension."

(Ib. p. 14.)

1170. "Thus, Mr. Spencer, who, as a psychologist, has treated the matter in a masterly manner, maintains this view of different aspects. 'For what,' he says, 'is objectively a change in a superior nerve-center, is subjectively a feeling, and the duration under the one aspect measures the duration of it under the other.' And the same thing is repeated in other passages. But this is no explanation, as Mr. Spencer, himself, tacitly recognizes when he later adds, 'though accumulated observations and experiments have led us by a very indirect series of inferences to the belief that mind and nervous action are the subjective and objective faces of the same thing, we remain utterly incapable of seeing and even of imagining how the two are related.'

(Ib. p. 17.)

1171. "We can have no consciousness without a material substance, the brain, nor without the activity of the brain."

(Ib. p. 45.)

1172. "Now, that matter, of which consciousness is the reality, must be subject to the laws which govern matter. One of these laws is the law of inertia. According to this, matter cannot of itself change its own state. Matter at rest, must forever remain at rest, unless something outside of itself disturbs it and puts it in motion. Matter in motion must forever persist in motion till something outside of it checks it. Matter exhibited under one property, must forever be exhibited under that property, unless some external force causes it to be exhibited under another. Whatever be the state of matter at a given moment, it must always remain in that state until outside agencies effect a change. This is a universal law, it has no exception. To this law then, the 'matter of the mind' must be subject.

Let us apply it and see what it means. It means this: that no change of any kind, chemical or physical, can occur in the protoplasm of the brain, without the interference of outside agencies; that no vibration or pulsation can occur among the protoplasmic molecules of any cell, unless some cause external

to that cell acts upon them; that for the undulations of the molecules—of which consciousness is the reality—to occur, some external force is requisite to start them into activity; in other words, for consciousness to be present, it is necessary that each cell should be stimulated by something external to that cell. The activity of the molecules of no cell can appear spontaneously, and, hence, neither can the reality of that activity, or consciousness. Consciousness, then, is passive, not active; it is conditioned existence, not unconditioned; it is a link in a series of events."

(Ib. pp. 94-95.)

1173. "Such is the inevitable result to which our reasoning leads us. If consciousness depends on matter being disturbed, it must be passive. This is a logical consequence of our premises, from which there is no escape. But if our thoughts are passive,—if they are merely the molecular disturbances in themselves, and cannot arise spontaneously,—it must be that the stimulus required for their production, cannot be applied in any definite manner at haphazard, but only through the anatomical mechanism of the brain,—only through the nerve conductors developed for the purpose. The channels by which stimuli from without reach the cells of the brain, are the centripetal nerves; and any succession of ideas can only occur by reason of the neural 'currents,' wherever originated, being reflected from one cell to another, along the anatomical connections which join the cells; and any objective expression of an idea can only take place by reason of the current passing again from the brain to the organs of expression, which are the muscles. In other words, under normal conditions every muscular action, every idea, sensation or emotion, requires for its production some stimulus originating outside of its own nervous center,—that is, it is reflex."

(Ib. pp. 95-96.)

1174. "To this reflex view there are logical consequences from which I see no escape. From a theory that a mental process is the reality of the reflex physiological process to the doctrine of automatism, is a step which we are compelled by the force of logical necessity to take, or rather, the two doctrines are essentially the same. For any doctrine which removes our thoughts from the control of a hypothetical agent, which is independent of external influences, and confines them to certain channels in which they are propelled, directly or indirectly, by stimuli (external or internal,) is practically automatism.

Under the reflex view, spontaneity, in the sense that any idea or state of mind can arise, except as the resultant of some other

idea by which it is conditioned, is impossible. Reflex is, consequently, equivalent to automatic."

(Ib. p. 100.)

1175. "When it is said that mental processes are automatic, I do not conceive that it is necessarily meant that we are identical with or like machines in every particular.

For instance, human beings grow and generate other human beings, functions not possessed by machines. When it is said that we are automata, or, that our mental processes are automatic, I understand that all that is meant is that our thoughts, sensations, volitions and actions, follow in certain grooves or channels which have their analogies and equivalents in the anatomical mechanism of the brain, and that the presence of every state of mind is conditioned by the anatomical structure and physiological working of the brain. Automatism is then synonymous with reflex action."

(Ib. pp. 105-106.)

1176. "There is one thing which must not be overlooked, and this is, that whatever powers of self-determination we may have, every action is determined by the strongest motive. However we may act, we cannot act contrary to the strongest motive; for the moment we conclude to act in opposition to what was the strongest motive, the new motive, whatever it be, if it be only the desire to show that we have the power to do so, becomes the strongest motive, overwhelming the preceding, and determining action. Whatever motive determines action is the strongest,—else it would not so determine us,—and we are compelled to act according to it."

(Ib. p. 141.)

1177. "The gentlest and most practical of all the Apostles of Christ, left this deep truth:

"If any man offend not in word, the same is a perfect man, and able to bridle the whole body;" and his Master, himself, said, "For every idle word that men do speak they shall give account in the day of judgment; for by thy words thou shalt be condemned."

(Haig's Symbolism, p. 11.)

1178. "Thinking is internal reasoning, or, reasoning to ourselves. Reasoning is external thinking, or thinking expressed in signs, words, symbols, intelligible to others. The one is private and peculiar to the individual man; the other is the same thing when made common to all mankind possessed of language and sufficient intelligence to comprehend it."

(Ib. p. 1.)

1179. "But these conventional words, which are called the names of the external things, are themselves the only things about which we can reason or hold any discussion, or have

any question in truth; because they are the only things or objects which men have, or can have, in common. Words are the only common objects which men possess jointly, or can compare together, in any possible question or discussion that can be raised between man and man. Man's mind must start with a symbol or symbols."

(Ib. p. 3.)

1180. "Our reasoning must begin and end with words—our reason has no other mutual instrument and no other mutual object; and though the faith of every man is quite fixed, and as certain to himself as the rock on which it stands, that language is not all that exists in the universe, yet it is all that exists in human cognition—it is all that men can compare together in every question and every discussion, and it forms every possible conclusion at which men can jointly arrive by their most earnest and careful thinking and reasoning."

(Ib. p. 4.)

1181. "Therefore, in the first place, it is clear that the reasoner always assumes the existence of the thing about which he proposes to reason. In the second place, it is equally certain that the reasoner cannot propose any question concerning the thing to be discussed, without he first assumes the possibility of such question. Thus, certain Existences and Possibilities—or, to use their logical names, Categories and Predicables—are unavoidably necessary, and must always be assumed in every discussion, before men can possibly think or reason together in any way whatever."

(Ib. pp. 4-5.)

1182. "Ideas are mental to the individual, but must be verbal to more than one man, or to mankind—all ideas are WORDS IN THE MIND."

(Ib. p. 77.)

1183. "But what the mind holds is not the outer world, but the minute undulations, motions and forms which reach the brain; but which motions exist only in the nerves of each man's own body caused by the outer world."

(Ib. p. 107.)

1184. "Logic is merely the laws by which we properly substitute one verbal expression for another, as its equivalent, in ordinary language."

(Ib. p. 148.)

1185. "How and by what means Man is to distinguish true symbols—true words from false words—thus becomes, to our human intellect, the question of questions; the end and object of all purely intellectual thinking and reasoning—the great riddle of philosophy—the science of Truth."

(Ib. p. 160.)

1186. "Thus we think of things and speak of thoughts, but can only reason to others about words."

(Ib. p. 163.)

1187. "Men can, therefore, neither speak nor reason of either things or thoughts themselves, but only of general words; not because words are everything in nature, but because they are everything in general human cognition and human reasoning."

(Ib. p. 165.)

1188. "Words, then, are something; they are signs of thoughts: thoughts are something; they are signs of things; and as we can know nothing of other men's thoughts of things, we can only discuss other men's words for their thoughts of things, and we can discover and invent new thoughts of things and express them in some new, clear, orderly and intelligible words."

(Ib. p. 186.)

1189. "But how are we to distinguish true axioms from the false ones, since false axioms have been assumed as self-evident? I know no answer to this except by comparing and reflecting upon the logical deductions and relations which we can make from them."

(Ib. p. 216.)

1190. "Logical inference is always hypothetical in form, and depends on the premises being true; but the inference itself, is absolute truth; the connection and mental deduction are absolute and necessary, and the conscientious intellect must distinguish between the necessary and the possible."

(Ib. p. 217.)

1191. "But there is, in fact, and can be, no such real existence as a general thing, or general idea apart from the word—the general term accepted by mankind in general."

(Ib. p. 226.)

1192. "In my opinion, the phrenologists have given far the best analysis of the human mind, and its natural powers and tastes and capacities and sentiments."

(Ib. p. 391.)

1193. "Of course, for philosophers, or men of science, to argue and reason with one another without defining their words, is about as reasonable and useful as might be the mutual confabulations of two men who do not understand each other's language, and are blind to each other's signs."

(Ib. p. 398.)

1194. "However, it is a most idle work to reason with indefinite words. That ought to be contemptible to the man of intellect."

(Ib. p. 410.)

1195. "When men of science speak of certain curves and motions and forces and atoms, they are talking of words whose scientific meanings are fixed; and they can crucify a falsity with logic. But metaphysics and politics and sociology are not yet scientific, and therefore, the spirit of falsehood can in such pseudo-sciences well contend with the spirit of truth, by means of false or ambiguous words."

(Ib. p. 437.)

1196. "And when we examine the likenesses or resemblances of the words of all languages to each other, we ultimately find that we can reduce them all to the two classes of nouns and verbs, for things and actions."

(Ib. p. 438.)

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